Channel-Aware Random Access Control for Distributed Estimation in Sensor Networks

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***Abstract*—A cross-layered slotted ALOHA protocol is proposed and analyzed for distributed estimation in sensor networks. Suppose that the sensors in the network record local measurements of a common event and report the data back to the fusion center through direct transmission links. We employ a channel-aware transmission control where the transmission probability of each sensor is chosen according to the quality of its local observation and transmission channels. As opposed to maximizing the system throughput, our goal is to design transmission control policies that optimize the estimation performance. Two transmission control strategies are proposed: the maximum mean-square-error (MSE) reduction (MMR) scheme and the suboptimal two-mode MSE-reduction (TMMR) scheme. The MMR maximizes the MSE-reduction of the estimate after each time slot. However, this method requires knowledge of the number of active sensors and the accumulated estimation performance in each time slot, which must be provided through feedback from the fusion center. In TMMR, the sensors switch between two predetermined transmission control functions without explicit knowledge of the estimation performance and the number of active sensors in each time slot. Moreover, we notice that, if new observations are made by the sensors in each time slot, diversity combining techniques can be employed to fully exploit the data that the sensors measure over their idle time slots. Specifically, we perform selective combining on the observations that are made in between transmissions. As a result, we are able to exploit both the spatial and temporal diversity gains inherent in the multi-sensor system.**

***Index Terms*—Cooperative communications, distributed estimation, diversity combining, medium access control, sensor networks, statistical inference.**

I. Introduction

Wireless sensor networks (WSN) [1], [2] typically consist of a large number of low-cost low-power devices that have the ability to sense, to compute and to communicate. The sensors are often deployed in large scale over wide areas or hostile environments making it prohibitive to perform human maintenance or battery replacement. The dense deployment of sensors also increases network congestion and reduces the peruser throughput in wireless networks [3]. These issues pose strict constraints on both energy and bandwidth utilization. Interestingly, in WSN, the sensors are often linked through a common application and work cooperatively towards a common goal. Therefore, one can exploit the cooperative nature of the sensors to improve the efficiency of resource utilization and the sensing performance [2]. In particular, we focus on the design of efficient random access protocols for distributed estimation in sensor networks.

Distributed estimation refers to the application where sensors record local measurements of a common event and report the data back to a fusion center where a global estimate of the event is computed. These problems are central to many sensor network applications [1], such as positioning [4], temperature control, and environmental monitoring [5], etc., and have been studied extensively in the past under various communication constraints, e.g., [6]–[10]. Most of these works focus on the design of local sensor quantization schemes and data fusion strategies while abstracting away the effects of medium access control (MAC). However, efficient MAC designs are crucial to achieving good estimation performance. In fact, as the number of sensors increases, channel congestion may reduce the amount of data that is delivered to the fusion center in a fixed amount of time and will eventually lead to large estimation errors.

The main contribution of this paper is to devise a cross-layered sensor network MAC protocol to efficiently retrieve data from the sensors and to rapidly improve the quality of the estimates. We propose a channel-aware slotted ALOHA protocol where the transmission probability of each sensor is assigned according to both the reliability of the local observation and the quality of the local transmission channel. In the past, channel-aware transmission control policies have been proposed for conventional slotted ALOHA systems in [11] and [12] to maximize the system throughput. However, in the distributed estimation problem, the sensors are transmitting information about a common event and the objective is to obtain an accurate estimate of the physical quantity of interest in the sensor field. Conventional MAC protocols [11], [12] that maximize system throughput may not necessarily lead to accurate estimates since the observations that are delivered to the fusion center may not be reliable.

In this paper, we adopt the distributed estimation model studied in [13] and [14] where each sensor transmits an amplified version of its analog measurements to the fusion center through a noisy fading channel. The model is similar to the amplify-and-forward (AF) cooperative transmission scheme described in [15]–[17]. Based on this model and given a fixed set of transmission probabilities, we first compute the expressions for the mean-square-error (MSE) distortion of the estimate. A channel-aware slotted ALOHA protocol is then derived to maximize the reduction of MSE after each transmission. This strategy is referred to as the *maximum MSE-reduction (MMR)* method. Two sensor systems are considered: 1) *the repeated*

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transmission (RT) system and 2) *the transmit once (TO) system*. In the RT system, the sensors are allowed to transmit repeatedly in each time slot as long as the observations and transmission channels are sufficiently reliable. In the TO system, each sensor is only allowed to transmit once regardless of whether or not the transmission was successful, i.e., whether or not a collision occurred. The RT system yields better estimation performance, but the TO system is more energy efficient.

In the MMR method, a channel-aware transmission control function is computed at the beginning of each time slot based on the instantaneous knowledge of the number of active sensors and the estimation performance (i.e., MSE) achieved up to that point. Yet, this real-time information may not be attainable in practice. Therefore, we propose a suboptimal *two-mode MSE-reduction (TMMR)* method to approximate the performance of MMR without explicit knowledge of the system parameters mentioned before. In the TMMR method, the sensors switch between two predetermined transmission control functions based only on the local estimates of the system parameters. It is interesting to note that the channel-aware transmission control policies are in the form of a thresholding function where a sensor transmits if and only if the effective local signal-to-noise ratio (SNR) (which is a function of the observation and transmission channel gains) reaches a certain threshold (see Section III). Both the MMR and TMMR methods outperform MAC protocols that do not exploit the advantages of cross-layered channel-awareness, e.g., conventional slotted ALOHA or TDMA. Certainly, channel-aware transmission control policies can also be derived under other estimation criteria, such as the maximum likelihood estimation or the Bayes estimation, etc. It is reasonable for the resultant transmission policy to also take on the form of a thresholding function but the optimal threshold may differ from the solutions obtained in this paper.

In the MMR and TMMR methods, we assume that each sensor transmits their most recent observation when it gains access to the channel. However, since the sensors are allowed to make independent measurements of the sensor field in each time slot, local processing can be performed to exploit the diversity of these independent observations while awaiting for transmission. Many diversity combining techniques [18], [19] can be employed in this case, such as selective combining, threshold combining, maximal-ratio combining or equal-gain combining. In this paper, we consider the selective combining as an example to illustrate the effectiveness of the proposed strategy. Here, each sensor is allowed to record the W most recent observations and chooses the most reliable observation to transmit when it has access to the channel. This method is referred to as the *Enhanced MMR Method with Selective Combining (EMMR-SC)*.

The remainder of this paper is organized as follows. In Section II, we describe the distributed estimation model that we consider and introduce the RT and TO transmission systems in detail. In Sections III and IV, we utilize the MMR and the TMMR methods to derive the proposed transmission control functions. The EMMR-SC method is then described in Section V. Numerical simulations and performance comparisons are given in Section VI. Finally, we conclude in Section VII.

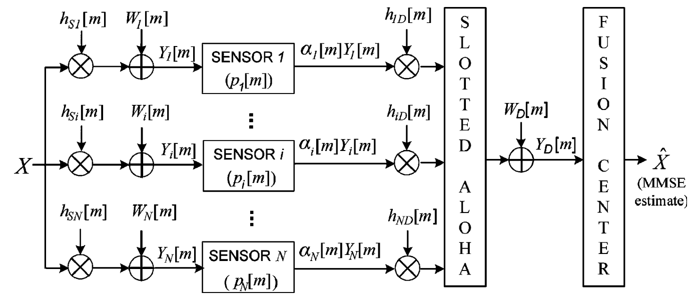

Fig. 1. System model.

II. System Model

Consider a wireless sensor network with N sensors, denoted by the set $\mathcal{S} = \{1, \ldots, N\}$, that are deployed to estimate a common parameter X, as shown in Fig. 1. Suppose that each sensor observes a local measurement of X and reports it to the fusion center through direct transmission links, similar to the model given in [13] and [14]. Instead of assuming the availability of centralized scheduling, we adopt a slotted ALOHA random access protocol where time is divided into time slots of equal length and the sensors transmit in each time slot with independent probabilities.

Suppose that X is a complex random variable with mean 0 and variance σ_X^2. The observation made by sensor i during the mth time slot is modeled as

$$Y_i[m] = h_{Si}[m]X + W_i[m] \tag{1}$$

where $h_{Si}[m]$ is the observation channel coefficient at sensor i that models the reliability of sensor observations and $W_i[m] \sim \mathcal{CN}(0, \sigma_W^2)$ is the additive white Gaussian noise (AWGN). $\{h_{Si}[m], \forall i, m\}$ are independent and identically distributed (i.i.d.) over time and across sensors. If sensor i transmits in the mth time slot, it will emit an amplified version of its local measurement to the fusion center, i.e., $\alpha_i[m]Y_i[m]$, where $\alpha_i[m]$ is the amplification gain. The amplify-and-forward scheme is similar to that considered in the literature on cooperative communications, e.g., [15]–[17], and is also considered for the distributed estimation problem in [13]. With the knowledge of $h_{Si}[m]$ at sensor i, the gain $\alpha_i[m]$ is given by

$$\alpha_i[m] = \sqrt{\frac{P}{E\left[|Y_i[m]|^2 | h_{Si}[m]\right]}} \tag{2}$$

which is chosen to satisfy the individual power constraints

$$E\left[\left|\alpha_i[m]Y_i[m]\right|^2 \middle| h_{Si}[m]\right] = P \tag{3}$$

for all i. Without loss of generality, we assume that $P = 1$. In this paper, we fix the average transmission power of each sensor and focus on deriving transmission control policies to improve the estimation performance. Although power control can also be considered to further improve energy efficiency, it will not be discussed in this paper.

Let us consider the *collision channel model* where the transmission from a sensor to the fusion center is successful only when no other sensors are transmitting in the same time slot. If more than one sensor is transmitting, the transmissions will collide and none of the messages will be received by the fusion center. On the other hand, if sensor i is the only sensor transmitting in the mth time slot, the signal arriving at the destination (i.e., the fusion center) will be

$$\begin{aligned} Y_D[m] &= \alpha_i[m]h_{iD}[m]Y_i[m] + W_D[m] \\ &= \alpha_i[m]h_{iD}[m]h_{Si}[m]X \\ &\quad + \alpha_i[m]h_{iD}[m]W_i[m] + W_D[m] \end{aligned} \tag{4}$$

where $h_{iD}[m]$ is the transmission channel coefficient of sensor i and $W_D[m] \sim \mathcal{CN}(0, \sigma_W^2)$ is the AWGN at the fusion center. We assume that $\{h_{iD}[m], \forall i, m\}$ are i.i.d. over time and across sensors as well. The SNR of the received signal $Y_D[m]$ is given by

$$\delta_i[m] \triangleq \frac{\alpha_i^2[m]|h_{Si}[m]h_{iD}[m]|^2\sigma_X^2}{(\alpha_i^2[m]|h_{iD}[m]|^2 + 1)\sigma_W^2}. \tag{5}$$

This is referred to as the *effective local SNR* of sensor i, which is the SNR of the signal received at the fusion center given that sensor i is the only one transmitting.

Suppose that $p_i[m]$ is the probability that sensor i transmits in the mth time slot. Under the collision channel model, the signal transmitted by a sensor is successfully received by the fusion center if and only if no other sensor is transmitting in the same time slot, i.e., there is no interference from other sensors. The probability that sensor i successfully transmits in the mth time slot is

$$P_i^{\text{suc}}[m] \triangleq p_i[m]\prod_{j\neq i}(1 - p_j[m]). \tag{6}$$

Assume that each sensor, say sensor i, has knowledge of only the local channel state information (CSI), i.e., $h_{Si}[m]$ and $h_{iD}[m]$, while the fusion center has knowledge of the CSI of all sensors,[1] i.e., $h_{Si}[m]$, $h_{iD}[m]$ for all i. With local CSI, the transmission probability of each sensor can be adjusted locally according to the realization of the channel coefficients in each time slot.

