Blind MAI and ISI Suppression for DS/CDMA Systems Using HOS-Based Inverse Filter Criteria

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***Abstract*—Cumulant-based inverse filter criteria (IFC) using second- and higher order statistics (HOS) proposed by Tugnait *et al.* have been widely used for blind deconvolution of discrete-time multi-input multi-output (MIMO) linear time-invariant systems with non-Gaussian measurements through a multistage successive cancellation procedure, but the deconvolved signals turn out to be an unknown permutation of the driving inputs. In this paper, a multistage blind equalization algorithm (MBEA) is proposed for multiple access interference (MAI) and intersymbol interference (ISI) suppression of multiuser direct sequence/code division multiple access (DS/CDMA) systems in the presence of multipath. The proposed MBEA, which processes the chip waveform matched filter output signal without requiring any path delay information, includes blind deconvolution processing using IFC followed by identification of the estimated symbol sequence with the associated user through using a user identification algorithm (UIA). Then, some simulation results are presented to support the proposed MBEA and UIA. Finally, some conclusions are drawn.**

***Index Terms*—Cumulant-based inverse filter criteria, higher order statistics, multistage blind equalization algorithm.**

I. Introduction

CODE division multiple access (CDMA) has been a central technique in multiuser communications due to efficient spectrum utilization, release from frequency management, low mobile station's transmit power (through power control and soft handoff), wide variety of data rates, high multipath resolution and high trunking efficiency, etc. In addition to additive white Gaussian noise, two major interference problems encountered in the receiver design of CDMA systems are multiple access interference (MAI), due to multiple users sharing the same channel, and intersymbol interference (ISI) resulting from multiple transmission paths between the transmitter and the receiver. Suppression of MAI and removal of ISI are crucial to the performance [capacity and bit error rate (BER)] of multiuser detection of CDMA systems.

A number of detection algorithms for the suppression of MAI for CDMA systems have been reported in the open literature. Optimum receivers, such as maximum likelihood (ML) detectors [1]–[3] and minimum error probability detectors [1], [2], [4] have been reported that are near–far resistant, but their computational complexity grows exponentially with number of active users. Therefore, suboptimal detectors with lower computational complexity such as linear detectors have been reported [1], [2], [5]–[9]. Lupas and Verdu [5] proposed a decorrelating (linear) detector that completely suppresses the unwanted users at the expense of noise enhancement; therefore, it is near–far resistant as the minimum error probability receiver. Minimum mean square error (MMSE) linear detectors [1]–[3], [6] perform as the decorrelating detector when noise variance approaches zero and as the single-user matched filter when powers of unwanted users approach zero. Honig *et al.* [7] proposed a minimum output energy (MOE) detector that also corresponds to an MMSE linear detector, and both of them can maximize signal-to-interference-plus-noise ratio (SINR). On the other hand, some decision-driven detectors with computational complexity comparable to linear detectors have been reported such as successive cancellation detectors [1] and multistage detectors [8], decorrelating decision-feedback detectors [9], and MMSE decision-feedback detectors [1], [2] that are suited to high SNR channels with power imbalance, but their performance is generally not superior to linear detectors. However, the detectors mentioned above assume the absence of multipath effects and require some prior information such as signature sequences, relative signal arrival time delays between users, noise variance and signal powers of users, etc.

Recently, many algorithms for simultaneously suppressing MAI and removing ISI have been reported [10]–[13] for CDMA systems in the presence of multipath. Tsatsanis and Giannakis [10] proposed an MMSE decorrelating receiver for asynchronous DS/CDMA systems. Their MMSE receiver is near–far resistant, but it requires signature waveform (convolution of multipath and signature sequence) of all the active users given in advance. They also proposed an MMSE receiver for direct sequence spread spectrum (DS/SS) systems in multipath [11], including estimation of the signature waveform using a subspace-based algorithm. Tsatsanis [12] also proposed a near–far resistant MOE receiver for asynchronous DS/CDMA systems, assuming that a path of the desired user is known ahead of time. Then, Tsatsanis and Xu [13] further proposed a blind minimum variance (MV) receiver that is near–far resistant with performance close to the MMSE decorrelating receiver for high SNR, and estimation of the multipath channel of the desired user is also included. On the other hand, a number of subspace-based algorithms were reported [14]–[16] for estimation of multipath channels for CDMA systems. Multipath channels of all the active users can be estimated by projecting the desired user's signature waveform into noise subspace. Usually, singular value decomposition (SVD) of

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correlation matrices with huge dimension must be performed by subspace-based methods for finding the noise subspace, and therefore, their practical use is limited due to large computational complexity.

Higher order statistics (HOS) [17], known as cumulants, have been successfully used for blind deconvolution (equalization) of nonminimum-phase linear time-invariant (LTI) single-input single-output (SISO) and multi-input multi-output (MIMO) systems [18]–[28]. Chi and Wu [18] proposed a family of SISO inverse filter criteria (SISO-IFC) for equalization of SISO systems using HOS, which includes Tugnait's IFC [19] and Shalvi and Weinstein's IFC [20] as special cases. Tugnait [22], Inouye [23], Feng and Chi [24], and Chi and Chen [25], [26] extended these SISO-IFC to those for blind deconvolution of MIMO systems. Yeung and Yau [27] proposed a super-exponential algorithm (SEA) for blind deconvolution of MIMO systems, which is also an extension of Shalvi and Weinstein's SEA for blind deconvolution of SISO systems [21]. Loubaton and Regalia [28] proposed a deflation algorithm for blind deconvolution of MIMO systems. Based on a state space description of the lossless lattice structure for the inverse filter, the deflation algorithm processes prewhitened measurements so that one of the input signals can be obtained.

A common fact regarding the preceding IFC, SE algorithm, and deflation algorithm for MIMO systems is that the deconvolved signals obtained through a multistage successive cancellation (MSC) procedure turn out to be an unknown permutation of the driving inputs. This fact prevents their use in the MAI suppression and ISI removal for CDMA systems, although received signals for CDMA channels in the presence of multipath can be modeled as an MIMO system. The reason for this is that as any one of the above approaches is employed, the match of each estimated symbol sequence $\hat{u}_l[n]$ (obtained at the lth stage of the MSC procedure) with the associated user j (with a known signature sequence $c_j[n]$) who transmitted it is unknown due to the fact mentioned above. In this paper, we propose a user identification algorithm (UIA) for identifying the match of $\hat{u}_l[n]$ and $c_j[n]$. Then, a multistage blind equalization algorithm (MBEA) is proposed for blind MAI suppression and ISI removal of asynchronous DS/CDMA systems in multipath including blind deconvolution processing using IFC followed by the use of the proposed UIA.

The paper is organized as follows. Section II presents the MIMO signal model for asynchronous DS/CDMA channels. Section III presents IFC for DS/CDMA systems. The proposed UIA and MBEA are presented in Section IV. Then, some simulation results are presented in Section V to support the efficacy of the proposed UIA and MBEA. Finally, we draw some conclusions.

II. MIMO Signal Model for Asynchronous DS/CDMA Channels

Consider a K-user asynchronous DS/CDMA system. Assume that $c_i[n]$ is the signature sequence (a binary sequence of $\{+1, -1\}$) of user i with spreading factor equal to $P(\geq K)$, and

$$\mathcal{R} = \{c_i[n],\ i = 1, 2, \ldots, K,\ n = 0, 1, \ldots, P-1\} \quad (1)$$

is the set of the K active users' signature sequences. Let $u_i[n]$ denote the symbol stream of user i, and let

$$s_i[n] = \sum_{k=-\infty}^{\infty} u_i[k] c_i[n - kP]. \quad (2)$$

The received signal $y(t)$ is a superposition of signals $y_i(t)$ from the K users as follows [2], [10], [12], [13]:

$$y(t) = \sum_{i=1}^{K} y_i(t) + w(t) \quad (3)$$

where $w(t)$ is additive white Gaussian noise, and

$$y_i(t) = \sum_{k=-\infty}^{\infty} s_i[k] g_i(t - kT_c) \quad (4)$$

where T_c is chip duration, and $g_i(t)$ denotes the channel of user i, including the transmitter filter $\psi(t)$ (chip waveform), multipath channel $m_i(t)$, and the receiver filter $\psi^*(-t)$ given by

$$g_i(t) = \psi(t) \circledast m_i(t) \circledast \psi^*(-t) \quad (5)$$

where $\circledast$ denotes the continuous-time convolution operator, the superscript "$*$" denotes the complex conjugation, and $m_i(t)$ can be expressed as

$$m_i(t) = \sum_{k=1}^{M_i} B_{i,k} \delta(t - \tau_{i,k}) \quad (6)$$

in which $B_{i,k}$, $\tau_{i,k}$, and M_i are the amplitude of the kth path, propagation delay of the kth path and number of paths, respectively, of user i.

The discrete-time signal $y[n]$ can be obtained by sampling $y(t)$ with sampling interval T_c as follows:

$$\begin{aligned} y[n] &= y(nT_c) \\ &= \sum_{i=1}^{K} y_i[n] + w[n] \\ &= \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} u_i[k] h_i[n - kP] + w[n] \end{aligned} \quad (7)$$

where $y_i[n] = y_i(nT_c)$, $w[n] = w(nT_c)$, and $h_i[n]$ is the signature waveform of user i given by

$$h_i[n] = c_i[n] * g_i[n] = \sum_{k=0}^{P-1} c_i[k] g_i[n - k] \quad (8)$$

where the discrete-time multipath channel $g_i[n]$ is given by

$$g_i[n] = g_i(nT_c). \quad (9)$$

Discrete-time MIMO model for $y[n]$ can be obtained through polyphase decomposition [2], [10], [12], [13]. Let

$$\begin{aligned} y^{(j)}[n] &= y[nP + j - 1] \\ y_i^{(j)}[n] &= y_i[nP + j - 1] \\ w^{(j)}[n] &= w[nP + j - 1] \end{aligned}$$

be type I polyphase components of $y[n]$, $y_i[n]$ and $w[n]$, respectively. Let

$$\mathbf{y}[n] = \left(y^{(1)}[n], y^{(2)}[n], \ldots, y^{(P)}[n]\right)^T = (y[nP], y[nP+1], \ldots, y[nP+P-1])^T \quad (10)$$

$$\mathbf{y}_i[n] = \left(y_i^{(1)}[n], y_i^{(2)}[n], \ldots, y_i^{(P)}[n]\right)^T = (y_i[nP], y_i[nP+1], \ldots, y_i[nP+P-1])^T \quad (11)$$

$$\mathbf{u}[n] = (u_1[n], u_2[n], \ldots, u_K[n])^T \quad (12)$$

$$\mathbf{w}[n] = \left(w^{(1)}[n], w^{(2)}[n], \ldots, w^{(P)}[n]\right)^T = (w[nP], w[nP+1], \ldots, w[nP+P-1])^T. \quad (13)$$

Note that $\mathbf{w}[n]$ is a white Gaussian vector random process. Then, the received signal $y[n]$ given by (7) can be expressed as

$$\mathbf{y}[n] = \sum_{i=1}^{K} \mathbf{y}_i[n] + \mathbf{w}[n] \quad (14)$$

$$= \mathbf{H}[n] * \mathbf{u}[n] + \mathbf{w}[n] = \sum_{k=-\infty}^{\infty} \mathbf{H}[k]\mathbf{u}[n-k] + \mathbf{w}[n] \quad (15)$$

where $\mathbf{H}[n]$ is a $P \times K$ impulse response matrix with the (i, k)th entry equal to

$$\mathrm{h}_k^{(i)}[n] = h_k[nP+i-1] \quad (16)$$

and

$$\mathbf{y}_i[n] = \mathbf{h}_i[n] * u_i[n] = \sum_{k=-\infty}^{\infty} \mathbf{h}_i[k]u_i[n-k] \quad (17)$$

in which

$$\mathbf{h}_i[n] = \left(\mathrm{h}_i^{(1)}[n], \mathrm{h}_i^{(2)}[n], \ldots, \mathrm{h}_i^{(P)}[n]\right)^T \quad (18)$$

is the ith column of $\mathbf{H}[n]$. Next, let us present IFC for estimating $\mathbf{u}[n]$ with $\mathbf{y}[n]$.