From (4), we define the normalized received signal as

$$Y'_D[m] = \frac{Y_D[m]}{\alpha_i[m]h_{Si}[m]h_{iD}[m]} = X + W'_i[m]$$

where $W'_i[m]$ is the normalized noise with variance

$$\sigma^2_{W'_i[m]} = \sigma_W^2 \frac{|\alpha_i[m]h_{iD}[m] + 1|^2}{|\alpha_i[m]h_{Si}[m]h_{iD}[m]|^2} = \frac{\sigma_X^2}{\delta_i[m]}. \qquad \text{(by (5))}$$

Suppose that, in the first M time slots, the fusion center successfully receives I packets from the sensors $k_1, k_2, \ldots, k_I$ at time instants $m_1, m_2, \ldots, m_I$, where $1 \leq m_1 < m_2 < \cdots < m_I \leq M$. For convenience, we define $\{\Delta[m]\}_{m=1}^M$ as the sequence of SNRs of the received signals, i.e.,

$$\Delta[m] = \begin{cases} \delta_{k_i}[m_i], & \text{for } m \in \{m_1, \ldots, m_I\} \\ 0, & \text{otherwise.} \end{cases}$$

[1]When X is a collaborative source, such as in positioning or tracking applications, the source may emit training symbols enabling the sensors to estimate the CSI. When X is a non-collaborative source, one could place reference points in the vicinity of the source allowing sensors to estimate the CSI by comparing measurements made on the references with that of the source.

Based on the messages successfully received in the first M time slots, the fusion center computes the linear minimum MSE (MMSE) estimate of X, which is given by

$$\hat{X} = \sigma_X^2 \mathbf{1}^T(\mathbf{1}\sigma_X^2\mathbf{1}^T + \mathbf{C}_{\mathbf{W}'})^{-1}\mathbf{Y}'_D$$

where $\mathbf{Y}'_D = [Y'_D[m_1],\ Y'_D[m_2], \ldots,\ Y'_D[m_I]]^T$ is the vector of signals received from the sensors $k_1, k_2, \ldots, k_I$ in the successful time slots, $\mathbf{1} = [1,\ 1,\ \ldots,\ 1]^T$ is the all-one vector, and $\mathbf{C}_{\mathbf{W}'} = \text{diag}\,(\sigma^2_{W'_{k_1}[m_1]}, \sigma^2_{W'_{k_2}[m_2]}, \ldots, \sigma^2_{W'_{k_I}[m_I]})$ is the covariance matrix of the normalized noise. The MSE of the estimate is given by

$$\begin{aligned} &\mathrm{E}\left[\left|X - \hat{X}\right|^2 \middle| \begin{array}{c} \text{sensors } k_1, \ldots, k_I \text{ succeed,} \\ \{h_{Si}[m], h_{iD}[m], \forall i, m\} \end{array}\right] \\ &= \sigma_X^2 - \sigma_X^2\mathbf{1}^T(\mathbf{1}\sigma_X^2\mathbf{1}^T + \mathbf{C}'_{\mathbf{W}})^{-1}\mathbf{1} \\ &\overset{(a)}{=} \sigma_X^2 - \sigma_X^4\mathbf{1}^T\left(\mathbf{C}'^{-1}_{\mathbf{W}} - \frac{\sigma_X^2\mathbf{C}'^{-1}_{\mathbf{W}}\mathbf{1}\mathbf{1}^T\mathbf{C}'^{-1}_{\mathbf{W}}}{1 + \sigma_X^2\mathbf{1}^T\mathbf{C}'^{-1}_{\mathbf{W}}\mathbf{1}}\right)\mathbf{1} \\ &= \frac{\sigma_X^2}{1 + \sum_{i=1}^{I}\delta_{k_i}[m_i]} = \frac{\sigma_X^2}{1 + \delta_T[M]} \end{aligned} \tag{7}$$

where (a) follows from the Woodbury's identity [20]. We define

$$\delta_T[M] \triangleq \sum_{m=1}^{M}\Delta[m] = \sum_{i=1}^{I}\delta_{k_i}[m_i] \tag{8}$$

as the total *accumulated SNR* after M time slots. Please note that the expectation in (7) is not taken over the channel coefficients since they are assumed to be known at the fusion center.

The proposed channel-aware transmission control protocol will be derived for two sensor systems: 1) the *RT System* and 2) the *TO System*. In the RT system, the sensors are allowed to transmit in each time slot regardless of whether or not they have already transmitted in the previous time slots. In the TO system, each sensor is only allowed to transmit once regardless of the success or failure of the transmission. The latter scheme is energy efficient while the former scheme achieves a lower MSE distortion.

Given the local CSI at each sensor, our goal is to derive channel-aware transmission probabilities that maximize the decrease in MSE after each time slot. It is worthwhile to mention that, in conventional slotted ALOHA systems where the CSI is not exploited to determine the transmission probabilities, the sensors transmit with a fixed probability $p_i[m] = 1/N$ in each time slot, regardless of the observation or transmission channel parameters. This probability is known to maximize the throughput of slotted ALOHA in a network of N nodes but may not result in the best estimation performance.

III. Maximum MSE-Reduction (MMR) Method

In this section, we describe the proposed MMR method where the transmission probabilities, e.g., $\{p_i[m], \forall i, m\}$, are derived to maximize the MSE reduction in each time slot. The method relies on the knowledge of the accumulated SNR before the current transmission takes place (i.e., $\delta_T[m-1]$) and the effective local SNR of the current time slot (i.e., $\delta_i[m]$). Therefore, $p_i[m]$ can be written as a function of $\delta_i[m]$ and $\delta_T[m-1]$, i.e.,

$$p_i[m] = g_{\rm MMR}(\delta_i[m], \delta_T[m-1]). \tag{9}$$

Since the sensors are identical, the MMR transmission control function $g_{\rm MMR}(\cdot,\cdot)$ is common for all sensors. To simplify the notations, we shall omit the time index m since the policy depends only on the actual values of δ_T and δ_i. Therefore, the transmission probability is expressed as $p_i = g_{\rm MMR}(\delta_i, \delta_T)$. The value of δ_i can be computed at sensor i, since we assume that local CSI is available, but the value of δ_T must be sent to the sensors from the fusion center through feedback at the beginning of each time slot. In the following, we derive the MMR method for both the RT and the TO systems.

A. MMR for the RT System

From (7), the MSE of the estimate after the current time slot will become $\sigma_X^2/(1+\delta_T+\delta_k)$ if sensor k succeeds in transmission and remains equal to $\sigma_X^2/(1+\delta_T)$ if no sensor succeeds. Therefore, given δ_T and $\{\delta_k, \forall k\}$, the average MSE can be written as

$$\begin{aligned}
&{\rm E}\left[\left|X-\hat{X}\right|^2 \middle| \delta_T, \delta_k, \forall k\right]\\
&=\sum_{k=1}^{N} P_k^{\rm suc}\frac{\sigma_X^2}{1+\delta_T+\delta_k}+\left(1-\sum_{k=1}^{N}P_k^{\rm suc}\right)\frac{\sigma_X^2}{1+\delta_T}\\
&=\sum_{k=1}^{N} p_k\prod_{j\neq k}(1-p_j)\frac{\sigma_X^2}{1+\delta_T+\delta_k}\\
&\quad+\left(1-\sum_{k=1}^{N}p_k\prod_{j\neq k}(1-p_j)\right)\frac{\sigma_X^2}{1+\delta_T}
\end{aligned} \tag{10}$$

where $P_k^{\rm suc}$ is the probability that sensor k transmits successfully, as given in (6). Since the sensors do not have knowledge of the other sensors' effective local SNR values, the transmission control function $g_{\rm MMR}(\cdot,\cdot)$ is derived by minimizing the average MSE conditioned on δ_T. Suppose we are given an arbitrary transmission control function $g(\cdot,\cdot)$. From (9), (10), and the fact that $\{\delta_k, \forall k\}$ are i.i.d., the average MSE achieved with $g(\cdot,\cdot)$ is given by

$$\begin{aligned}
&{\rm E}\left[\left|X-\hat{X}\right|^2\middle|\delta_T\right]={\rm E}\left[{\rm E}\left[\left|X-\hat{X}\right|^2\middle|\delta_T,\delta_k,\forall k\right]\middle|\delta_T\right]\\
&=\sum_{k=1}^{N}{\rm E}\left[g(\delta_k,\delta_T)\frac{\sigma_X^2}{1+\delta_T+\delta_k}\middle|\delta_T\right]\prod_{j\neq k}{\rm E}[1-g(\delta_j,\delta_T)|\,\delta_T]\\
&+\frac{\sigma_X^2}{1+\delta_T}-\sum_{k=1}^{N}{\rm E}\left[g(\delta_k,\delta_T)\frac{\sigma_X^2}{1+\delta_T}\middle|\delta_T\right]\prod_{j\neq k}{\rm E}[1-g(\delta_j,\delta_T)|\,\delta_T]\\
&=-N\gamma(1-\bar{p})^{N-1}+\frac{\sigma_X^2}{1+\delta_T}
\end{aligned} \tag{11}$$

where $\bar{p} = {\rm E}[g(\delta_k,\delta_T)|\delta_T]$ and $\gamma = {\rm E}\left[g(\delta_k,\delta_T)(\delta_k\sigma_X^2/(1+\delta_T+\delta_k)(1+\delta_T))\middle|\,\delta_T\right]$, for all k. We remove the user index from $\bar{p}$ and γ since $\{\delta_k,\forall k\}$ are i.i.d. random variables and, thus, the averages $\bar{p}$ and γ are identical for all k. It is worthwhile to notice that, since

$$\frac{\delta_k\sigma_X^2}{(1+\delta_T+\delta_k)(1+\delta_T)} = \frac{\sigma_X^2}{1+\delta_T} - \frac{\sigma_X^2}{1+\delta_T+\delta_k} \tag{12}$$

is equal to the MSE reduction when sensor k's message is successfully received at the fusion center, the parameter γ can be viewed as the average MSE-reduction given that this occurs.

Lemma 1: Suppose that δ_k is a continuous random variable with distribution function F_δ. For a fixed value of $\bar{p}$, the MSE in (11) is minimized if

$$g(\delta_k,\delta_T) = \begin{cases} 1, & \text{if } \delta_k \geq \beta \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where $\beta = F_\delta^{-1}(1-\bar{p})$ and $F_\delta^{-1}(y) = \min\{x : F_\delta(x) = y\}$.

Lemma 1 follows directly from the fact that (12) increases monotonically with δ_k and, thus, γ is maximized for fixed $\bar{p}$ if $g(\cdot,\cdot)$ takes on the form in (13). This shows that, to maximize the MSE-reduction after each time slot, the sensors should adopt a transmission control that takes on the form of a thresholding function. That is, a sensor transmits with probability 1 if the effective local SNR (e.g., δ_i) exceeds a certain threshold and remains silent otherwise. This result is intuitive and is consistent with the transmission control policies derived in [11], [12] for throughput maximization.

Given the optimal form of $g(\cdot,\cdot)$ for a fixed value of $\bar{p}$, as shown in Lemma 1, our search for $g_{\rm MMR}(\cdot,\cdot)$ is reduced to finding the average transmission probability $\bar{p}_{\rm MMR}$ or, in other words, the optimal threshold $\beta_{\rm MMR}$ that maximizes the average MSE-reduction. Notice that the value of β affects two parameters in (11), namely, γ and $\bar{p}$. In fact, the increase of β will cause γ to decrease while causing $(1-\bar{p})$ to increase. The minimum value of (11) is obtained by setting the derivative to 0, i.e.,

$$\begin{aligned}
&\frac{\partial}{\partial\beta}\left\{-N\gamma(1-\bar{p})^{N-1}\right\}\Big|_{\beta=\beta_{\rm MMR}}\\
&=\frac{\partial}{\partial\beta}\left\{\int_\beta^\infty\frac{-Ny\sigma_X^2 f_\delta(y)dy}{(1+\delta_T+y)(1+\delta_T)}\left(1-\int_\beta^\infty f_\delta(y)dy\right)^{N-1}\right\}\Big|_{\beta=\beta_{\rm MMR}}\\
&=0
\end{aligned}$$

where f_δ is the density function of δ_k. By Leibnitz's rule, it follows that

$$\beta_{\rm MMR} = \frac{\sigma_X^2(1+\delta_T)}{\sigma_X^2 - \frac{(1+\delta_T)(N-1)\gamma_{\rm MMR}}{(1-\bar{p}_{\rm MMR})}} - (1+\delta_T) \tag{14}$$

where, by (13)

$$\begin{aligned}
\bar{p}_{\rm MMR} &= {\rm E}[g_{\rm MMR}(\delta_k,\delta_T)|\delta_T]\\
&= \int_0^\infty g_{\rm MMR}(y,\delta_T)f_\delta(y)dy = 1-F_\delta(\beta_{\rm MMR})
\end{aligned} \tag{15}$$

and

$$\gamma_{\rm MMR} = \int_{\beta_{\rm MMR}}^\infty \frac{y\sigma_X^2}{(1+\delta_T+y)(1+\delta_T)}f_\delta(y)dy. \tag{16}$$

The value of $\beta_{\rm MMR}$ is found by solving the fixed point equation in (14) and is computed at the beginning of each time slot since it depends on the instantaneous knowledge of the accumulated SNR of the signals gathered up to that point, i.e., δ_T. Therefore, it is more accurate to express the optimal threshold with the time index, i.e., $\beta_{\rm MMR}[m]$. Some properties of $\beta_{\rm MMR}[m]$ are given in the following proposition. The proof can be found in Appendix A.

Proposition 1: $\beta_{\rm MMR}[m]$ is monotonically non-decreasing with m and is bounded as

$$\beta_{\rm L} \triangleq F_\delta^{-1}\left(1-\frac{1}{N}\right) < \beta_{\rm MMR}[m] < \beta_{\rm U} \triangleq N{\rm E}\left[\delta\cdot 1_{\{\delta > F_\delta^{-1}(1-1/N)\}}\right] \quad (17)$$

where $1_{\{\cdot\}}$ is the indicator function.

Since $\beta_{\rm MMR}[m]$ varies in each time slot, the average transmission probability given in (15), i.e., $\bar{p}_{\rm MMR}[m]$, is also a function of time and is bounded as follows.

Corollary 1:

$$\bar{p}_{\rm L} \triangleq 1-F_\delta(\beta_{\rm U}) < \bar{p}_{\rm MMR}[m] < \bar{p}_{\rm U} \triangleq 1-F_\delta(\beta_{\rm L}) = \frac{1}{N}. \quad (18)$$

The proof follows directly from (15) and the fact that F_δ is a monotonically non-decreasing function. In the following, we give an example of these bounds for the case where the effective local SNR values are exponentially distributed.

Example: (Exponentially Distributed Local SNRs): Let us consider the case where the transmission channel is noiseless and has a constant gain, e.g., $h_{iD}[m] = 1, \forall i$. Then, by assuming that the observation channel coefficients, $\{h_{Si}[m], \forall i\}$, are i.i.d. with distribution $\mathcal{CN}(0, \sigma_S^2)$, the effective local SNR values, $\{\delta_i[m], \forall i\}$, can be modeled as exponential random variables with mean $\theta = \sigma_S^2\sigma_X^2/\sigma_W^2$. Consequently, we have

$$\bar{p}_{\rm MMR} = 1 - F_\delta(\beta_{\rm MMR}) = e^{-\beta_{\rm MMR}/\theta} \quad (19)$$

and

$$\gamma_{\rm MMR} = \int_{\beta_{\rm MMR}}^{\infty} \frac{y\sigma_X^2}{(1+\delta_T+y)(1+\delta_T)}\frac{1}{\theta}e^{-y/\theta}dy. \quad (20)$$

By substituting (19) and (20) into (14), we can solve for the values of $\beta_{\rm MMR}$ numerically.

For $N = 20$ and $\theta = 5$, we plot the solutions of $\beta_{\rm MMR}[m]$ for different values of $\delta_T[m-1]$ in Fig. 2, along with its upper and lower bounds. The lower bound follows directly from Proposition 1 while a tighter upper bound is derived in closed-form as shown in the following proposition. The proof is given in Appendix B.

Proposition 2:

$$\beta_{\rm L} \triangleq \theta \ln N < \beta_{\rm MMR}[m] < \beta_{\rm U} \triangleq \theta \ln\left[1+\frac{1+\ln N}{\ln N}(N-1)\right]. \quad (21)$$

Similar to Corollary 1, the average transmission probabilities can also be bounded as follows.

Corollary 2:

$$\bar{p}_{\rm L} \triangleq \frac{1}{1+\frac{1+\ln N}{\ln N}(N-1)} < \bar{p}_{\rm MMR}[m] < \bar{p}_{\rm U} \triangleq \frac{1}{N}. \quad (22)$$

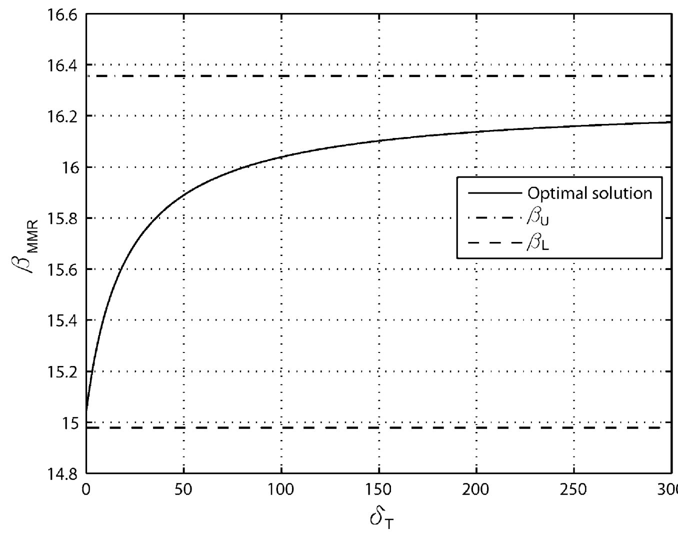

Fig. 2. Threshold values $\beta_{\rm MMR}$ versus δ_T, along with the upper bound $\beta_{\rm U}$ and lower bound $\beta_{\rm L}$, for $N = 20, \theta = 5$ and $\sigma_X^2 = \sigma_W^2 = 1$.

It is worthwhile to notice that the average transmission probability initially starts at a value close to the upper bound $1/N$ (which is the probability that maximizes the throughput in conventional slotted ALOHA systems) and decreases later on. This shows that, in the early stage of the process, it is desirable to successfully receive as many messages as possible. However, as δ_T increases, the messages transmitted by less reliable sensors (i.e., those that have unreliable observations or bad transmission channels) do not contribute much to the MSE-reduction while causing congestion to other sensors. Hence, the demand for higher throughput is overcome by the need for more reliable data and, thus, $\bar{p}_{\rm MMR}$ decreases. ■

B. MMR for the TO System

The MMR method can also be applied to the TO system, where each sensor is only allowed to transmit once regardless of the success or failure of the transmission attempt. In contrast to the RT system, the number of active sensors changes over time since each sensor in the TO system becomes inactive once it has transmitted. Therefore, in addition to $\delta_i[m]$ and $\delta_T[m-1]$, the transmission control threshold $\beta_{\rm MMR}[m]$ in the TO system must also depend on $N[m]$, which denotes the number of active sensors at the beginning of the mth time slot. The values of $N[m]$ and $\delta_T[m-1]$ are sent to the sensors from the fusion center as a control signal at the beginning of each time slot. Similarly, we can solve for the value of $\beta_{\rm MMR}[m]$ by substituting (15) and (16) into (14) with N replaced by $N[m]$. Notice that, when $N[m] = 1$, we have $\beta_{\rm MMR}[m] = 0$, in which case the sensor will transmit with probability 1.

As described before, the MMR method relies on the knowledge of $N[m]$ and $\delta_T[m-1]$ at the beginning of each time slot, which must be provided by the fusion center. However, the feedback from the fusion center may not always be available. Hence, we propose in the following section a suboptimal method that does not utilize the explicit knowledge of these system parameters.

IV. TMMR Method

In this section, we propose the suboptimal TMMR method where the transmission control at each sensor depends only on the local effective SNR. In other words, the transmission probability of sensor i in the mth time slot is given by $p_i[m] = g_{\rm TMMR}(\delta_i[m])$. Specifically, instead of computing the MMR transmission control threshold (i.e., $\beta_{\rm MMR}[m]$) at the beginning of each time slot, the sensors switch between two threshold values (namely, the upper and lower bounds of $\beta_{\rm MMR}[m]$) depending on the *estimated* values of $\delta_T[m-1]$ and $N[m]$. Since the estimated values of $\delta_T[m-1]$ and $N[m]$ do not depend on any real-time channel information or system state, the time for which the sensors switch from one threshold value to the other can be computed offline. The method is also derived for both RT and TO systems, respectively, and is detailed in the following.

A. TMMR for the RT System

From Proposition 1, we know that $\beta_{\rm MMR}[m]$ is monotonically non-decreasing with respect to $\delta_T[m-1]$ and m. In fact, as illustrated in Fig. 2, the value of $\beta_{\rm MMR}[m]$ starts initially at a value close to $\beta_{\rm L}$ and converges towards a value that is close to $\beta_{\rm U}$ as time progresses. In the TMMR method, we assume that the sensors do not have explicit knowledge of $\delta_T[m-1]$ and, thereby, are not able to compute the values of $\beta_{\rm MMR}[m]$ in each time slot. Instead, each sensor first assigns the transmission control threshold as $\beta_{\rm TMMR}[m] = \beta_{\rm L}$ and switches to $\beta_{\rm TMMR}[m] = \beta_{\rm U}$ after a certain number of time slots. That is, for a switching time M_s, we set $\beta_{\rm TMMR}[m] = \beta_{\rm L}$, for $m < M_s$, and set $\beta_{\rm TMMR}[m] = \beta_{\rm U}$, for $m \geq M_s$. The values of $\beta_{\rm L}$ and $\beta_{\rm U}$ do not depend on the time index m and can be obtained analytically [or numerically for a tighter upper bound (see Appendix A)] without knowing the value of $\delta_T[m-1]$. However, the problem remains as to when we should switch from one value to the other, i.e., the value of M_s.