III. IFC FOR DS/CDMA SYSTEMS

For ease of later use, let cum$\{x_1, x_2, \ldots, x_M\}$ denote the joint cumulant of random variables $x_1, x_2, \ldots, x_M$, and

$$\text{cum}\{x{:}\, p, \ldots\} = \text{cum}\{x_1 = x, x_2 = x, \ldots, x_p = x, \ldots\}$$

$$C_{p,q}\{x\} = \text{cum}\{x{:}\, p, x^*: q\}$$

$\|\mathbf{A}\|(\|\mathbf{a}\|)$: Euclidean-norm of matrix $\mathbf{A}$ (vector $\mathbf{a}$)

$\mathbf{A}(z)(\mathbf{a}(z))$: z-transform of matrix $\mathbf{A}[n]$ (vector $\mathbf{a}[n]$)

$$\mathbf{1}_{n,k} = (\xi_1 = 0, \ldots, \xi_{k-1} = 0, \xi_k = 1, \xi_{k+1} = 0, \ldots, \xi_n = 0)^T, \quad 1 \le k \le n$$

$$\mathcal{E}\{\mathbf{b}[n]\} = \sum_n \|\mathbf{b}[n]\|^2.$$

Assume that we are given a set of measurements $\mathbf{y}[n]$, $n = 0, 1, \ldots, N-1$ given by (15) under the following assumptions:

A1) $u_i[n]$ is zero-mean, i.i.d., non-Gaussian and statistically independent of $u_k[n]$ for all $k \neq i$, and

$$\kappa_i(p, q) = \frac{C_{p,q}\{u_i[n]\}}{|C_{1,1}\{u_i[n]\}|^{(p+q)/2}} \neq 0 \quad (19)$$

for a chosen (p, q), where p and q are non-negative integers, and $p + q \ge 3$.

A2) The MIMO system $\mathbf{H}[n]$ is exponentially stable, i.e., $\|\mathbf{H}[n]\| < a\beta^{|n|}$ for some $0 < a < \infty$ and $0 < \beta < 1$.

Assume that $\mathbf{v}[n] = (v^{(1)}[n], v^{(2)}[n], \ldots, v^{(P)}[n])^T$ ($P \times 1$ vector) is an FIR inverse filter with region of support $[L_1, L_2]$ (i.e., $\mathbf{v}[n] \neq (\mathbf{0})$ for $n = L_1, L_1+1, \ldots, L_2$). Then, the inverse filter output $e[n]$ can be expressed as

$$\begin{aligned} e[n] &= \mathbf{v}^T[n] * \mathbf{y}[n] \\ &= \sum_{i=1}^{P} v^{(i)}[n] * y^{(i)}[n] \\ &= \sum_{i=1}^{P} \sum_{k=L_1}^{L_2} v^{(i)}[k] y[(n-k)P + i - 1] \\ &= \sum_{k=0}^{q_v} v[k] y[(n-L_1)P + P - 1 - k] \\ &= \boldsymbol{\nu}^T \boldsymbol{y}[n] \end{aligned} \quad (20)$$

where

$$q_v = (L_2 - L_1 + 1)P - 1 \quad (21)$$

$$\boldsymbol{\nu} = (v[0], v[1], \ldots, v[q_v])^T = \left(v^{(P)}[L_1], v^{(P-1)}[L_1], \ldots, v^{(1)}[L_1], \ldots, v^{(P)}[L_2] \; v^{(P-1)}[L_2], \ldots, v^{(1)}[L_2]\right)^T \quad (22)$$

$$\boldsymbol{y}[n] = (y[(n-L_1)P + P - 1], y[(n-L_1)P + P - 2] \; \ldots, y[(n-L_1)P + P - 1 - q_v])^T. \quad (23)$$

Note that (20) is nothing but the type II polyphase decomposition for $e[n]$ and is referred to as multirate convolution [11].

By (15) and (20), $e[n]$ can also be expressed as

$$e[n] = \mathbf{f}^T[n] * \mathbf{u}[n] + \mathbf{v}^T[n] * \mathbf{w}[n] \quad (24)$$

where

$$\mathbf{f}^T[n] = (f_1[n], \ldots, f_K[n]) = \mathbf{v}^T[n] * \mathbf{H}[n] \quad (25)$$

is the overall (multi-input single-output) channel from $\mathbf{u}[n]$ to $e[n]$.

A. IFC

A family of IFC is defined as

$$J_{p,q}(\boldsymbol{\nu}) = \frac{|C_{p,q}\{e[n]\}|}{|C_{1,1}\{e[n]\}|^{(p+q)/2}} \quad (26)$$

where p and q are non-negative integers, and $p + q > 2$. Tugnait [22] finds the optimum $\boldsymbol{\nu}$ by maximizing IFC $J_{p,q}(\boldsymbol{\nu})$ for

$(p, q) = (2, 1)$ and $(p, q) = (2, 2)$ based on the following theorem.

Theorem 1 [22]: Assume that $\mathbf{y}[n]$ is the output of an MIMO system given by (15) in the absence of noise ($\mathbf{w}[n] = \mathbf{0}$) under the assumptions A1) and A2). Then, $J_{p,q}(\boldsymbol{\nu})$ with $(p, q) = (2, 1)$ or $(p, q) = (2, 2)$ are maximum as $L_1 \to -\infty$ and $L_2 \to \infty$ if and only if $\mathbf{f}[n] = \alpha \cdot \mathbf{1}_{K,j} \cdot \delta[n - \tau]$, i.e.,

$$e[n] = \alpha u_j[n - \tau] \tag{27}$$

or $\max\{J_{p,q}(\boldsymbol{\nu})\} = |\kappa_j(p, q)|$, where

$$j = \arg\max_i \{|\kappa_i(p, q)|\} \tag{28}$$

and $\alpha \neq 0$ and τ (integer) are unknown scale factor and time delay, respectively. □

Chi and Chen [25] extended Theorem 1 [$(p, q) = (2, 1)$ and $(p, q) = (2, 2)$] for all the other admissible choices of p and q with $p + q > 2$. Theorem 1 also implies that as $|\kappa_i(p, q)| = \kappa$, $i = 1, 2, \ldots, K$, the optimum $e[n]$ can be the estimate of any one of the K inputs. In addition to the global optimum inverse filter associated with $(p, q) = (2, 1)$ and $(p, q) = (2, 2)$ presented in Theorem 1, among local maxima and minima (stationary points) of IFC $J_{p,q}(\boldsymbol{\nu})$ for $p + q > 2$, there are K stable local maxima as SNR is infinite, which are presented in the following theorem.

Theorem 2: Assume that $\mathbf{y}[n]$ is the output of an MIMO system given by (15) in the absence of noise ($\mathbf{w}[n] = \mathbf{0}$) under the assumptions A1), A2), $L_1 \to -\infty$, and $L_2 \to \infty$. There are K stable local maxima for $J_{p,q}(\boldsymbol{\nu})$, where each is associated with an $\mathbf{f}[n] = \alpha_j \cdot \mathbf{1}_{K,j} \cdot \delta[n - \tau_j]$, $j \in \{1, 2, \ldots, K\}$, and the other local maxima are unstable equilibria for the following cases.

C1) $p + q > 2$ as $\mathbf{y}[n]$ is real.

C2) $p = q \geq 2$ as $\mathbf{y}[n]$ is complex.

C3) $p + q > 2$, $p \neq q$, $C_{1,1}\{u_l[n]\} = \sigma_u^2$, and $C_{p,q}\{u_l[n]\} = \gamma \neq 0$, $l = 1, 2, \ldots, K$ as $\mathbf{y}[n]$ is complex. □

The proof of Theorem 2 is given in Appendix A, which needs the following lemma in the proof.

Lemma 1 [30]: Assume that $r > 1$. Then

$$(\epsilon - \varepsilon)^r + (\epsilon + \varepsilon)^r > 2\epsilon^r, \qquad \text{for } 0 < \varepsilon < \epsilon \tag{29}$$

$$(\epsilon - \varepsilon)^r + \varepsilon^r < \epsilon^r, \qquad \text{for } 0 < \varepsilon < \epsilon/2. \tag{30}$$

Because $J_{p,q}(\boldsymbol{\nu})$ is a highly nonlinear function of $\boldsymbol{\nu}$, one has to resort to iterative gradient-type optimization algorithms that, initialized by a chosen $\boldsymbol{\nu} = \boldsymbol{\nu}^{(0)}$, can only find a stable local optimum $\boldsymbol{\nu}$ associated with one input signal $u_{j_0}[n]$, $j_0 \in \{1, 2, \ldots, K\}$ by Theorem 2.

Chi and Chen [26] proved the fact that as SNR is finite, the optimum inverse filter $\boldsymbol{\nu}$ by maximizing $J_{p,q}(\boldsymbol{\nu})$ is the same as that obtained by the SEA [27] for the cases C1) ($p + q > 2$ as $\mathbf{y}[n]$ is real) and C2) ($p = q \geq 2$ as $\mathbf{y}[n]$ is complex) in Theorem 2. Based on this fact, they proposed a fast iterative gradient-type algorithm [26, Alg. 2] with guaranteed convergence that basically updates $\boldsymbol{\nu}$ at the ith iteration by solving the same set of linear equations (formed of correlations of $\mathbf{y}[n]$ and cross cumulants of $\mathbf{y}[n]$ and the inverse filter output $e[n]$ obtained at the $(i - 1)$th iteration), as used by the SEA. Therefore, the former converges almost as fast as the latter with similar computational load. Moreover, three worthy remarks regarding the use of the IFC $J_{p,q}(\boldsymbol{\nu})$ are as follows.