Interestingly, by observing the relation between $\beta_{\rm MMR}[m]$ and $\delta_T[m-1]$ (see, e.g., Fig. 2), we can find a value $\delta_T^\star$ for which $\beta_{\rm MMR}[m]$ is closely approximated by $\beta_{\rm U}$ when $\delta_T[m-1] > \delta_T^\star$. In this case, the TMMR method should apply the switch from $\beta_{\rm L}$ to $\beta_{\rm U}$ when $\delta_T[m-1]$ exceeds $\delta_T^\star$. However, the value of $\delta_T[m-1]$ is not known explicitly in this case and, therefore, the average value is used instead. This average value can be computed as follows.

Notice that, before the switch occurs (i.e., for $m < M_s$), all sensors apply the threshold $\beta_{\rm TMMR}[m] = \beta_{\rm L} = F_\delta^{-1}(1-1/N)$. In this case, the transmission probability of sensor i is

$$p_i[m] = g_{\rm TMMR}(\delta_i[m]) = \begin{cases} 1, & \text{if } \delta_i[m] > F_\delta^{-1}\left(1-\frac{1}{N}\right) \\ 0, & \text{otherwise} \end{cases}$$

and the average transmission probability is $\bar{p}_{\rm TMMR}[m] = 1/N$. For $m < M_s$, the average accumulated SNR that the fusion center obtains over the first $m-1$ time slots is computed as

$$\begin{aligned}\hat{\delta}_T[m-1] &= {\rm E}\left[\sum_{\ell=1}^{m-1}\sum_{i=1}^{N} P_i^{\rm suc}[\ell]\delta_i[\ell]\right] \\ &= \sum_{\ell=1}^{m-1}\sum_{i=1}^{N} {\rm E}\left[p_i[\ell]\delta_i[\ell]\right]\prod_{j\neq i}{\rm E}\left[1-p_j[\ell]\right] \\ &= (m-1)\cdot N\cdot\int_{\beta_{\rm L}}^{\infty} y f_\delta(y)dy\cdot(1-\bar{p}_{\rm L})^{N-1} \qquad (23)\end{aligned}$$

where $P_i^{\rm suc}[\ell]$ [given by (6)] is the probability that sensor i transmits successfully in the ℓth time slot. With $\delta_T^\star$ as the switching threshold, the switch occurs when m reaches the value

$$M_s = \inf\left\{m : \hat{\delta}_T[m-1] > \delta_T^\star\right\}.$$

That is, we set $\beta_{\rm TMMR}[m] = \beta_{\rm U}$ when $m \geq M_s$. To summarize, in the TMMR method, the transmission control function is given by

$$g_{\rm TMMR}(\delta_i[m]) = \begin{cases} 1, & \text{if } \delta_i[m] > \beta_{\rm TMMR}[m] \\ 0, & \text{otherwise} \end{cases}$$

where

$$\beta_{\rm TMMR}[m] = \begin{cases} \beta_{\rm L}, & \text{if } m < M_s \\ \beta_{\rm U}, & \text{if } m \geq M_s. \end{cases}$$

It is worthwhile to notice that the switching time M_s can be computed offline without real-time information of $\delta_T[m-1]$. This reduces considerably the computational requirements at the sensors as compared with the MMR method.

B. TMMR for the TO System

The TMMR method can be applied to the TO system as well. Similarly, we start out by having all sensors transmit using the lower bound in (17) as the transmission threshold and switch to the upper bound after a certain number of time slots. However, it is important to note that the bounds given in Propositions 1 and 2 are derived for a fixed number of sensors N. Therefore, in the TO system, where the number of active sensors $N[m]$ varies in each time slot, the upper and lower bounds will also vary with m. Let $\beta_{\rm U}(N[m])$ and $\beta_{\rm L}(N[m])$ be the upper and lower bounds of $\beta[m]$ when there are $N[m]$ sensors active. That is, from (17), we have

$$\beta_{\rm L}(N[m]) \triangleq F_\delta^{-1}\left(1-\frac{1}{N[m]}\right)$$

and

$$\beta_{\rm U}(N[m]) \triangleq N[m]{\rm E}\left[\delta\cdot 1_{\{\delta > F_\delta^{-1}(1-1/N[m])\}}\right].$$

Following the procedures in the RT system and assuming that $N[m]$ is known in each time slot, the TMMR method should set $\beta_{\rm TMMR}[m] = \beta_{\rm L}(N[m])$, when $m < M_s$, and $\beta_{\rm TMMR}[m] = \beta_{\rm U}(N[m])$, when $m \geq M_s$. Unfortunately, without feedback from the fusion center, the actual value of $N[m]$ cannot be obtained by the sensors and, therefore, must be estimated locally at the beginning of each time slot in addition to computing $\hat{\delta}_T[m-1]$.

Suppose that $\hat{N}[m]$ is the estimated number of active sensors at the beginning of the mth time slot. Before the switch occurs, i.e., when $m < M_s$, each sensor will transmit using the threshold $\beta_{\rm L}(\hat{N}[m])$, which results in the average transmission probability $\bar{p}_{\rm TMMR}[m] = 1/\hat{N}[m]$. When $m \geq M_s$, the threshold then switches to $\beta_{\rm U}(\hat{N}[m])$, which is an upper bound of $\beta_{\rm MMR}[m]$ when we assume that $\hat{N}[m]$ sensors are active and the average transmission probability becomes $\bar{p}_{\rm MMR} = 1 - F_\delta(\beta_{\rm U}(\hat{N}[m]))$.

Initially, we assume that all sensors are informed of the initial number of active sensors N and let $\hat{N}[1] = N$. In the mth time slot, the estimated number of active sensors, i.e., $\hat{N}[m]$, is obtained by maximizing the probability that this number of sensors is still active. Specifically, before the switch occurs, each sensor will transmit using the threshold $\beta_{\rm L}(\hat{N}[m])$ and the probability that n sensors remain active at the beginning of the mth time slot is

$$P_{\text{active}}^{[m]}(n) = \binom{N}{n}\left[\prod_{\ell=1}^{m-1}(1-\bar{p}_{\text{TMMR}}[\ell])\right]^{n}\left[1-\prod_{\ell=1}^{m-1}(1-\bar{p}_{\text{TMMR}}[\ell])\right]^{N-n} \tag{24}$$

where the average transmission probability $\bar{p}_{\text{TMMR}}[\ell] = 1 - F_\delta\left(\beta_{\text{L}}(\hat{N}[\ell])\right)$. The estimated number of active sensors is then

$$\hat{N}[m] = \arg\max_{n} P_{\text{active}}^{[m]}(n). \tag{25}$$

The probability $P_{\text{active}}^{[m]}(n)$ (and, thus, $\hat{N}[m]$) corresponding to the mth time slot is computed recursively since the average transmission probabilities $\bar{p}_{\text{TMMR}}[1], \bar{p}_{\text{TMMR}}[2], \ldots, \bar{p}_{\text{TMMR}}[m-1]$ depend on the estimated values $\hat{N}[1], \hat{N}[2], \ldots, \hat{N}[m-1]$, respectively. Intuitively, the estimate in (25) is chosen to be the number of sensors that is most likely to remain active in the mth time slot. Similar to (23), the average SNR accumulated at the fusion center after $m-1$ time slots (and before the switch occurs) is equal to

$$\hat{\delta}_T[m-1] = \sum_{\ell=1}^{m-1}\hat{N}[\ell]\cdot\int_{\beta_{\text{I}}(\hat{N}[\ell])}^{\infty} y f_\delta(y)dy\cdot(1-\bar{p}_{\text{TMMR}}[\ell])^{\hat{N}[\ell]-1}.$$

The switching time M_s is then chosen as the time for which the average accumulated SNR exceeds $\delta_T^\star$, i.e.,

$$M_s = \inf\left\{m : \hat{\delta}_T[m-1] > \delta_T^\star\right\}.$$

Notice that M_s can also be computed offline in this case since it does not depend on the actual values of $N[m]$ or $\delta_T[m-1]$ in each time slot. After the switch occurs, i.e., for $m \geq M_s$, we set $\beta_{\text{TMMR}}[m] = \beta_{\text{U}}(\hat{N}[m])$, where $\hat{N}[m]$ is also obtained from (24) and (25) with

$$\bar{p}_{\text{TMMR}}[\ell] = \begin{cases} 1 - F_\delta\left(\beta_{\text{L}}(\hat{N}[\ell])\right), & \text{for } \ell < M_s \\ 1 - F_\delta\left(\beta_{\text{U}}(\hat{N}[\ell])\right), & \text{for } \ell \geq M_s. \end{cases}$$

The TMMR method proposed in this section does not utilize the explicit knowledge of $\delta_T[m-1]$ and $N[m]$ in each time slot, therefore, the performance slightly degrades compared to the MMR method. However, as we show later in Section VI, the decrease in performance is small and is often worth the tradeoff in order to avoid feedback and to reduce the computational complexity at the sensors.

V. Enhanced MMR With Selective Combining

In this section, we improve upon the MMR and TMMR methods discussed in previous sections by incorporating diversity combining techniques on the local observations at each sensor.

In the methods proposed previously, each sensor computes an effective local SNR at the beginning of each time slot (based on the knowledge of the local observation and transmission channel coefficients) and transmits only if the effective local SNR exceeds a certain threshold. However, even though the sensors make a new observation in each time slot, only the observation made at the time of transmission is sent to the fusion center while those made in the idle time slots are discarded. This is clearly inefficient since the discarded observations also contain information of the physical quantity of interest. To fully exploit diversity in the temporal domain, we propose the use of diversity combining techniques on the observations accumulated over a certain time window and send the combined value to the fusion center when the sensor gains access to the channel. Many diversity combining techniques [18] have been proposed in the literature, such as maximal ratio combining, selective combining, threshold combining and equal gain combining, all of which can be applied to the proposed system. In the following, we shall consider only the EMMR-SC as an example to illustrate the effectiveness of the proposed class of strategies.

A. EMMR-SC in the RT System

Suppose that each sensor maintains a buffer of size W to record the set of most recent observations (including the observation made in the current time slot). This is similar to the sliding window technique where the oldest observation is replaced by the newest observation when the buffer is full. With selective combining, each sensor transmits the observation with the best channel quality among the ones recorded in the buffer. To simplify the fusion process, we assume that the buffer is cleared out once a transmission occurs and the observations are accumulated again starting from the next time slot. This assumption is not necessary since one may simply drop the observation that was transmitted and leave the others in the buffer until newer observations arrive. However, this induces correlation among the signals received at the fusion center and complicates the fusion process.[2] We would also like to remark that, with selective combining, each sensor actually needs only a size-1 buffer to record the observation with the best quality, i.e., the new observation obtained in each time slot is compared with the observation recorded in the buffer and only the one with the best quality is stored, similar to the case of priority queues [21,Ch. 3]. Nevertheless, a window (or buffer) size greater than 1 is necessary for other combining techniques, such as maximal ratio combining or equal gain combining, and, therefore, will be used in the following discussions to maintain generality.

Since the buffer is cleared out after each transmission, the number of observations recorded in the buffer can be modeled as a random variable that takes on the integer values between 1 and W. Let $L_i[m] \in \{1, 2, \ldots, W\}$ be the number of observations recorded by sensor i up to, and including, time slot m. In the mth time slot, the recorded set of observations is denoted by

$$\mathcal{Y}_i[m] = \{Y_i[m-L_i[m]+1], Y_i[m-L_i[m]+2], \ldots, Y_i[m]\}.$$

If sensor i is to transmit in slot m, the observation with the best quality (i.e., the one with the largest observation channel gain) among the set $\mathcal{Y}_i[m]$ will be sent. The selected observation is denoted by $Y_i^{(L_i[m])}[m]$ and the corresponding observation channel coefficient is

$$h_{Si}^{(L_i[m])}[m] = h_{Si}[n], \text{ where } n = \arg\max_{k\in\{m-\text{L}_i[m]+1,\ldots,m\}} |h_{Si}[k]|.$$

[2]Note that (7) will no longer hold in this case.

If the maximum channel gain is achieved by more than one sensor, a sensor will be chosen randomly out of this set. However, this occurs with probability zero when the channel is assumed to be a continuous random variable. If $Y_i^{(L_i[m])}[m]$ is transmitted without collision, the fusion center receives the signal

$$Y_D[m] = \alpha_i^{(L_i[m])}[m] \cdot h_{iD}[m] \cdot Y_i^{(L_i[m])}[m] + W_D[m]$$

where

$$\alpha_i^{(L_i[m])}[m] = \sqrt{\frac{P}{\mathrm{E}\left[\left|Y_i^{(L_i[m])}[m]\right|^2 \middle| h_{Si}^{(L_i[m])}[m]\right]}}$$

is chosen to satisfy the power constraint in (3). Here, we also set $P = 1$ without loss of generality. Similar to (5), the SNR of the received signal is given by

$$\delta_i^{(L_i[m])}[m] = \frac{\left(\alpha_i^{(L_i[m])}[m]\right)^2 \cdot \left|h_{Si}^{(L_i[m])}[m] \cdot h_{iD}[m]\right|^2}{\left(\alpha_i^{(L_i[m])}[m] \cdot |h_{iD}[m]|\right)^2 + 1} \cdot \frac{\sigma_X^2}{\sigma_W^2}. \tag{26}$$

Following the same approach as in Section III, we can show that the transmission control function, which is now a function of $\delta_i^{(L_i[m])}[m]$ and $\delta_T[m-1]$, also takes on the form of a thresholding function, i.e., the transmission probability is given by

$$\begin{aligned} p_i[m] &= g_{\mathrm{EMMR}}\left(\delta_i^{(L_i[m])}[m], \delta_T[m-1]\right) \\ &= \begin{cases} 1, & \text{if } \delta_i^{(L_i[m])}[m] \geq \beta_{\mathrm{EMMR}}[m] \\ 0, & \text{if } \delta_i^{(L_i[m])}[m] < \beta_{\mathrm{EMMR}}[m]. \end{cases} \end{aligned} \tag{27}$$

To evaluate $\beta_{\mathrm{EMMR}}[m]$ using the fixed point equation in (14), we must first obtain the distribution function $F_{\delta[m]}$ and the density function $f_{\delta[m]}$ of $\delta_i^{(L_i[m])}[m]$. Notice that the sensor index is omitted from $F_{\delta[m]}$ and $f_{\delta[m]}$ but the time index is preserved since $\delta_i^{(L_i[m])}[m]$ is now time-dependent. The distribution function of $\delta_i^{(L_i[m])}[m]$ is computed as follows:

$$\begin{aligned} F_{\delta[m]}(y) &= \Pr\left(\delta_i^{(L_i[m])}[m] \leq y\right) \\ &= \sum_{\ell=1}^{W} \Pr\left(\delta_i^{(L_i[m])}[m] \leq y \middle| L_i[m] = \ell\right) \cdot \Pr\left(L_i[m] = \ell\right) \\ &= \sum_{\ell=1}^{W} \pi_\ell[m] \cdot F_{\delta|\ell}(y) \end{aligned} \tag{28}$$

where $\pi_\ell[m] \triangleq \Pr(L_i[m] = \ell)$ is the probability that ℓ observations are recorded in slot m and

$$F_{\delta|\ell}(y) \triangleq \Pr\left(\delta_i^{(L_i[m])}[m] \leq y \middle| L_i[m] = \ell\right) \tag{29}$$

is the conditional distribution function of $\delta_i^{(L_i[m])}[m]$ given $L_i[m] = \ell$. Please note that, when given $L_i[m] = \ell$, the distribution of $\delta_i^{(L_i[m])}[m]$ no longer depends on the time index m since the observations made in each time slot are i.i.d.. Therefore, the time index is omitted in (29).

It is worthwhile to notice that, given $L_i[m] = \ell$, we have $L_i[m+1] = 1$ if sensor i transmits in time slot m (regardless of whether or not it was successful), and $L_i[m+1] = \min\{\ell + 1, W\}$ if sensor i remains silent. The average probability that sensor i transmits, given $L_i[m] = \ell$, is denoted by

$$\begin{aligned} \bar{p}_{\mathrm{EMMR}}^{(\ell)}[m] &\triangleq \Pr\left(\delta_i^{(L_i[m])}[m] > \beta_{\mathrm{EMMR}}[m] \middle| L_i[m] = \ell\right) \\ &= \Pr\left(L_i[m+1] = 1 | L_i[m] = \ell\right). \end{aligned}$$

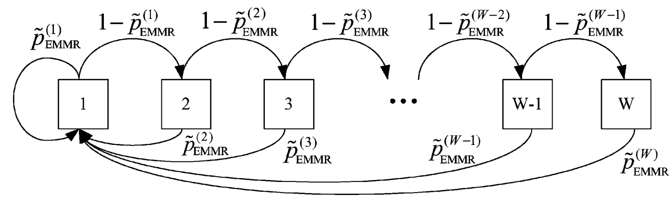

Fig. 3. Finite-state Markov chain of $\{L_i[m]\}_{m=M}^{\infty}$ for sufficiently large M.

Since $\beta_{\mathrm{EMMR}}[m]$ is monotonically increasing and bounded, which can be shown by following the proof in Proposition 1, the sequence $\{\beta_{\mathrm{EMMR}}[m]\}_{m=1}^{\infty}$ will converge to a constant $\tilde{\beta}_{\mathrm{EMMR}}$ and, thus, $\bar{p}_{\mathrm{EMMR}}^{(\ell)}[m]$ will also converge to the constant $\tilde{p}_{\mathrm{EMMR}}^{(\ell)}$. Consequently, given that $L_i[m] = \ell$ and for m sufficiently large, the probability that $L_i[m+1] = 1$ can be approximated as $\tilde{p}_{\mathrm{EMMR}}^{(\ell)}$ and the probability that $L_i[m+1] = \min\{\ell+1, W\}$ as $1 - \tilde{p}_{\mathrm{EMMR}}^{(\ell)}$. Hence, the sequence of random variables $\{L_i[m]\}_{m=M}^{\infty}$, for large M, can be approximated as a finite-state Markov chain as shown in Fig. 3. The set of probabilities $\{\pi_1[m], \ldots, \pi_W[m]\}$ converges to the steady-state distribution of the Markov chain. The set of steady-state probabilities is denoted by $\{\tilde{\pi}_1, \ldots, \tilde{\pi}_W\}$ and is computed from the following set of equations [22]:

$$\tilde{\pi}_1 = \left[1 + \sum_{\ell=2}^{W-1} \prod_{i=1}^{\ell-1}\left(1 - \tilde{p}_{\mathrm{EMMR}}^{(i)}\right) + \prod_{\ell=1}^{W-1}\left(1 - \frac{\tilde{p}_{\mathrm{EMMR}}^{(\ell)}}{\tilde{p}_{\mathrm{EMMR}}^{(W)}}\right)\right]^{-1}$$

$$\tilde{\pi}_W = \prod_{\ell=1}^{W-1}\left(1 - \frac{\tilde{p}_{\mathrm{EMMR}}^{(\ell)}}{\tilde{p}_{\mathrm{EMMR}}^{(W)}}\right)\tilde{\pi}_1$$

and

$$\tilde{\pi}_\ell = \prod_{i=1}^{\ell-1}\left(1 - \tilde{p}_{\mathrm{EMMR}}^{(i)}\right)\tilde{\pi}_1 \quad \text{for } \ell = 2, 3, \ldots, W-1. \tag{30}$$

Hence, as time increases, the distribution function $F_{\delta[m]}(y)$ converges to

$$\tilde{F}_\delta(y) = \sum_{\ell=1}^{W} \tilde{\pi}_\ell F_{\delta|\ell}(y). \tag{31}$$

An approximated value of $\beta_{\mathrm{EMMR}}[m]$ is obtained by substituting (31) into (14). However, the approximation is less accurate when m is small, resulting in performance degradation during the early time slots, which can be observed from the simulations in Section VI. This is eventually overcome by the increase of diversity gains in later time slots.

B. EMMR-SC in the TO System

In the TO system, the sensors become inactive once a transmission occurs. Therefore, if the sensor remains to be active at the beginning of the mth time slot, the number of recorded observations will be equal to either the time index m or the window size W (whichever is smaller). Consequently, given that sensor i is active in the mth time slot, the distribution of $\delta_i^{(L_i[m])}[m]$ is given by

$$F_{\delta[m]}(y) = \begin{cases} F_{\delta[m]|m}(y), & \text{if } m \leq W \\ F_{\delta[m]|W}(y), & \text{otherwise.} \end{cases} \quad (32)$$

Similarly, by substituting (32) into (14) and by replacing N with $N[m]$, we can find the optimum value of $\beta_{\text{EMMR}}[m]$. In contrast to the RT system, no approximation is done here and the derived transmission threshold is accurate.

VI. Numerical Simulations and Performance Comparisons

In this section, numerical simulations of the proposed strategies are given for two cases: 1) the case with exponentially distributed local SNRs and 2) the case with Rayleigh distributed channel gains. The MMR, TMMR, and EMMR-SC methods are compared with three other MAC protocols: 1) the conventional slotted ALOHA scheme with no channel-aware transmission control; 2) the TDMA scheme; and 3) the optimal scheduling scheme. In the slotted ALOHA scheme, the sensors transmit with probability $1/N$ in each time slot regardless of the channel realizations. In TDMA, the sensors transmit in a round-robin fashion in the order of their indices. In the optimal scheduling scheme, the sensor with the largest effective local SNR is scheduled to transmit in each time slot. This scheme is optimal in the sense that it minimizes the MSE of the estimate at the fusion center and serves as a performance lower bound for other strategies.

In the following simulations, we let $N = 20$ and let X be a circularly symmetric complex Gaussian random variable with zero mean and unit variance, i.e., $X \sim \mathcal{CN}(0,1)$. The results shown in this section are obtained by averaging over 1500 independent trials. The simulations have been conducted using (7) to compute the MSE in each trial.

1) Example I—(Exponentially Distributed SNRs): In this example, we assume that $\{\delta_i[m], \forall i, m\}$ are i.i.d. exponentially distributed with mean $\theta = \sigma_S^2\sigma_X^2/\sigma_W^2 = 5$. This corresponds to the case where the transmission channels are noiseless with constant gain $h_{iD}[m] = 1$, for all i, m, and the observation channel coefficients $\{h_{Si}[m], \forall i, m\}$ are i.i.d. with distribution $\mathcal{CN}(0, \sigma_S^2)$. In Figs. 4 and 5, we show the MSE performance of the RT and the TO systems, respectively. In both systems, we observe that both the MMR and the TMMR methods significantly outperform the conventional slotted ALOHA scheme, but lose to the optimal scheduling due to collision.