R1) All of the input signal estimates can be obtained through using a K-stage successive cancellation procedure [22], [27], [29], which will be presented in Section III-B. At each stage, one input estimate is obtained using an iterative gradient-type optimization algorithm such as Chi and Chen's fast algorithm [26, Alg. 2].

R2) The phase ambiguity in the estimated input signal resultant from the complex scale factor α [see (27)] can be overcome using differential coding techniques such as the differential phase shift keying (DPSK) modulation scheme [3] that encodes the information of phase differences between successive symbol transmissions.

R3) $J_{2,2}(\boldsymbol{\nu})$ is mainly used in digital communications because $\kappa_i(p, q) \neq 0$, $i = 1, 2, \ldots, K$ for $(p, q) = (2, 2)$, whereas $\kappa_i(p, q) = 0$ for $p + q = 3$ in most situations. Other choices of p and q are possible in some signal processing areas such as blind source separation [31] as long as $\kappa_i(p, q) \neq 0$ and $p + q > 2$. However, in practice, $C_{p,q}\{e[n]\}$ and $C_{1,1}\{e[n]\}$ needed by $J_{p,q}(\boldsymbol{\nu})$ must be replaced by the associated sample averages obtained from the given finite data. The smaller the $p + q$, the smaller the variance of the sample average associated with $C_{p,q}\{e[n]\}$ and, therefore, the better the performance of IFC in general. Moreover, the computational complexity of IFC is lower for smaller $p + q$. Therefore, among all admissible choices of (p, q), the one with the smallest $p + q$ is suggested.

B. MSC Procedure [22], [27], [29]

This subsection presents how to extract all the input signals using the IFC. With the obtained inverse filter output $e[n]$ using $J_{p,q}(\boldsymbol{\nu})$, $\mathbf{h}_j[k]$ can be estimated as [22], [27], [29]

$$\hat{\mathbf{h}}_j[k] = \frac{E\{\mathbf{y}[n]e^*[n - k]\}}{E\{|e[n]|^2\}}. \tag{31}$$

Substituting (27) into (31) yields

$$\hat{\mathbf{h}}_j[n] = \frac{1}{\alpha}\mathbf{h}_j[n + \tau] \qquad \text{as } \mathbf{w}[n] = \mathbf{0}. \tag{32}$$

Therefore, the contribution in $\mathbf{y}[n]$ due to $u_j[n]$ can be estimated as

$$\hat{\mathbf{y}}_j[n] = \hat{\mathbf{h}}_j[n] * e[n] \quad \text{[by (17)]} \tag{33}$$

$$= \hat{\mathbf{h}}_j[n] * \hat{\mathbf{v}}^T[n] * \mathbf{y}[n] \tag{34}$$

$$= \mathbf{h}_j[n] * u_j[n] \quad \text{as } \mathbf{w}[n] = \mathbf{0}. \tag{35}$$

Canceling $\hat{\mathbf{y}}_j[n]$ from the data $\mathbf{y}[n]$ yields

$$\tilde{\mathbf{y}}[n] = \mathbf{y}[n] - \hat{\mathbf{y}}_j[n] \quad \text{[by (14)]} \tag{36}$$

$$= \mathbf{y}[n] - \hat{\mathbf{h}}_j[n] * \hat{\mathbf{v}}^T[n] * \mathbf{y}[n] \tag{37}$$

that corresponds to the outputs of a $P \times (K-1)$ system driven by $(K-1)$ inputs $u_i[n]$, $i = 1, \ldots, j-1, j+1, \ldots, K$.

The IFC $J_{p,q}(\boldsymbol{\nu})$ given by (26) can be employed to estimate $\{u_i[n], i = 1, 2, \ldots, K\}$ through an MSC procedure [22], [27], [29] with the following signal processing procedure at the lth stage.

- S1) Find a local optimum $\boldsymbol{\nu}$ (and $\hat{\mathbf{v}}[n]$) of $J_{p,q}(\boldsymbol{\nu})$ using an iterative gradient-type optimization algorithm and the associated $e_l[n] = e[n]$ and $\overline{\mathbf{h}}_l[n] = \hat{\mathbf{h}}_j[n]$ (jth column of $\mathbf{H}[n]$).
- S2) Update $\mathbf{y}[n]$ by $\tilde{\mathbf{y}}[n]$ given by (37).

Two worthy remarks regarding the MSC procedure are as follows.

- R4) The obtained $\{e_l[n], \overline{\mathbf{h}}_l[n]\}$, $l = 1, 2, \ldots, K$ are estimates of $\{u_j[n], \mathbf{h}_j[n]\}$, $j \in \{1, 2, \ldots, K\}$ up to an unknown scale factor and an unknown time delay. Estimates $\hat{u}_i[n]$ with higher signal power E_i (strong users) defined as [11]
$$E_i = \frac{E\left\{\|\mathbf{y}_i[n]\|^2\right\}}{P} \tag{38}$$
are usually obtained prior to those with lower E_i (weak users).
- R5) Imperfect cancellation in S2) results in error propagation accumulated in the ensuing stages. Moreover, it is possible that two input estimates, said to be $e_{l_1}[n]$ and $e_{l_2}[n]$ ($l_2 > l_1$), are associated with the same (unknown) user j due to error propagation, whereas $e_{l_1}[n]$ is the more reliable estimate, and thus, the redundant $e_{l_2}[n]$ can be ignored.

C. Initial Condition for IFC

Assume that the user of interest is user 1; it is preferred that $u_1[n]$ is estimated at an early stage of the MSC procedure due to error propagation effects [see R5)]. Next, let us present an initial condition $\boldsymbol{\nu}^{(0)}$ that is helpful for obtaining the estimate $\hat{u}_1[n]$ at an early stage of the MSC procedure.

Assume that each multipath channel $g_j[n]$ is an FIR channel of order equal to $q_g \le P$ that occurs in most asynchronous DS/CDMA channels [10], [13]. Let

$$\mathbf{C}_j = \begin{bmatrix} c_j[P-1] & \cdots & c_j[P-1-q_v] \\ \vdots & & \vdots \\ c_j[P-1-q_g] & \cdots & c_j[P-1-q_v-q_g] \\ \vdots & & \vdots \\ c_j[(\mathcal{L}+1)P-1] & \cdots & c_j[(\mathcal{L}+1)P-1-q_v] \\ \vdots & & \vdots \\ c_j[(\mathcal{L}+1)P-1-q_g] & \cdots & c_j[(\mathcal{L}+1)P-1-q_v-q_g] \end{bmatrix} \tag{39}$$

where $\mathcal{L} = \lfloor (q_v + q_g)/P \rfloor$, and $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. Note that matrix $\mathbf{C}_j$ is of full rank if $q_g < P$ [13].

Tsatsanis [12] proposed a constrained criterion for estimating $u_1[n]$ that minimizes the mean-output-energy given by

$$J_{\text{MOE}}(\boldsymbol{\nu}) = E\left\{|e[n]|^2\right\} \tag{40}$$

subject to

$$f_1[n] = g_1[k_0] \cdot \delta[n - n_0] \tag{41}$$

which is equivalent to the following linear equations:

$$\mathbf{C}_1 \boldsymbol{\nu} = \mathbf{1}_{(\mathcal{L}+1)\cdot(q_g+1),\, n_0\cdot(q_g+1)+k_0+1} \tag{42}$$

where $0 \le n_0 \le \mathcal{L}$, and $0 \le k_0 \le q_g$. Remark that $g_1[k_0] \ne 0$ is required, i.e., a path of the desired user must be known in advance and that a distortionless signal $g_1[k_0] \cdot u_1[n - n_0]$ of $u_1[n]$ always exists in the MOE equalizer output, regardless of interference signals and noise.

For the suppression of interference signals, Schodorf and Williams [32] proposed a constrained criterion by minimizing $J_{\text{MOE}}(\boldsymbol{\nu})$ for blind deconvolution of synchronous CDMA channels in the absence of multipath subject to the following decorrelating constraint:

$$\mathbf{f}[n] = \mathbf{1}_{K,1} \cdot \delta[n]. \tag{43}$$

Li and Fan [33] proposed a constrained constant modulus algorithm (CMA) that minimizes the following cost function:

$$J_{\text{CMA}}(\boldsymbol{\nu}) = \tfrac{1}{4} E\left\{\left(|e[n]|^2 - \zeta\right)^2\right\} \tag{44}$$

where ζ is the constant modulus of $u_1[n]$ subject to

$$f_1[n_0] = 1, \qquad n_0 \in \{0, 1, \ldots, \mathcal{L}\}. \tag{45}$$

To obtain the optimum causal inverse filter (i.e., $L_1 = 0$), Tugnait and Li [34] proposed a constrained IFC by maximizing $J_{2,2}(\boldsymbol{\nu})$ subject to

$$\mathcal{U}^H \mathcal{T} \mathcal{R}_{yy} \tilde{\boldsymbol{\nu}}^* = \mathbf{0} \tag{46}$$

where the superscript "H" denotes complex conjugate transpose, $\mathcal{U}$ is a $P(L_2+1) \times (P(L_2+1) - 2P)$ matrix whose columns are an orthonormal basis for the orthogonal complement of a $P(L_2+1) \times 2P$ matrix formed by $c_1[n]$, $\mathcal{T}$ is a $P(L_2+1) \times P(L_2+1)$ permutation matrix, $\mathcal{R}_{yy}$ is an $(L_2+1) \times (L_2+1)$ block matrix with (i, j)th block element equal to $E\{\mathbf{y}[n+j-i]\mathbf{y}^H[n]\}$, $\mathbf{0}$ is a $(P(L_2+1) - 2P) \times 1$ zero vector, and

$$\tilde{\boldsymbol{\nu}} = \left(\mathbf{v}^T[0], \mathbf{v}^T[1], \ldots, \mathbf{v}^T[L_2]\right)^T. \tag{47}$$

Tugnait and Li [35] also proposed a constrained CMA by minimizing $J_{\text{CMA}}(\boldsymbol{\nu})$ with $\zeta = 1$ subject to the constraint (46).

The optimum equalizers reported in [33] and those reported in [34] and [35] are obtained by an iterative projection stochastic gradient algorithm, whereas the latter two are further used as the initial conditions of the unconstrained $J_{2,2}(\boldsymbol{\nu})$ and $J_{\text{CMA}}(\boldsymbol{\nu})$, respectively.