In the RT system, we show in Fig. 4 that the TMMR method has comparable performance with respect to the MMR method, even though $\delta_T[m-1]$ is not explicitly known. However, in the TO system shown in Fig. 5, the MMR outperforms the TMMR since the performance is further degraded by the error of the estimation in $\hat{N}[m]$. In fact, the threshold $\beta_{\text{TMMR}}[m]$ in the TMMR method is often underestimated in the early stages and, thus, increases the amount of transmissions that fail due to collision. As a result, fewer sensors remain active in later time slots and, thereby, limits the eventual MSE performance of TMMR.

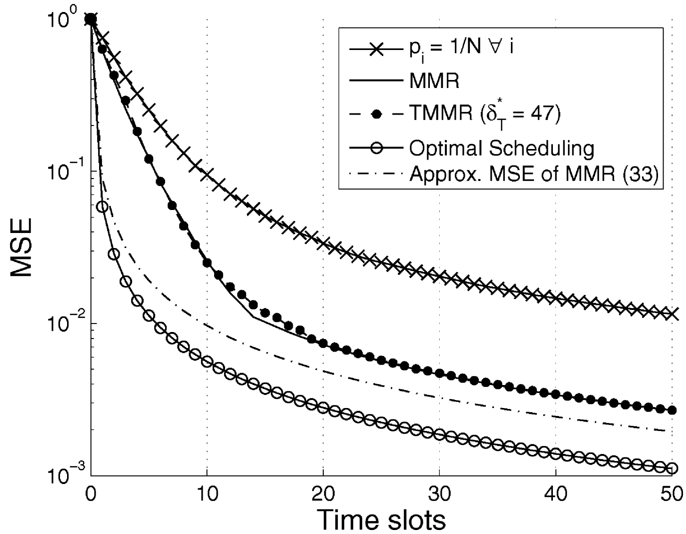

Fig. 4. Performance of the proposed MMR and TMMR methods for Example I in the RT system.

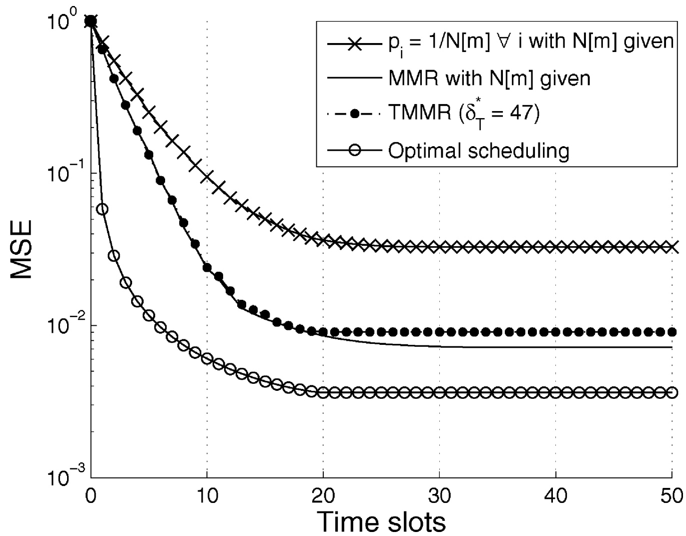

Fig. 5. Performance of the proposed MMR and TMMR methods for Example I in the TO system.

Notice that, in the TO system, the average MSE saturates as time increases since all sensors eventually transmit and become inactive. However, in the RT system, the MSE of the estimate steadily decreases with m, as shown in Fig. 4. For m sufficiently large, we can approximate the transmission threshold as $\beta_{\text{MMR}}[m] \approx \beta_{\text{U}}$ and the average accumulated SNR as

$$\begin{aligned} \mathrm{E}\left[\delta_T[m]\right] &\approx mN \int_{\beta_{\text{U}}}^{\infty} \delta \cdot \frac{1}{\theta} \cdot e^{-\delta/\theta} d\delta \cdot (1 - \bar{p}_{\text{L}})^{N-1} \\ &\approx m\theta \left(1 + \ln \bar{p}_{\text{L}}\right) \left(1 - \bar{p}_{\text{L}}\right)^{N-1} \end{aligned}$$

which follows from derivations similar to (23). Then, the average MSE can be approximated as

$$\begin{aligned} \mathrm{E}\left[\frac{\sigma_X^2}{1 + \delta_T[m]}\right] &\approx \frac{\sigma_X^2}{1 + \mathrm{E}\left[\delta_T[m]\right]} \\ &\approx \frac{\sigma_X^2}{1 + m\theta \left(1 + \ln \bar{p}_{\text{L}}\right) \left(1 - \bar{p}_{\text{L}}\right)^{N-1}}. \quad (33) \end{aligned}$$

This can be viewed as an approximation for both the MMR and TMMR performance curves for large m and is plotted in Fig. 4 as a reference. We can see, from (33), that the MSE is inversely proportional to m when m is sufficiently large.

2) Example II—(Rayleigh Distributed Channel Gains): In this example, we assume that the observation channel coefficients $\{h_{Si}[m], \forall i, m\}$ and the transmission channel coefficients $\{h_{iD}[m], \forall i, m\}$ are both i.i.d. random processes with distributions $\mathcal{CN}(0, \sigma_S^2)$ and $\mathcal{CN}(0, \sigma_D^2)$, respectively. The distribution of $\delta_i[m]$ is given in the following Lemma, with the proof provided in Appendix C.

Lemma 2:

$$F_\delta(y) = 1 - \sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}} K_1\left(\sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right) \times \exp\left\{-\frac{y\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\} \quad (34)$$

and

$$f_\delta(y) = \frac{2\sigma_W^2}{\sigma_X^2\sigma_S^2\sigma_D^2} \exp\left\{-\frac{\sigma_W^2 y(\sigma_D^2 + \sigma_S^2\sigma_X^2)}{\sigma_X^2\sigma_S^2\sigma_D^2}\right\} \times \left[(2y+1)\sigma_W^2 K_0\left(\sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right) + (\sigma_D^2+\sigma_S^2\sigma_X^2)\sqrt{\frac{\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}} K_1\left(\sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right)\right] \quad (35)$$

where K_i is the ith order modified Bessel function of second kind.

The threshold $\beta_{\mathrm{MMR}}[m]$ in the MMR method can be evaluated numerically by substituting (34) and (35) into the fixed point equation given by (14). The upper and lower bounds of $\beta_{\mathrm{MMR}}[m]$, which are used in the TMMR method, can be obtained similarly from Proposition 1. The simulation results are shown in Figs. 6 and 7 for $\sigma_S^2 = \sigma_D^2 = \sigma_W^2 = 1$. Similarly, both the MMR and the TMMR methods outperform the conventional slotted ALOHA system but lose to the one with optimal scheduling. More interestingly, the two schemes also outperform TDMA where sensors transmit in the order of their indices regardless of their local SNRs. This shows that the advantage of channel-awareness more than compensates for the loss due to collision in random access networks. In both the RT and the TO systems, the TMMR is clearly inferior to the MMR method, which is not the case in Example I. This is because, in this example, the upper bound of $\beta_{\mathrm{MMR}}[m]$ given in Proposition 1 is used instead of the tighter upper bound given in Proposition 2. Recall that the latter bound is applicable only to the model of Example I.

To further improve the estimation performance, we employ the EMMR-SC method that combines the observations recorded in the buffer to exploit temporal diversity. To obtain the transmission threshold, we first compute the asymptotic distribution function $\tilde{F}_\delta(y) = \sum_{\ell=1}^{W} \tilde{\pi}_\ell \cdot F_{\delta|\ell}(y)$, where the values of $\tilde{\pi}_1, \tilde{\pi}_2, \ldots, \tilde{\pi}_W$ follow from (30) and $F_{\delta|\ell}(y)$ is given in the following.

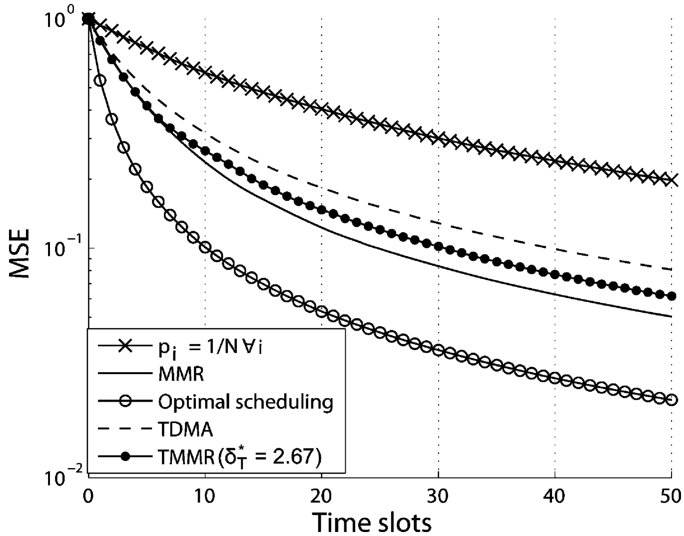

Fig. 6. Performance of the proposed MMR and TMMR methods for Example II in the RT system.

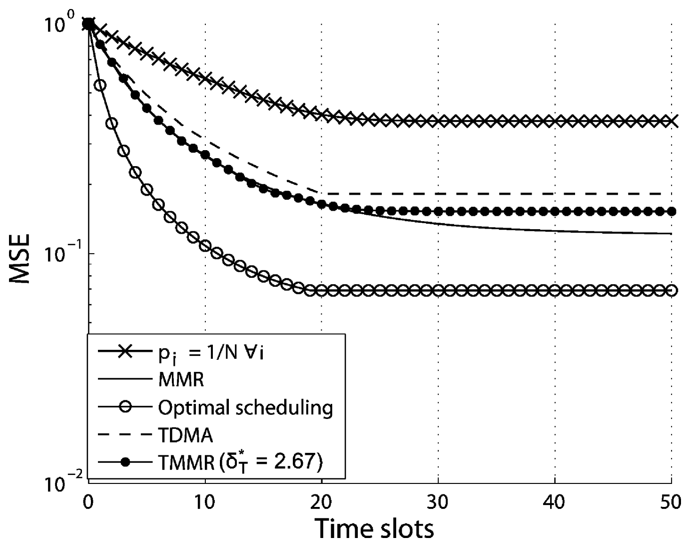

Fig. 7. Performance of the proposed MMR and TMMR methods for Example II in the TO system.

Lemma 3: The conditional distribution function of $\delta_i^{L_i[m]}$ given $L_i[m] = \ell$ is

$$F_{\delta[m]|\ell}(y) = 1 + \sum_{r=1}^{\ell} \binom{\ell}{r} (-1)^r \sqrt{\frac{4r\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}} \times K_1\left(\sqrt{\frac{4r\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right) \exp\left\{-\frac{ry\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\}. \quad (36)$$

The proof is given in Appendix D.

The MSE performance of EMMR-SC is shown in Figs. 8 and 9 for the RT and the TO systems, respectively. In Fig. 8, the performance of EMMR-SC for $W = 2, 5, 8$ is compared with the MMR method proposed in Section III. The MMR method is equivalent to the EMMR-SC method when $W = 1$. In the RT system, shown in Fig. 8, the EMMR-SC schemes do not perform as well as the MMR in the early time slots because the distribution function given in (31) holds only when m is

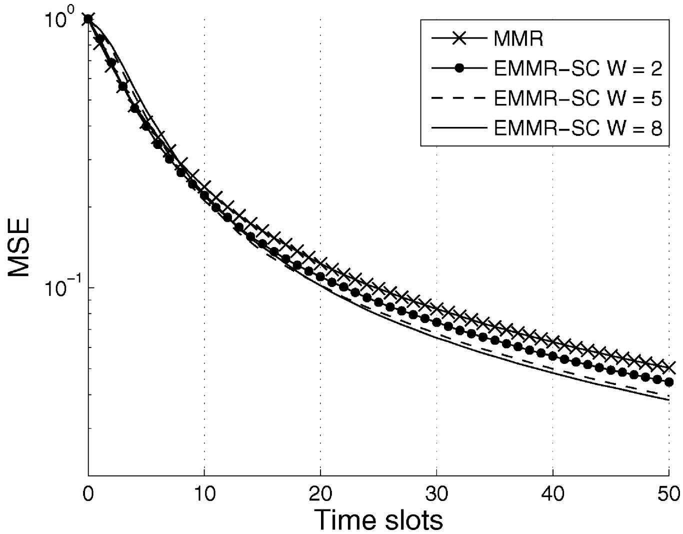

Fig. 8. Performance of the EMMR-SC method for $W = 2, 5, 8$ in the RT system.

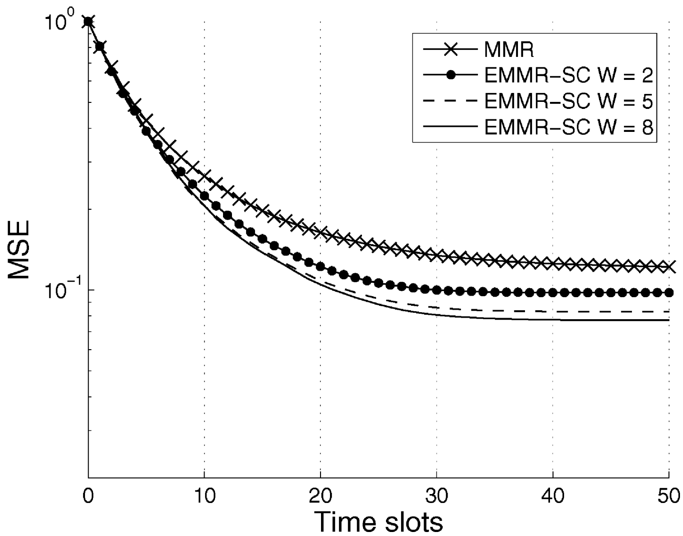

Fig. 9. Performance of the EMMR-SC method for $W = 2, 5, 8$ in the TO system.

sufficiently large. Nonetheless, the EMMR-SC method eventually outperforms the MMR in later time slots since the diversity order increases due to the selective combining performed on the buffered set of observations. The MSE decreases faster as W increases but the improvement is limited for large W. In Fig. 9, the performance of the EMMR-SC for $W = 2, 5, 8$ is also compared with the MMR method in the TO system. We notice that, in the TO system, the EMMR-SC schemes outperform the MMR method even in early time slots since the actual distribution in (32) is used instead of the asymptotic approximation. Similarly, the gain improves as W increases, but is limited for large W.

VII. Conclusion

In this paper, we studied the performance of the distributed estimation problem in a cooperative slotted ALOHA system with channel-aware sensors. It is shown that the transmission probability assignment that results in maximal throughput does not yield desirable estimation performance. Therefore, by exploiting the local sensor information, i.e., the sensor's measurement gain and the transmission channel gain, we showed that a lower distortion can be achieved in each time slot. Specifically, we found that the optimal transmission control function takes on the form of a thresholding function where the sensors transmit only if their effective local SNR values exceed a certain threshold. Two methods were proposed to derive the transmission control thresholds: the MMR and the suboptimal TMMR methods. For the MMR method, sensors compute a new threshold at the beginning of each time slot based on the knowledge of the number of active sensors and the accumulated estimation performance. In the TMMR method, sensors do not require the explicit knowledge of the above mentioned system parameters, but switch between two predetermined thresholds based only on the local estimates of these parameters. Even so, we found that TMMR achieves a reasonably good performance, one that is comparable with the MMR scheme. Additionally, to improve upon the MMR and TMMR methods, the EMMR-SC method was presented which effectively exploits both the spatial and temporal diversities and thus outperforms the MMR method at the expense of increased computational complexity.

Appendix A
Proof of Proposition 1

To show that $\beta_{\rm MMR}[m]$ is monotonically non-decreasing as m increases, it is sufficient to show that $\beta_{\rm MMR}[m]$ is monotonically non-decreasing with respect to $\delta_T[m-1]$ since the accumulated SNR cannot decrease with time. In fact, it can be easily shown that the right-hand side (RHS) of (14) is non-decreasing as δ_T increases. Thus, when δ_T increases, $\beta_{\rm MMR}$ must also increase in order to satisfy the equality in (14). Hence, $\beta_{\rm MMR}$ is monotonically non-decreasing with respect to δ_T and m.

Since $\beta_{\rm MMR}$ is non-decreasing with respect to δ_T, the smallest value of $\beta_{\rm MMR}$ is obtained when $\delta_T = 0$. In this case, we have

$$\begin{aligned}\gamma_{\rm MMR} &= \int_{\beta_{\rm MMR}}^{\infty} \frac{y\sigma_X^2}{1+y} f_\delta(y)dy \\ &\overset{(a)}{>} \int_{\beta_{\rm MMR}}^{\infty} \frac{\beta_{\rm MMR}\sigma_X^2}{1+\beta_{\rm MMR}} f_\delta(y)dy \\ &= \frac{\beta_{\rm MMR}\sigma_X^2}{1+\beta_{\rm MMR}} [1 - F_\delta(\beta_{\rm MMR})]\end{aligned}$$

where (a) follows from the fact that $y/(1+y)$ is monotonically increasing with respect to y, for $y \geq 0$. For $\delta_T = 0$, it follows from (14), (15), and the previous bound on $\gamma_{\rm MMR}$ that

$$\begin{aligned}\beta_{\rm MMR} &= \left. \frac{\sigma_X^2(1+\delta_T)}{\sigma_X^2 - \frac{(1+\delta_T)(N-1)\gamma_{\rm MMR}}{(1-\bar{p}_{\rm MMR})}} - (1+\delta_T) \right|_{\delta_T=0} \\ &> \frac{\sigma_X^2}{\sigma_X^2 - (N-1)\frac{\beta_{\rm MMR}\sigma_X^2}{1+\beta_{\rm MMR}}\frac{[1-F_\delta(\beta_{\rm MMR})]}{F_\delta(\beta_{\rm MMR})}} - 1 \\ &\Rightarrow F_\delta(\beta_{\rm MMR}) > 1 - \frac{1}{N} \\ &\Rightarrow \beta_{\rm MMR} > F_\delta^{-1}\left(1 - \frac{1}{N}\right) \equiv \beta_{\rm L}.\end{aligned}$$

Thus, we have obtained the lower bound.

Since $\beta_{\rm MMR}$ is monotonically non-decreasing, the largest value is obtained when $\delta_T \to \infty$. Therefore, it is sufficient to obtain an upper bound on $\beta_{\rm MMR}$ as $\delta_T \to \infty$. To obtain the upper bound on $\beta_{\rm MMR}$, we first recognize the fact that

$$\begin{aligned}\gamma_{\rm MMR} &= \int_{\beta_{\rm MMR}}^{\infty} \frac{y\sigma_X^2}{(1+\delta_T+y)(1+\delta_T)} f_\delta(y)dy \\ &< \int_{\beta_{\rm MMR}}^{\infty} \frac{y\sigma_X^2}{(1+\delta_T+\beta_{\rm MMR})(1+\delta_T)} f_\delta(y)dy \\ &= \frac{\sigma_X^2}{(1+\delta_T+\beta_{\rm MMR})(1+\delta_T)} {\rm E}\left[\delta \cdot 1_{\{\delta>\beta_{\rm MMR}\}}\right]. \quad (37)\end{aligned}$$

Then, for δ_T sufficiently large, such that

$$\sigma_X^2 - \frac{N-1}{F_\delta(\beta_{\rm MMR})} \frac{\sigma_X^2}{1+\delta_T+\beta_{\rm MMR}} {\rm E}[\delta \cdot 1_{\{\delta>\beta_{\rm MMR}\}}] > 0,$$

it follows from (14), (15), and (37) that

$$\begin{aligned}\beta_{\rm MMR} &< \frac{\sigma_X^2(1+\delta_T)}{\sigma_X^2 - \frac{N-1}{F_\delta(\beta_{\rm MMR})}\frac{\sigma_X^2}{1+\delta_T+\beta_{\rm MMR}}{\rm E}[\delta\cdot 1_{\{\delta>\beta_{\rm MMR}\}}]} \\ &\quad - (1+\delta_T) \\ &< \frac{\frac{N-1}{F_\delta(\beta_{\rm MMR})}{\rm E}[\delta\cdot 1_{\{\delta>\beta_{\rm MMR}\}}]}{1 - \frac{N-1}{F_\delta(\beta_{\rm MMR})}\frac{1}{1+\delta_T+\beta_{\rm MMR}}{\rm E}[\delta\cdot 1_{\{\delta>\beta_{\rm MMR}\}}]}.\end{aligned}$$

By taking $\delta_T \to \infty$, we have

$$\beta_{\rm MMR} < \frac{N-1}{F_\delta(\beta_{\rm MMR})} {\rm E}\left[\delta \cdot 1_{\{\delta>\beta_{\rm MMR}\}}\right] < N{\rm E}[\delta \cdot 1_{\{\delta>\beta_{\rm L}\}}] \quad (38)$$

where the last inequality is obtained by substituting $\beta_{\rm MMR}$ with $\beta_{\rm L}$. Notice that $F_\delta(\beta_{\rm L}) = 1-(1/N)$. The upper bound is thus obtained.

Remark 1: Please note that a tighter upper bound, denoted by $\beta'_{\rm U}$, can be found numerically by solving the fixed point equation

$$\beta'_{\rm U} F_\delta(\beta'_{\rm U}) = (N-1){\rm E}\left[\delta \cdot 1_{\{\delta>\beta'_{\rm U}\}}\right] \quad (39)$$

instead of replacing $\beta_{\rm MMR}$ with $\beta_{\rm L}$ as done in (38). The reasoning is that, by restating the first inequality in (38) as

$$\beta_{\rm MMR} F_\delta(\beta_{\rm MMR}) < (N-1){\rm E}[\delta \cdot 1_{\{\delta>\beta_{\rm MMR}\}}]$$

we can see that, as $\beta_{\rm MMR}$ increases, the left-hand-side (LHS) increases while the RHS decreases. Therefore, the largest value of $\beta_{\rm MMR}$ that satisfies the above inequality is the solution $\beta'_{\rm U}$ that is obtained from the fixed point equation in (39).

APPENDIX B
PROOF OF PROPOSITION 2

In Proposition 2, we provide lower and upper bounds for $\beta_{\rm MMR}[m]$ for the case where the local SNRs are exponentially distributed with mean θ. The lower bound $\beta_{\rm L}$ follows directly from Proposition 1 while the upper bound $\beta_{\rm U}$ is derived as follows.