Consider the following decorrelating constraint:

$$\mathbf{f}[n] = \mathbf{1}_{K,1} \cdot g_1[k_0] \cdot \delta[n - n_0] \tag{48}$$

which is equivalent to the following linear equations:

$$\begin{aligned} \mathcal{C}\boldsymbol{\nu} &= \left[\mathbf{1}^T_{(\mathcal{L}+1)\cdot(q_g+1),\, n_0\cdot(q_g+1)+k_0+1} \mathbf{0}_1^T \cdots \mathbf{0}_{K-1}^T\right]^T \\ &= \mathbf{1}_{(\mathcal{L}+1)\cdot(q_g+1)\cdot K,\, n_0\cdot(q_g+1)+k_0+1} \end{aligned} \tag{49}$$

where $\mathbf{0}_i$, $i = 1, 2, \ldots, K-1$ are $(\mathcal{L}+1)\cdot(q_g+1)\times 1$ zero vectors, and

$$\boldsymbol{\mathcal{C}} = \left[\mathbf{C}_1^T \mathbf{C}_2^T \cdots \mathbf{C}_K^T\right]^T. \tag{50}$$

Then, the least squares (LS) solution for $\boldsymbol{\nu}$ by minimizing

$$\begin{aligned}&\mathcal{E}\{\mathbf{f}[n] - \mathbf{1}_{K,1}\cdot g_1[k_0]\cdot\delta[n-n_0]\}\\ &\qquad = \left\|\boldsymbol{\mathcal{C}}\boldsymbol{\nu} - \mathbf{1}_{(\mathcal{L}+1)\cdot(q_g+1)\cdot K,\, n_0\cdot(q_g+1)+k_0+1}\right\|^2\end{aligned} \tag{51}$$

is given by

$$\boldsymbol{\nu}_{\rm LS}(n_0, k_0) = \boldsymbol{\mathcal{C}}^+ \cdot \mathbf{1}_{(\mathcal{L}+1)\cdot(q_g+1)\cdot K,\, n_0\cdot(q_g+1)+k_0+1} \tag{52}$$

where $\boldsymbol{\mathcal{C}}^+$ is the Moore–Penrose pseudo inverse of $\boldsymbol{\mathcal{C}}$. SVD is an efficient approach for obtaining $\boldsymbol{\nu}_{\rm LS}(n_0, k_0)$. When $\boldsymbol{\mathcal{C}}$ is an overdetermined or exact system [i.e., $(\mathcal{L}+1)\cdot(q_g+1)\cdot K \geq q_v + 1$] and $\boldsymbol{\mathcal{C}}$ is of full column rank, $\boldsymbol{\mathcal{C}}^+ = (\boldsymbol{\mathcal{C}}^H\boldsymbol{\mathcal{C}})^{-1}\boldsymbol{\mathcal{C}}^H$, and $\boldsymbol{\nu}_{\rm LS}(n_0, k_0)$ is unique. Otherwise, $\boldsymbol{\nu}_{\rm LS}(n_0, k_0)$ is the minimum norm LS solution.

The initial condition $\boldsymbol{\nu}^{(0)}$ for IFC $J_{p,q}(\boldsymbol{\nu})$ is suggested as follows:

$$\boldsymbol{\nu}^{(0)} = \boldsymbol{\nu}_{\rm LS}\left(\tilde{n}_0, \tilde{k}_0\right) \tag{53}$$

where

$$\left(\tilde{n}_0, \tilde{k}_0\right) = \underset{(n_0\in\mathcal{N}_0,\, k_0\in\mathcal{K}_0)}{\arg\max}\{J_{p,q}(\boldsymbol{\nu}_{\rm LS}(n_0, k_0)) \leq |\kappa_1(p, q)|\} \tag{54}$$

in which $\mathcal{N}_0 \subseteq \{0, 1, \ldots, \mathcal{L}\}$, and $\mathcal{K}_0 \subseteq \{0, 1, \ldots, q_g\}$. A worthy remark regarding the proposed initial condition (53) is as follows.

R6) The $\boldsymbol{\nu}^{(0)}$ given by (53) not only minimizes the error squares of the decorrelating constraint (48) [indexed by (n_0, k_0)] associated with the desired user (user 1) but also maximizes the associated $J_{p,q}$ [bounded by $|\kappa_1(p, q)|$] with respect to $(n_0, k_0) \in (\mathcal{N}_0, \mathcal{K}_0)$. Therefore, a transmission path with "large" magnitude [i.e., large $|g_1[\tilde{k}_0]|$ in $\mathbf{f}[n]$ given by (48)] associated with the desired user is detected without need of any prior multipath information of the desired user. Consequently, $\hat{u}_1[n] = e[n]$ by maximizing IFC $J_{p,q}(\boldsymbol{\nu})$ can always be obtained at an early stage of the MSC procedure, even when user 1 is a weak user, as long as the near–far ratios (NFRs) defined as

$$\mathrm{NFR}_i = E_i/E_1 \tag{55}$$

are not too high for all $i \neq 1$.

The output SINR, which is denoted by SINR_1, for user 1 has been a widely used performance measure of equalization algorithms in digital communications. The computation of SINR_1 for the obtained $\hat{u}_1[n]$ through the MSC procedure is summarized in Appendix B. Moreover, a UIA is needed to identify user number at each stage of the MSC procedure until $u_1[n]$ is estimated. Next, let us present a UIA for identifying $e_l[n]$ obtained at the lth stage of the MSC procedure with the associated user j (with signature sequence $c_j[n]$) and an MBEA for the MAI suppression and ISI removal of asynchronous DS/CDMA channels in the presence of multipath.

IV. User Identification and MBEA

First of all, let us present three facts on which the UIA to be presented below is based. The first fact is about the relation between the phase and higher order moments of a stable sequence. Let $a[n]$ (i.e., $\sum_n |a[n]| < \infty$) be a stable sequence with a certain amplitude spectrum $|A(\omega)|$. Define

$$\mathcal{F}(a[n]) = \int_{-\pi}^{\pi} |A(\omega)|\cdot[\phi_a(\omega)]^2\, d\omega, \tag{56}$$

$$\Lambda(a[n]) = \sum_{n=-\infty}^{\infty} |a[n]|^{2r} \tag{57}$$

where $\phi_a(\omega) = \arg\{A(\omega)\}$ with linear phase term removed [i.e., the linear term in the Taylor series expansion of $\arg\{A(\omega)\}$ is equal to zero], and $r \geq 2$. Note that

$$\Lambda(\alpha a[n-\tau]) = |\alpha|^{2r}\Lambda(a[n]) \tag{58}$$

implying that $\Lambda(a[n])$ is invariant for any linear phase change in $\arg\{A(\omega)\}$ as long as $|\alpha| = 1$. Chien *et al.* [38] have shown the following fact for real $a[n]$.

F1) The smaller the $\mathcal{F}(a[n])$, the larger the $\Lambda(a[n])$. In other words, $\Lambda(a[n])$ is maximum as $\phi_a(\omega) = 0$ for all ω.

Following the same procedure for proving F1) as presented in [38], one can easily show that F1) is also true if $a[n]$ is complex. The second fact regarding properties of signature sequences [1], [2] is as follows.

F2) Each signature sequence $c_k[n] \in \mathcal{R}$ is basically a pseudo-random (approximate allpass) sequence with approximate random phase and autocorrelation function $c_k[n] * c_k[-n] \simeq P\delta[n]$ (or $|c_k(\omega)|^2 \simeq P$) and uncorrelated with $c_i[n] \in \mathcal{R}$ for $i \neq k$. Moreover, $c_i[n] = c_k[n]$ if $\phi_{c_i}(\omega) = \phi_{c_k}(\omega)$ (with linear phase term removed).

The third fact is regarding an "entropy measure" of a stable nonzero sequence $b[n]$ defined as

$$\Gamma_{k,m}(b[n]) = \frac{\sum_{n=-\infty}^{\infty} |b[n]|^k}{\left(\sum_{n=-\infty}^{\infty} |b[n]|^m\right)^{k/m}}, \qquad 1 \leq m < k. \tag{59}$$

F3) $0 < \Gamma_{k,m}(b[n]) = \Gamma_{k,m}(\alpha b[n-\tau]) \leq 1$ and $\Gamma_{k,m}(\alpha\delta[n-\tau]) = 1$ (minimum entropy) for all $\alpha \neq 0$ and integer τ [36], [37]. The smaller the entropy of $b[n]$ (or the closer $b[n]$ to $\alpha\delta[n-\tau]$), the closer $\Gamma_{k,m}(b[n])$ to unity.

Next, let us present the UIA, its analysis, and the MBEA, respectively.

A. UIA

Assume that $e_l[n]$ and $\overline{\mathbf{h}}_l[n]$ are the optimum estimates of $\alpha_j u_j[n-\tau_j]$ and $\mathbf{h}_j[n+\tau_j]/\alpha_j$, respectively, which are obtained using the IFC $J_{p,q}(\boldsymbol{\nu})$ at the lth stage of the MSC procedure, where the user number j is unknown [see R4)]. Let $\overline{h}_l[n]$ be the (chip rate) signature waveform estimate associated with $\overline{\mathbf{h}}_l[n]$, i.e.,

$$\overline{h}_l[nP+i-1] = \overline{\mathrm{h}}_l^{(i)}[n] \quad \text{[by (16)]} \tag{60}$$

where $\overline{\mathrm{h}}_l^{(i)}[n]$ is the ith entry of $\overline{\mathbf{h}}_l[n]$. Therefore

$$\overline{h}_l[n] \simeq \frac{1}{\alpha_j} h_j[n+\tau_j P]. \tag{61}$$

Let

$$a_{l,k}[n] = \frac{\overline{h}_l[n]}{||\overline{h}_l[n]||} * c_k[-n], \qquad c_k[n] \in \mathcal{R} \tag{62}$$

where $||\overline{h}_l[n]||^2 = \mathcal{E}\{\overline{h}_l[n]\}$ denotes the energy of $\overline{h}_l[n]$. Then, $\mathcal{F}(a_{l,k}[n])$ [see (56)] can be expressed as

$$\begin{aligned}\mathcal{F}(a_{l,k}[n]) \simeq & \frac{1}{\alpha_j ||\overline{h}_l[n]||} \cdot \int_{-\pi}^{\pi} |G_j(\omega)| \\ & \cdot [\phi_{g_j}(\omega) + \phi_{c_j}(\omega) - \phi_{c_k}(\omega)]^2 \, d\omega \\ \simeq & \frac{1}{||h_j[n]||} \cdot (\mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3) \quad \text{[by (8) and (61)]}\end{aligned} \tag{63}$$

where

$$\mathcal{F}_1 = \int_{-\pi}^{\pi} |G_j(\omega)| \cdot [\phi_{c_j}(\omega) - \phi_{c_k}(\omega)]^2 \, d\omega \tag{64}$$

$$\mathcal{F}_2 = \int_{-\pi}^{\pi} |G_j(\omega)| \cdot [\phi_{g_j}(\omega)]^2 \, d\omega \tag{65}$$

$$\mathcal{F}_3 = 2\int_{-\pi}^{\pi} |G_j(\omega)| \cdot \phi_{g_j}(\omega) \cdot [\phi_{c_j}(\omega) - \phi_{c_k}(\omega)] \, d\omega. \tag{66}$$

Note that $\mathcal{F}_2$ is a constant [not function of $\phi_{c_k}(\omega)$] and that $|\mathcal{F}_3| \ll \mathcal{F}_1$ when $\phi_{c_j}(\omega) \neq \phi_{c_k}(\omega)$ due to approximate random phases $\phi_{c_j}(\omega)$ and $\phi_{c_k}(\omega)$ by F2). Therefore, $\mathcal{F}(a_{l,k}[n])$ is minimum and $\Lambda(a_{l,k}[n])$ is maximum when $c_k[n] = c_j[n]$ by F1), which suggests the following UIA.

UIA:

- Q1) Calculate $\Lambda(a_{l,k}[n]) \, \forall \, c_k[n] \in \mathcal{R}$ using (57) and (62).
- Q2) Identify $e_l[n]$ with $\hat{u}_j[n]$ where the user j (with the signature sequence $c_j[n]$) is decided by

$$j = \arg\max_k \{\Lambda(a_{l,k}[n]), \, \forall \, c_k[n] \in \mathcal{R}\}. \tag{67}$$

B. Analysis of the UIA

Because the identification of $e_l[n]$ with $\hat{u}_j[n]$ depends on the difference between $\Lambda(a_{l,j}[n])$ and $\Lambda(a_{l,k}[n])$ for all $k \neq j$, the larger the difference, the better the performance of the proposed UIA. Next, let us analyze how $\Lambda(a_{l,k}[n])$ for $k = j$ and $k \neq j$ depend on the spreading gain P and the multipath channel $g_j[n]$.

Substituting (61) into (62) yields

$$\begin{aligned}a_{l,k}[n] &= \frac{\overline{h}_l[n]}{||\overline{h}_l[n]||} * c_k[-n] \\ &\simeq \frac{h_j[n+\tau_j P]}{||h_j[n+\tau_j P]||} * c_k[-n] \quad \text{[by (61)]} \\ &= \frac{1}{||h_j[n]||} g_j[n+\tau_j P] * c_j[n] * c_k[-n] \quad \text{[by (8)]}.\end{aligned} \tag{68}$$

Furthermore, from F2) and (68)

$$a_{l,j}[n] \simeq \frac{P}{||h_j[n]||} g_j[n+\tau_j P] \tag{69}$$

which implies the following first-order approximation $\tilde{g}_j[n]$ to $g_j[n]$:

$$\tilde{g}_j[n] = a_{l,j}[n-\tau_j P] \tag{70}$$

up to a scale factor. Moreover, it can be easily shown from F2) and (8) that

$$||h_j[n]||^2 \simeq P||g_j[n]||^2 \tag{71}$$

and

$$||g_j[n] * c_j[n] * c_k[-n]||^2 \simeq P^2 ||g_j[n]||^2. \tag{72}$$

From (57), (59), (68), (71), and (72), it can be easily shown that

$$\Lambda(a_{l,k}[n]) \simeq \begin{cases} P^r \Gamma_{2r,2}(g_j[n] * c_j[n] * c_j[-n]), & k = j \\ P^r \Gamma_{2r,2}(g_j[n] * c_j[n] * c_k[-n]), & k \neq j \end{cases} \tag{73}$$

$$\simeq \begin{cases} P^r \Gamma_{2r,2}(g_j[n]), & k = j \\ P^r \Gamma_{2r,2}(g_j[n] * c_j[n] * c_k[-n]), & k \neq j \end{cases} \tag{74}$$

where we have used the fact that $g_j[n]*c_j[n]*c_j[-n] \simeq P g_j[n]$ by F2) in the derivation of (74). One can easily infer, from (74), F2) and F3), that $\Lambda(a_{l,k}[n]) \ll \Lambda(a_{l,j}[n])$ for $k \neq j$ because the entropy of $g_j[n]$ is much smaller than that of $g_j[n] * c_j[n] * c_k[-n]$. Therefore, we have the following remark.

- R7) The performance of the proposed UIA is better for larger P and $\Gamma_{2r,2}(g_j[n])$. Because the proposed UIA employs the characteristics of pseudo-random codes [see F2)], it cannot be applied as nonpseudo-random codes are used.

C. MBEA

Assume that the user of interest is user 1. We would like to estimate $u_1[n]$ using the IFC through the MSC procedure. The proposed MBEA includes the following four steps:

MBEA:

- V1) Set $\ell = 1$ (stage number).
- V2) Find a local optimum $\boldsymbol{\nu}$ (and $\hat{\mathbf{v}}[n]$) of $J_{p,q}(\boldsymbol{\nu})$ using an iterative gradient-type optimization algorithm with the initial condition $\boldsymbol{\nu}^{(0)}$ given by (53), and the associated $e_l[n] = e[n]$ and $\overline{\mathbf{h}}_l[n]$ [see (31)].
- V3) Update $\mathbf{y}[n]$ by $\tilde{\mathbf{y}}[n] = \mathbf{y}[n] - \overline{\mathbf{h}}_l[n] * \hat{\mathbf{v}}^T[n] * \mathbf{y}[n]$ [see (37)].
- V4) Decide the user number j using the proposed UIA. If $j \neq 1$, update ℓ by $\ell + 1$ and $\mathcal{C}$ [given by (50)] by the one with $\mathbf{C}_j$ removed and then go to (V2); otherwise, $\hat{u}_1[n] = e_\ell[n]$ has been obtained at the stage ℓ.

Three remarks regarding the proposed MBEA are worth mentioning.

- R8) The obtained $\hat{u}_1[n]$ is free from error propagation as $\ell = 1$. As the power of user 1 is sufficient, $\hat{u}_1[n]$ can always be obtained for $\ell = 1$ due to the proposed initial condition $\boldsymbol{\nu}^{(0)}$ given by (53) used in V2) [see

R6)]. However, as $\text{NFR}_i = E_i/E_1$ [see (55)] for all $i \neq 1$ are high (the desired user 1 is a weak user), it may happen that $\ell > 1$. The smaller the ℓ, the more accurate the obtained estimate $\hat{u}_1[n]$. In other words, ℓ also provides some information for power control, i.e., higher demand of raising the power of the desired user for larger ℓ.

R9) As the data length N is not sufficient (i.e., small N) for reliably estimating $C_{p,q}\{e[n]\}$, the resultant maximum $J_{p,q}(\boldsymbol{\nu}) > |\kappa_1(p,q)|$ for the chosen $(\tilde{n}_0, \tilde{k}_0)$ may happen in step V2). One can remove $\tilde{n}_0$ and $\tilde{k}_0$ from the set $\mathcal{N}_0$ and the set $\mathcal{K}_0$, respectively, and then repeat V2) until the resultant maximum $J_{p,q}(\boldsymbol{\nu}) \leq |\kappa_1(p,q)|$.

R10) As $p + q \geq 3$ for real $\mathbf{y}[n]$ [i.e., case C1) in Theorem 2] and $p = q \geq 2$ for complex $\mathbf{y}[n]$ [i.e., case C2) in Theorem 2], Chi and Chen's iterative algorithm [26, Alg. 2] is suggested for obtaining a local optimum $\boldsymbol{\nu}$ in V2) that, as mentioned in Section III-A, is a fast gradient-type IFC algorithm with convergence speed and computational load similar to those of the SEA [27] and with guaranteed convergence. As reported in [26], this algorithm was employed to process the output signal of a discrete-time MIMO model obtained from signature waveform matched filter output signals, assuming at least one path delay $\tau_{i,k}$ for each user known in advance, whereas the proposed MBEA processes the chip waveform matched filter output signal $y[n]$ given by (7) with no need of path delay information. The desired input estimates $\hat{u}_1[n]$ in [26] is usually obtained at the first stage of the MSC procedure due to a specific initial condition used for the inverse filter, whereas user identification is never involved.

Next, let us present some simulation results to support the proposed UIA and MBEA, respectively.

V. Simulation Results

Two simulation examples [one for a six-user ($K = 6$) and the other for a three-user ($K = 3$) asynchronous DS/CDMA system of three paths associated with each user (i.e., $M_i = 3, \forall i$)] are to be presented. Spreading sequences $c_i[n]$ were taken from a set of $P + 2$ Gold codes of length $P = 31$ and $P = 15$ for the two respective examples, and the rectangular chip waveform

$$\psi(t) = \begin{cases} \sqrt{\dfrac{1}{T_c}}, & 0 \leq t < T_c \\ 0, & \text{otherwise} \end{cases} \tag{75}$$

was used in the two examples.

The proposed MBEA with $(p, q) = (2, 2)$ was employed for estimating $u_1[n]$ (i.e., the desired user is user 1) where Chi and Chen's fast iterative algorithm [26, Alg. 2] was used in V2), and the proposed UIA with $r = 2$ was used in V4). The (symbol rate) signature waveform estimate $\hat{\mathbf{h}}_j[n]$ was obtained by (31) for $-6 \leq n \leq 6$ in V2) of the MBEA.

For comparison, Tsatsanis and Xu's blind MV receiver [13] was also employed for each simulation case. The MV receiver estimates $u_1[n]$ (user of interest) by

$$\hat{u}_1[n] = \mathbf{v}_{\text{MV}}^H \overline{\boldsymbol{y}}[n] \tag{76}$$

where

$$\overline{\boldsymbol{y}}[n] = (y[nP],\, y[nP+1],\, \ldots,\, y[nP+P+q_g-1])^T \tag{77}$$

and

$$\mathbf{v}_{\text{MV}} = \overline{\mathbf{R}}^{-1}\overline{\mathbf{C}}_1\left(\overline{\mathbf{C}}_1^H \overline{\mathbf{R}}^{-1}\overline{\mathbf{C}}_1\right)^{-1}\hat{\mathbf{g}}_1 \tag{78}$$

in which $\overline{\mathbf{R}} = E\{\overline{\boldsymbol{y}}[n]\overline{\boldsymbol{y}}^H[n]\}$, $\hat{\mathbf{g}}_1$ is an estimate of $\mathbf{g}_1 = (g_1[0], g_1[1], \ldots, g_1[q_g])^T$ obtained as the eigenvector of $\overline{\mathbf{C}}_1^H \overline{\mathbf{R}}^{-1}\overline{\mathbf{C}}_1$ associated with the smallest eigenvalue μ, and

$$\overline{\mathbf{C}}_1 = \begin{bmatrix} c_1[0] & 0 & \cdots & 0 & 0 \\ c_1[1] & c_1[0] & \ddots & \vdots & 0 \\ \vdots & c_1[1] & \ddots & 0 & \vdots \\ c_1[P-1] & \vdots & \ddots & c_1[0] & 0 \\ 0 & c_1[P-1] & & c_1[1] & c_1[0] \\ 0 & 0 & \ddots & \vdots & c_1[1] \\ \vdots & \vdots & \ddots & c_1[P-1] & \vdots \\ 0 & 0 & \cdots & 0 & c_1[P-1] \end{bmatrix}. \tag{79}$$

Moreover, it has been shown in [13] that

$$\text{SINR}_1 = \frac{1}{\frac{1}{\mu \cdot C_{1,1}\{u_1[n]\} \cdot \|\hat{\mathbf{g}}_1^H \mathbf{g}_1\|^2} - 1} \tag{80}$$

where $\hat{\mathbf{g}}_1$ is normalized by $\|\hat{\mathbf{g}}_1\|^2 = 1$. A remark is worth mentioning as follows.