First of all, by reorganizing the terms in (14), the fixed point equation becomes

$$\bar{p}_{\rm MMR} = 1 - \frac{(1+\delta_T)(1+\delta_T+\beta_{\rm MMR})}{\beta_{\rm MMR}\sigma_X^2}(N-1)\gamma_{\rm MMR}. \quad (40)$$

Then, by substituting (37) into (40), we have

$$\begin{aligned}\bar{p}_{\rm MMR} &> 1 - \frac{(1+\delta_T)(1+\delta_T+\beta_{\rm MMR})}{\beta_{\rm MMR}\sigma_X^2}(N-1) \\ &\quad \cdot \frac{\sigma_X^2}{(1+\delta_T+\beta_{\rm MMR})(1+\delta_T)}{\rm E}\left[\delta 1_{\{\delta>\beta_{\rm MMR}\}}\right] \\ &\overset{(a)}{=} 1 - \frac{N-1}{\beta_{\rm MMR}}(\theta+\beta_{\rm MMR})e^{-\beta_{\rm MMR}/\theta} \\ &\overset{(b)}{>} 1 - \frac{N-1}{\beta_{\rm L}}(\theta+\beta_{\rm L})e^{-\beta_{\rm MMR}/\theta}\end{aligned}$$

where (a) follows the fact that

$${\rm E}[\delta 1_{\{\delta>\beta_{\rm MMR}\}}] = \int_{\beta_{\rm MMR}}^{\infty} y f_\delta(y)dy = (\theta+\beta_{\rm MMR})e^{-\beta_{\rm MMR}/\theta}$$

and (b) holds since

$$\frac{\theta+\beta_{\rm MMR}}{\beta_{\rm MMR}} = \frac{\theta}{\beta_{\rm MMR}} + 1$$

is smaller for larger $\beta_{\rm MMR}$.

By substituting (19) into (40) and with $\beta_{\rm L} = \theta \ln N$, we have

$$e^{-\beta_{\rm MMR}/\theta} > 1 - \frac{1+\ln(N)}{\theta\ln(N)}(N-1)e^{-\beta_{\rm MMR}/\theta} \quad (41)$$

which leads to the upper bound that

$$\beta_{\rm MMR} < \beta_{\rm U} = \theta \ln\left[1 + \frac{1+\ln N}{\ln N}(N-1)\right].$$

This completes the proof.

APPENDIX C
PROOF OF LEMMA 2

From (5), we have

$$F_\delta(y) = \Pr\left(\frac{\alpha_i^2[m]|h_{Si}[m]h_{iD}[m]|^2\sigma_X^2}{(\alpha_i^2[m]|h_{iD}[m]|^2+1)\sigma_W^2} \le y\right) \quad (42)$$

where $\alpha_i[m] = 1/\sqrt{E\left[|Y_i[m]|^2|h_{Si}[m]\right]}$. For convenience, we shall omit the time index m. Then, by letting $a_{Si} = |h_{Si}|^2$, $a_{iD} = |h_{iD}|^2$ and from the fact that

$$\begin{aligned}&\left\{(a_{Si}, a_{iD})\colon \frac{a_{Si}a_{iD}\sigma_X^2}{(a_{iD}+a_{Si}\sigma_X^2+\sigma_W^2)\sigma_W^2} \le y\right\} \\ &= \left\{(a_{Si}, a_{iD})\colon \left(a_{Si} - \frac{y\sigma_W^2}{\sigma_X^2}\right)(a_{iD}-y\sigma_W^2) \le \frac{\sigma_W^2 y(y+1)}{\sigma_X^2}\right\} \\ &= \left\{(a_{Si}, a_{iD}) : a_{iD} < y\sigma_W^2\right\} \\ &\bigcup \left\{(a_{Si}, a_{iD}) : a_{Si} \le \frac{y\sigma_W^2(a_{iD}+\sigma_W^2)}{\sigma_X^2(a_{iD}-y\sigma_W^2)}, a_{iD} \ge y\sigma_W^2\right\}\end{aligned}$$

the distribution function can be computed as

$$\begin{aligned}F_\delta(y) &= \Pr\left(\frac{a_{Si}a_{iD}\sigma_X^2}{(a_{iD}+a_{Si}\sigma_X^2+\sigma_W^2)\sigma_W^2} \le y\right) \\ &= \Pr\left(a_{iD} < y\sigma_W^2\right) \\ &\quad + \Pr\left(a_{Si} \le \frac{y\sigma_W^2(a_{iD}+\sigma_W^2)}{\sigma_X^2(a_{iD}-y\sigma_W^2)}, a_{iD} \ge y\sigma_W^2\right) \\ &= I_1 + I_2\end{aligned}$$

where $I_1 = \int_0^{y\sigma_W^2} (1/\sigma_D^2) e^{-\lambda/\sigma_D^2} d\lambda = 1 - \exp\left\{-y\sigma_W^2/\sigma_D^2\right\}$ and

$$\begin{aligned}
I_2 =& \int_{y\sigma_W^2}^{\infty} \left(1-\exp\left\{\frac{-y\sigma_W^2(\lambda+\sigma_W^2)}{\sigma_S^2\sigma_X^2(\lambda-y\sigma_W^2)}\right\}\right)\cdot\frac{1}{\sigma_D^2}e^{-\lambda/\sigma_D^2}d\lambda \\
& \text{(with } \eta=\lambda - y\sigma_W^2) \\
=& \exp\left\{\frac{-y\sigma_W^2}{\sigma_D^2}\right\} - \frac{1}{\sigma_D^2}\int_0^{\infty} \exp\left\{-\frac{y\sigma_W^4(1+y)}{\sigma_S^2\sigma_X^2\eta} - \frac{\eta}{\sigma_D^2}\right\}d\eta \\
& \cdot\exp\left\{-\frac{y\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\} \\
\overset{(a)}{=}& \exp\left\{-\frac{y\sigma_W^2}{\sigma_D^2}\right\} - \sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}} \\
& \times K_1\left(\sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right)\exp\left\{-\frac{y\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\}
\end{aligned}$$

where (a) follows from the fact that

$$\int_0^{\infty} \exp\left(-\frac{u}{4x} - vx\right)dx = \sqrt{\frac{u}{v}}K_1(\sqrt{uv}), \quad \text{for } u\geq 0,\ v\geq 0.$$

Hence, we have

$$\begin{aligned}
F_\delta(y) &= I_1 + I_2 \\
&= 1 - \sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}K_1\left(\sqrt{\frac{4\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right) \\
&\quad \times \exp\left\{-\frac{y\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\}.
\end{aligned}$$

By taking the derivative of $F_\delta(y)$ with respect to y, we obtain the density function $f_\delta(y)$ given by (35).

Appendix D
Proof of Lemma 3

From (29), it follows that

$$F_{\delta|\ell}(y) = \Pr\left(\frac{(\alpha_i^{(\ell)}[m])^2\cdot|h_{Si}^{(\ell)}[m]\cdot h_{iD}[m]|^2}{(\alpha_i^{(\ell)}[m]\cdot|h_{iD}[m]|)^2+1}\cdot\frac{\sigma_X^2}{\sigma_W^2}\leq y\right)$$

where

$$\alpha_i^{(\ell)}[m] = \sqrt{\frac{P}{\mathrm{E}\left[\left|Y_i^{(\ell)}[m]\right|^2\middle| h_{Si}^{(\ell)}[m]\right]}}$$

and $P=1$. Let $a_{iD}[m] = |h_{iD}[m]|^2$ and

$$a_{Si}^{(\ell)}[m] = \left|h_{Si}^{(\ell)}[m]\right|^2 = \max\{|h_{Si}[m-\ell+1]|^2,\ldots,|h_{Si}[m]|^2\}$$

with the distribution functions $F_{a_{iD}}(y) = 1 - e^{-y/\sigma_D^2}$ and $F_{a_{Si}^{(\ell)}}(y) = \left(1-e^{-y/\sigma_S^2}\right)^{\ell}$, respectively. Then, by omitting the time index m and by following the procedures of Appendix C, we have

$$\begin{aligned}
F_{\delta|\ell}(y) &= \Pr\left(\frac{a_{Si}^{(\ell)}\cdot a_{iD}}{a_{iD}+a_{Si}^{(\ell)}\cdot\sigma_X^2+\sigma_W^2}\cdot\frac{\sigma_X^2}{\sigma_W^2}\leq y\right) \\
&= \Pr\left(a_{iD} < y\sigma_W^2\right) \\
&\quad + \Pr\left(a_{Si}^{(\ell)} \leq \frac{y\sigma_W^2(a_{iD}+\sigma_W^2)}{\sigma_X^2(a_{iD}-y\sigma_W^2)}, a_{iD}\geq y\sigma_W^2\right) \\
&= I_1 + I_2,
\end{aligned}$$

where $I_1 = 1 - \exp\left\{-y\sigma_W^2/\sigma_D^2\right\}$ and

$$\begin{aligned}
I_2 =& \int_{y\sigma_W^2}^{\infty}\left(1-\exp\left\{\frac{-y\sigma_W^2(\lambda+\sigma_W^2)}{\sigma_S^2\sigma_X^2(\lambda-y\sigma_W^2)}\right\}\right)^{\ell}\cdot\frac{1}{\sigma_D^2}e^{-\lambda/\sigma_D^2}d\lambda \\
=& \int_{y\sigma_W^2}^{\infty}\left(1+\sum_{r=1}^{\ell}\binom{\ell}{r}(-1)^r\exp\left\{\frac{-ry\sigma_W^2(\lambda+\sigma_W^2)}{\sigma_S^2\sigma_X^2(\lambda-y\sigma_W^2)}\right\}\right) \\
& \times\frac{1}{\sigma_D^2}e^{-\lambda/\sigma_D^2}d\lambda \\
=& \exp\left\{\frac{-y\sigma_W^2}{\sigma_D^2}\right\} + \frac{1}{\sigma_D^2}\sum_{r=1}^{\ell}\binom{\ell}{r}(-1)^r \\
& \times\int_0^{\infty}\exp\left\{-\frac{ry\sigma_W^4(1+y)}{\sigma_S^2\sigma_X^2\eta} - \frac{\eta}{\sigma_D^2}\right\}d\eta\exp\left\{-\frac{ry\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\} \\
=& \exp\left\{-\frac{y\sigma_W^2}{\sigma_D^2}\right\} + \sum_{r=1}^{\ell}\binom{\ell}{r}(-1)^r\sqrt{\frac{4r\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}} \\
& \times K_1\left(\sqrt{\frac{4r\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right)\exp\left\{-\frac{ry\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\}.
\end{aligned}$$

Thus

$$\begin{aligned}
F_{\delta|\ell}(y) =& I_1 + I_2 \\
=& 1+\sum_{r=1}^{\ell}\binom{\ell}{r}(-1)^r\sqrt{\frac{4r\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}K_1\left(\sqrt{\frac{4r\sigma_W^4 y(y+1)}{\sigma_X^2\sigma_S^2\sigma_D^2}}\right) \\
& \times\exp\left\{-\frac{ry\sigma_W^2}{\sigma_S^2\sigma_X^2} - \frac{y\sigma_W^2}{\sigma_D^2}\right\}.
\end{aligned}$$

The proof is complete.

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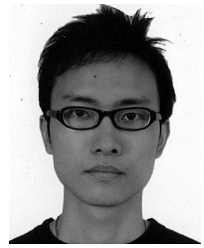

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