R11) The SINR_1 given by (80) (the limiting performance of the actual MV receiver) is true only as $N \to \infty$. In practice, all the statistics such as $\overline{\mathbf{R}}$ used by the MV receiver, and $C_{p,q}\{e[n]\}$ and $C_{1,1}\{e[n]\}$ used by the IFC, must be estimated from finite data (finite N) or replaced by the associated time averages. Therefore, for finite N, the SINR_1 of the actual MV receiver that depends on the estimated $\mathbf{v}_{\text{MV}}$ may deviate much from the theoretical SINR_1 given by (80).

Let us briefly present how to compute the SINR_1 of the actual MV receiver for finite data. Assume that $\hat{\mathbf{v}}_{\text{MV}}$ is obtained using (78) with $\overline{\mathbf{R}}$ replaced by $(1/N)\sum_{n=0}^{N-1}\overline{\boldsymbol{y}}[n]\overline{\boldsymbol{y}}^H[n]$. Then, it can be shown that the actual MV receiver output $\hat{u}_1[n]$ can be expressed as

$$\hat{u}_1(z) = \hat{\mathbf{v}}_{\text{MV}}^H \cdot \overline{\boldsymbol{y}}(z) = \tilde{\mathbf{f}}^T(z) \cdot \mathbf{u}(z) + \tilde{w}(z) \tag{81}$$

where

$$\tilde{\mathbf{f}}^T(z) = \hat{\mathbf{v}}_{\text{MV}}^H\left[\mathbf{H}^T(z) | z \cdot \mathbf{H}_{\text{t}}^T(z)\right]^T \tag{82}$$

$$\tilde{w}(z) = \hat{\mathbf{v}}_{\text{MV}}^H\left[\mathbf{w}^T(z) | z \cdot \mathbf{w}_{\text{t}}^T(z)\right]^T \tag{83}$$

where the matrix $\mathbf{H}_t(z)$ and the vector $\mathbf{w}_t(z)$ consist of the first q_g rows of $\mathbf{H}(z)$ and the first q_g components of $\mathbf{w}(z)$, respectively. The SINR_1 of the actual MV receiver can be obtained by (B.6) (see Appendix B) with $\overline{\mathbf{f}}_\ell(z)$ and $\overline{w}_\ell(z)$ replaced by $\tilde{\mathbf{f}}(z)$ and $\tilde{w}(z)$, respectively. Next, let us present Example 1.

Example 1—Six-User ($K = 6$) Case: In the example, each multipath channel $g_i[n]$ was generated using (5) and (9) with $B_{i,k} = [\boldsymbol{B}]_{i,k}$ and $\tau_{i,k} = [\boldsymbol{T}]_{i,k} T_c$, where

$$\boldsymbol{B}^T = \begin{bmatrix} 0.25 & -1.1 & 1.37 & 0.2 & -1.3 & 1.2 \\ -1.5 & 0.7 & -0.5 & -1.3 & 0.7 & -0.8 \\ -0.5 & -0.5 & 0.75 & 0.5 & -0.5 & 0.6 \end{bmatrix} \quad (84)$$

$$\boldsymbol{T}^T = \begin{bmatrix} 3.2 & 4.9 & 6.7 & 6.0 & 6.9 & 6.4 \\ 4.1 & 5.8 & 7.6 & 6.9 & 7.8 & 7.3 \\ 5.5 & 6.5 & 8.4 & 8.2 & 8.6 & 8.1 \end{bmatrix}. \quad (85)$$

Assuming that $E_1 \leq E_2 = E_3 = E_4 = E_5 = E_6$, where E_i is the signal power of user i defined by (38), i.e., $\text{NFR}_i = \text{NFR}$, $i = 2, 3, \ldots, 6$, measurements $\mathbf{y}[n]$ of length $N = 2500$ were generated using (15) with input signal $u_i[n] = A_i \xi_i[n]$, where A_i is the amplitude of $u_i[n]$, and $\xi_i[n]$ is an equiprobable i.i.d. binary random sequence of $\{+1, -1\}$ with $|\kappa_i(2, 2)| = 2$.

In V2) of the proposed MBEA, $\mathbf{v}[n]$ was a causal FIR filter of length $L_v = 3$ ($L_1 = 0$ and $L_2 = 2$), and $q_v = 31 \times 3 - 1 = 92$, multipath channel order $q_g = 10$ (which was also used for designing the actual MV receiver), $\mathcal{N}_0 = \{0, 1, \ldots, \mathcal{L} = \lfloor (q_v + q_g)/P \rfloor = \lfloor (92 + 10)/31 \rfloor = 3\}$, and $\mathcal{K}_0 = \{0, 1, \ldots, q_g = 10\}$ were used for the associated $\boldsymbol{\nu}^{(0)}[n]$ [see (53)]. One hundred independent runs were performed for different values of NFR (0, 2, 4, 6, 8, 10 dB) and different values of $\text{SNR}_1 = E_1/E\{|w[n]|^2\}$ (3, 5, 7, 9, 11, 13 dB), respectively.

The output SINR for user 1 (the weak user) associated with the theoretical nonblind MMSE equalizer (solid line), the ideal MV receiver (X) [calculated by (80) with theoretical $\overline{\mathbf{R}}$], the actual MV receiver (□), and the obtained inverse filter (◯) by the proposed MBEA are shown in Fig. 1(a)–(f) for NFR = 0, 2, 4, 6, 8, 10 dB, respectively. Fig. 2 shows the averages of ℓ denoted by $\hat{\ell}$ (the stage number that $\hat{u}_1[n]$ was obtained by the proposed MBEA) over the performed 100 independent runs for NFR = 0, 2, 4, 6, 8, 10 dB.

Some observations from Fig. 1(a)–(f) are as follows. The performance (output SINR_1) of the theoretical nonblind MMSE equalizer, that of the ideal MV receiver, and that of the actual MV receiver are insensitive to different values of NFR implying their high near–far resistance. On the other hand, the proposed MBEA, although its performance varies for different values of NFR, always performs better than the actual MV receiver. Their performance difference (output SINR difference) for NFR = 10 dB is around 2 dB, whereas for NFR = 0∼8 dB, it is larger for larger input SNR. On the other hand, significant output SINR difference between the actual MV receiver (for $N = 2500$) and the ideal MV receiver can also be observed from these figures. These simulation results are consistent with R11).

Again, one can observe, from Fig. 1(a)–(f), that the theoretical nonblind MMSE equalizer outperforms the ideal MV receiver by about 2 dB, whereas the performance of the inverse filter is very close to that of the theoretical nonblind MMSE equalizer for NFR = 0, 2, 4, 6 dB. Note, from Fig. 2, that $\hat{\ell} = 1$ for these

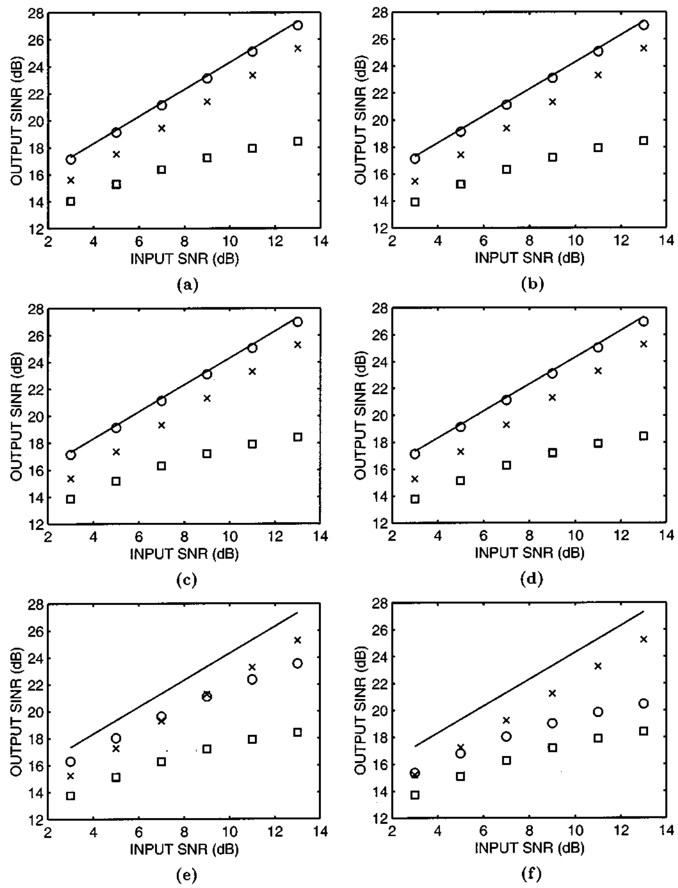

Fig. 1. Simulation results of Example 1. Output SINR for user 1 (the weak user) associated with the theoretical nonblind MMSE equalizer (solid line), the ideal MV receiver ("X"), the actual MV receiver ("□"), and the obtained inverse filter ("◯") through the proposed MBEA for (a) NFR = 0 dB, (b) NFR = 2 dB, (c) NFR = 4 dB, (d) NFR = 6 dB, (e) NFR = 8 dB, (f) NFR = 10 dB, respectively.

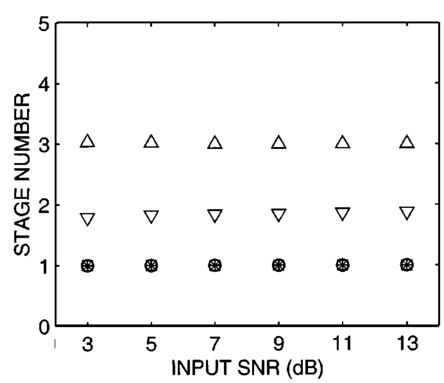

Fig. 2. Simulation results of Example 1. Averages ($\hat{\ell}$) of the stage number ℓ of the MBEA in obtaining the 100 estimates $\hat{u}_1[n]$ for NFR = 0 dB ("◯"), NFR = 2 dB ("+"), NFR = 4 dB ("X"), NFR = 6 dB ("□"), NFR = 8 dB ("▽"), and NFR = 10 dB ("△"), respectively.

values of NFR although user 1 is the weak user. These simulation results are consistent with R6). On the other hand, for the case of NFR = 8 dB, the performance of the proposed MBEA is slightly better than that of the ideal MV receiver for smaller input SNR (3, 5, and 7 dB), whereas the latter is slightly better than the former for larger input SNR (9, 11, and 13 dB). For the case of NFR = 10 dB, the performance of the proposed MBEA is inferior to that of the ideal MV receiver with larger output

SINR difference for larger input SNR. Moreover, the values of $\hat{\ell}(>1)$ for NFR $= 8$ dB are smaller than those for NFR $= 10$ dB, as shown in Fig. 2. These simulation results are consistent with R8) and support that the proposed initial condition $\boldsymbol{\nu}^{(0)}[n]$ [see (53)] can help the proposed MBEA obtain the desired estimate $\hat{u}_1[n]$ at an early stage [see R6) and R8)]. Larger ℓ indicates larger NFR that may be useful information for power control as mentioned in R8).

Remark that in all the simulation results in this example, perfect user identification (for either of the desired user identified at stage ℓ and any other users identified at stages $1, 2, \ldots, \ell - 1$ as $\ell > 1$) was achieved by the proposed UIA in V4) of the proposed MBEA, although $\hat{\ell} > 1$ happened for high NFR, and symbol error rates (SERs) of both the proposed MBEA and the actual MV receiver are zero because of high SINR_1($\geq$13 dB). The above simulation results support the efficacy of the proposed UIA and MBEA.

Example 2—Three-User ($K = 3$) Case: In the example, each multipath channel $g_i[n]$ was generated using (5) and (9). The path amplitudes $B_{i,k}$ [see (6)] were mutually independent, complex Gaussian with zero-mean. Measurements $\mathbf{y}[n]$ of length $\mathcal{N} = 5000$ for each realization were generated using (15) with input signals $u_i[n] = A_i \xi_i[n]$, where $\xi_i[n]$ is an equiprobable i.i.d. 4-QAM signal with $|\kappa_i(2, 2)| = 1$, and A_i is the amplitude of $u_i[n]$. In V2) of the proposed MBEA $\mathbf{v}[n]$ was a causal FIR filter of length $L_v = 5$ ($L_1 = 0$ and $L_2 = 4$) ($q_v = 15 \times 5 - 1 = 74$), multipath channel order $q_g = 15$ (which was also used for designing the actual MV receiver), and the integer sets $\mathcal{N}_0 = \{0, 1, \ldots, \mathcal{L} = 5\}$ and $\mathcal{K}_0 = \{0, 1, \ldots, q_g = 15\}$ were used for the associated $\boldsymbol{\nu}^{(0)}[n]$ [see (53)]. The first $N < \mathcal{N}$ measurements were used for the equalizer design, and the designed IFC equalizer using either of the proposed MBEA and the MV receiver was then employed to process the $\mathcal{N}$ synthetic data $\mathbf{y}[n]$ to calculate the SER over 200 independent runs for different values of input SNR_1 (3, 5, 7, 9, 11 dB). A case of equal powers and a case of unbalanced powers are considered as follows.

Case A—Equal Powers: In this case, path delays ($\tau_{i,k}$, $i = 1, 2, 3$, $k = 1, 2, 3$) are uniformly distributed over $[0, 10T_c]$, $E\{|B_{i,1}|^2\} = 1$, and $E\{|B_{i,k}|^2\} = 0.3$, $k = 2, 3$. The amplitudes A_i, $i = 1, 2, 3$ are adjusted such that $E_1 = E_2 = E_3$, i.e., $\mathrm{NFR}_2 = \mathrm{NFR}_3 = \mathrm{NFR} = 0$ dB.

Case B—Unbalanced Powers: In this case, path delays ($\tau_{i,k}$, $i = 1, 2, 3$, $k = 1, 2, 3$) are uniformly distributed over $[0, PT_c = 15T_c]$, and $E\{|B_{i,k}|^2\} = 1$, $k = 1, 2, 3$. The amplitudes A_i, $i = 1, 2, 3$ are adjusted such that $E_1 = 0.1E_2$ and $E_2 = E_3$, i.e., $\mathrm{NFR}_2 = \mathrm{NFR}_3 = \mathrm{NFR} = 10$ dB.

The SERs for user 1 associated with the proposed MBEA and the actual MV receiver are shown in Table I for $N = 400, 500$ for Case A and $N = 600, 700$ for Case B. One can see from Table I that the proposed MBEA performs much better than the actual MV receiver for both Cases A and B.

Remark that stage number averages $\hat{\ell}$ in Case A (equal powers) are between 1.09 and 1.23 for $N = 400$ and between 1.07 and 1.20 for $N = 500$, respectively. Note that values of $\hat{\ell}$ are much smaller than the mean value (2) of $\{1, 2, 3\}$ for all the three users. This indicates that the suggested initial condition $\boldsymbol{\nu}^{(0)}[n]$ given by (53) is helpful for obtaining $\hat{u}_1[n]$ at an early stage in V2) of the proposed MBEA for low NFR, as mentioned in R6). Moreover, perfect user identification was achieved by the proposed UIA in V4) of the proposed MBEA in all the $200 \times 5 \times 2 = 2000$ independent runs for Case A.

TABLE I
SIMULATION RESULTS OF EXAMPLE 2. SERS FOR USER 1 ASSOCIATED WITH THE PROPOSED MBEA AND THE ACTUAL MV RECEIVER

		SER ($\times 10^{-4}$)									
		Proposed MBEA					**MV Receiver**				
		Input SNR (dB)					Input SNR (dB)				
NFR	***N***	3	5	7	9	11	3	5	7	9	11
0 dB (Case A)	**400**	44.31	6.586	2.118	0.833	0.683	1457	548.5	165.0	40.86	8.434
	500	12.41	1.918	0.924	0.392	0.050	1428	521.5	148.6	33.52	5.843
10 dB (Case B)	**600**	57.75	43.51	10.97	0.293	0.031	2305	972.1	324.2	68.25	10.88
	700	89.80	7.440	1.175	0.110	0.030	2287	956.0	315.0	63.97	9.430

On the other hand, in Case B (severe near–far situation), stage number averages $\hat{\ell}$ are between 2.959 and 2.995 for $N = 600$ and between 2.97 and 2.99 for $N = 700$, respectively. Nevertheless, perfect user identification was also achieved in all the $200 \times 5 = 1000$ independent runs for $N = 700$. As for $N = 600$, the desired user was not correctly detected by the UIA in four runs (one for the case of SNR $= 9$ dB and three for the case of SNR $= 11$ dB) out of the $200 \times 5 = 1000$ independent runs, which, therefore, were excluded in the calculation of SERs and stage number averages $\hat{\ell}$. The above simulation results support the efficacy of the proposed UIA and MBEA.

VI. CONCLUSIONS

We have presented a family of IFC [see (26)] where $p+q > 2$ and the characteristics of their stationary points (see Theorem 2) for blind equalization of MIMO systems with Tugnait's IFC [$(p, q) = (2, 1)$ and $(p, q) = (2, 2)$] as special cases. Then, an MBEA for blind equalization of asynchronous DS/CDMA systems in the presence of multipath was presented that processes the chip waveform matched filter output signal $y[n]$ given by (7) without requiring any path delay information. The proposed MBEA includes iterative blind deconvolution processing using cumulant-based IFC with the suggested initial condition [see (53)] followed by user identification using the proposed UIA. Some simulation results were provided to support the efficacy of the proposed UIA and MBEA. The performance of the proposed MBEA is superior to Tsatsanis and Xu's blind MV receiver, as long as the given data are sufficient for reliably estimating the higher order cumulant $C_{p,q}\{e[n]\}$ needed by IFC. Further studies on the performance limit of the proposed MBEA with respect to data length (N) and number of active users (K) are left for future research.

APPENDIX A
PROOF OF THEOREM 2

Cases C1) and C2) can be proved by following the same procedure for the proof of Theorem 1, as presented in [22]. Therefore, we only prove Case C3).

Under the assumptions A1) and A2) and absence of noise $\mathbf{w}[n]$, it can be easily shown from (24) that [17]

$$C_{p,q}\{e[n]\} = \sum_{i=1}^{K} \left\{ C_{p,q}\{u_i[n]\} \sum_{k=-\infty}^{\infty} f_i^p[k](f_i^*[k])^q \right\}. \quad \text{(A.1)}$$

By (A.1), we can easily see

$$C_{1,1}\{e[n]\} = \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} C_{1,1}\{u_i[n]\} \cdot |f_i[k]|^2. \quad \text{(A.2)}$$

Consider the objective function

$$\begin{aligned}\tilde{J}_{p,q}(\boldsymbol{\nu}) &= J_{p,q}^2(\boldsymbol{\nu}) \\ &= \frac{|C_{p,q}\{e[n]\}|^2}{|C_{1,1}\{e[n]\}|^{p+q}} \\ &= \frac{C_{p,q}\{e[n]\}C_{q,p}\{e[n]\}}{|C_{1,1}\{e[n]\}|^{p+q}}. \end{aligned} \quad \text{(A.3)}$$

Taking partial derivative of $\tilde{J}_{p,q}(\boldsymbol{\nu})$ given by (A.3) with respect to $f_j[k]$ yields

$$\begin{aligned}\frac{\partial \tilde{J}_{p,q}}{\partial f_j[k]} = &\frac{1}{|C_{1,1}\{e[n]\}|^{2(p+q)}} \\ &\cdot \Bigg\{ |C_{1,1}\{e[n]\}|^{p+q} \cdot \Bigg(C_{p,q}\{e[n]\} \\ &\cdot \frac{\partial C_{q,p}\{e[n]\}}{\partial f_j[k]} + C_{q,p}\{e[n]\} \cdot \frac{\partial C_{p,q}\{e[n]\}}{\partial f_j[k]} \Bigg) \\ &- |C_{p,q}\{e[n]\}|^2 \cdot (p+q) \cdot |C_{1,1}\{e[n]\}|^{p+q-1} \\ &\cdot \frac{\partial C_{1,1}\{e[n]\}}{\partial f_j[k]} \Bigg\}. \end{aligned} \quad \text{(A.4)}$$

Then, taking partial derivatives of $C_{p,q}\{e[n]\}$ given by (A.1) and $C_{1,1}\{e[n]\}$ given by (A.2) with respect to $f_j[k]$ gives rise to

$$\frac{\partial C_{p,q}\{e[n]\}}{\partial f_j[k]} = p \cdot C_{p,q}\{u_j[n]\} \cdot f_j^{p-1}[k] \cdot (f_j^*[k])^q \quad \text{(A.5)}$$

and

$$\frac{\partial C_{1,1}\{e[n]\}}{\partial f_j[k]} = C_{1,1}\{u_j[n]\} \cdot f_j^*[k] \quad \text{(A.6)}$$

respectively. Note that $\partial \tilde{J}_{p,q}/\partial f_j[k] = 0$ implies $\partial J_{p,q}/\partial f_j[k] = 0$ by (A.3). Substituting (A.5) and (A.6) into $\partial \tilde{J}_{p,q}/\partial f_j[k] = 0$ given by (A.4) results in

$$\begin{aligned}&(C_{1,1}\{e[n]\})^{p+q} \cdot C_{p,q}\{e[n]\} \cdot q \cdot C_{q,p}\{u_j[n]\} \cdot f_j^{q-1}[k] \\ &\quad \cdot (f_j^*[k])^p + (C_{1,1}\{e[n]\})^{p+q} \cdot C_{q,p}\{e[n]\} \cdot p \\ &\quad \cdot C_{p,q}\{u_j[n]\} \cdot f_j^{p-1}[k] \cdot (f_j^*[k])^q \\ &= |C_{p,q}\{e[n]\}|^2 \cdot (p+q) \cdot (C_{1,1}\{e[n]\})^{p+q-1} \\ &\quad \cdot C_{1,1}\{u_j[n]\} \cdot f_j^*[k], \quad j = 1, 2, \ldots, K, k = -\infty \sim \infty \end{aligned} \quad \text{(A.7)}$$

which, after some manipulations, leads to

$$f_j^*[k] \Big(\mathcal{A}_j \cdot f_j^{q-1}[k] \cdot (f_j^*[k])^{p-1} + \tilde{\mathcal{A}}_j \cdot f_j^{p-1}[k] \cdot (f_j^*[k])^{q-1} - \mathcal{B}_j \Big) = 0 \quad \text{(A.8)}$$

where

$$\begin{aligned}\mathcal{A}_j &= q \cdot C_{1,1}\{e[n]\} \cdot C_{p,q}\{e[n]\} \cdot C_{q,p}\{u_j[n]\} \\ \tilde{\mathcal{A}}_j &= p \cdot C_{1,1}\{e[n]\} \cdot C_{q,p}\{e[n]\} \cdot C_{p,q}\{u_j[n]\} \\ \mathcal{B}_j &= |C_{p,q}\{e[n]\}|^2 \cdot (p+q) \cdot C_{1,1}\{u_j[n]\}. \end{aligned} \quad \text{(A.9)}$$

Equation (A.8) implies that a stationary point of $J_{p,q}(\mathbf{f}[k])$ must satisfy either $f_j[k] = 0$ or

$$\mathcal{A}_j \cdot f_j^{q-1}[k] \cdot (f_j^*[k])^{p-1} + \tilde{\mathcal{A}}_j \cdot f_j^{p-1}[k] \cdot (f_j^*[k])^{q-1} = \mathcal{B}_j. \quad \text{(A.10)}$$

By the assumptions $C_{1,1}\{u_j[n]\} = \sigma_u^2$ and $C_{p,q}\{u_j[n]\} = \gamma$, $j = 1, 2, \ldots, K$, the three quantities $\mathcal{A}_j$, $\tilde{\mathcal{A}}_j$, and $\mathcal{B}_j$ become constant. Therefore, $f_j[k]$ that satisfies (A.8) can be expressed as

$$f_j[k] = \begin{cases} \sqrt{\epsilon} \exp\{\sqrt{-1}\vartheta\}, & (j, k) \in \{(j_1, k_1), (j_2, k_2) \\ & \quad \ldots, (j_{\mathcal{M}}, k_{\mathcal{M}})\} \\ 0, & \text{otherwise} \end{cases} \quad \text{(A.11)}$$

where $\epsilon > 0$ and ϑ are real constants, and $\mathcal{M} \geq 0$ is the total number of nonzero elements of $\mathbf{f}[k]$.

Let

$$\begin{aligned}\mathcal{G}(\mathcal{M}) = \{\mathbf{f}[k] \colon \mathbf{f}[k] \text{ satisfies (A.8), i.e., } f_j[k] \\ j = 1, 2, \ldots, K, k = -\infty \sim \infty \\ \text{are given by (A.11)}\} \end{aligned} \quad \text{(A.12)}$$

be the set of all the stationary points of $J_{p,q}(\mathbf{f}[k])$, where $\mathcal{M} \geq 1$ (excluding the trivial solution $\mathbf{f}[k] = \mathbf{0}$). $J_{p,q}(\mathbf{f}[k])$ for all $\mathbf{f}[k] \in \mathcal{G}(\mathcal{M})$ can be easily shown, from (A.1) and (A.2), to be

$$\begin{aligned} J_{p,q}(\mathbf{f}[k]) &= \frac{\left| \sum_{i=1}^{K} C_{p,q}\{u_i[n]\} \sum_{k=-\infty}^{\infty} f_i^p[k](f_i^*[k])^q \right|}{\left(\sum_{i=1}^{K} C_{1,1}\{u_i[n]\} \sum_{k=-\infty}^{\infty} |f_i[k]|^2 \right)^{(p+q)/2}} \\ &= \frac{|\gamma| \cdot \left| \mathcal{M} \cdot \epsilon^{(p+q)/2} \cdot \exp\{\sqrt{-1} \cdot (p-q) \cdot \vartheta\} \right|}{(\sigma_u^2 \cdot \mathcal{M} \cdot \epsilon)^{(p+q)/2}} \\ &= \frac{|\gamma| \cdot \left| \mathcal{M} \cdot \epsilon^{(p+q)/2} \right|}{(\sigma_u^2 \cdot \mathcal{M} \cdot \epsilon)^{(p+q)/2}} \end{aligned} \quad \text{(A.13)}$$

which only depends on the nonzero magnitude ($\sqrt{\epsilon}$) of $f_j[k]$. The set $\mathcal{G}(\mathcal{M})$ can also be expressed as

$$\mathcal{G}(\mathcal{M}) = \mathcal{G}_1 \cup \mathcal{G}_2 \quad \text{(A.14)}$$

where $\mathcal{G}_1 \cap \mathcal{G}_2 = \phi$ (empty set), and

$$\mathcal{G}_1 = \left\{ \mathbf{f}[k] \colon \mathbf{f}[k] = \mathbf{1}_{K,j} \cdot \sqrt{\epsilon} \exp\{\sqrt{-1}\vartheta\}, j = 1, \ldots, K \right\}. \quad \text{(A.15)}$$

Note that $\mathcal{M} = 1$ for all $\mathbf{f}[k] \in \mathcal{G}_1$ and $\mathcal{M} \geq 2$ for all $\mathbf{f}[k] \in \mathcal{G}_2$. Next, let us prove that all the stationary points of $\mathcal{G}_2$ are unstable equilibria (i.e., neither local maxima nor local minima).

Assume that $\mathbf{f}[k] \in \mathcal{G}_2$ (with $\mathcal{M} \geq 2$) given by (A.11). Let us define $\check{\mathbf{f}}[k]$ such that

$$\left|\check{f}_j[k]\right|^2 = \begin{cases} |f_{j_1}[k_1]|^2 - \varepsilon = \epsilon - \varepsilon \\ \qquad 0 < \varepsilon < \epsilon,\ (j,\,k) = (j_1,\,k_1) \\ |f_{j_2}[k_2]|^2 + \varepsilon = \epsilon + \varepsilon \\ \qquad (j,k) = (j_2,k_2),\ (j_2,k_2) \neq (j_1,k_1) \\ |f_j[k]|^2, \quad \text{otherwise} \end{cases} \tag{A.16}$$

and $\arg\{\check{f}_j[k]\} = \arg\{f_j[k]\}$ for all $(j,\,k)$, where $\varepsilon > 0$. Note that $\check{\mathbf{f}}[k] \to \mathbf{f}[k]$ as $\varepsilon \to 0$. Then, substituting (A.16) into the first line of (A.13) yields

$$J_{p,\,q}\left(\check{\mathbf{f}}[k]\right) = \frac{|\gamma| \cdot \left|(\mathcal{M}-2)\cdot \epsilon^{(p+q)/2} + (\epsilon-\varepsilon)^{(p+q)/2} + (\epsilon+\varepsilon)^{(p+q)/2}\right|}{(\sigma_u^2 \cdot \mathcal{M} \cdot \epsilon)^{(p+q)/2}}. \tag{A.17}$$

By Lemma 1, it can be seen from (A.13) and (A.17) that

$$J_{p,\,q}(\mathbf{f}[k]) < J_{p,\,q}\left(\check{\mathbf{f}}[k]\right), \qquad \text{as } 0 < \varepsilon < \epsilon \tag{A.18}$$

[since $\mathcal{M} \geq 2$ and $(p+q)/2 > 1$], which implies that $\mathbf{f}[k]$ is not a stable local maximum. Therefore, all the stationary points of $\mathcal{G}_2$ are not local maxima.

Next, let us show that $\mathbf{f}[k] \in \mathcal{G}_2$ is not a local minimum either. Define $\tilde{\mathbf{f}}[k]$ such that

$$\left|\tilde{f}_j[k]\right|^2 = \begin{cases} |f_{j_1}[k_1]|^2 - \varepsilon \\ \quad = \epsilon - \varepsilon, & 0 < \varepsilon \leq \epsilon/2,\ (j,k) = (j_1,k_1) \\ \varepsilon, & (j,\,k) = (j_{\mathcal{M}+1},\,k_{\mathcal{M}+1}) \\ |f_j[k]|^2, & \text{otherwise} \end{cases} \tag{A.19}$$

and $\arg\{\tilde{f}_j[k]\} = \arg\{f_j[k]\}$ for all $(j,\,k)$, where $\varepsilon > 0$. Note that $\tilde{\mathbf{f}}[k] \to \mathbf{f}[k]$ as $\varepsilon \to 0$. Again, substituting (A.19) into the first line of (A.13), one can obtain

$$J_{p,\,q}\left(\tilde{\mathbf{f}}[k]\right) = \frac{|\gamma| \cdot \left|(\mathcal{M}-1)\cdot \epsilon^{(p+q)/2} + (\epsilon-\varepsilon)^{(p+q)/2} + \varepsilon^{(p+q)/2}\right|}{(\sigma_u^2 \cdot \mathcal{M} \cdot \epsilon)^{(p+q)/2}}. \tag{A.20}$$

By Lemma 1, it can be seen from (A.13) and (A.20) that

$$J_{p,\,q}(\mathbf{f}[k]) > J_{p,\,q}\left(\tilde{\mathbf{f}}[k]\right), \qquad \text{as } 0 < \varepsilon \leq \epsilon/2 \tag{A.21}$$

[since $\mathcal{M} \geq 2$ and $(p+q)/2 > 1$], which implies that $\mathbf{f}[k]$ is not a stable local minimum. Therefore, all the stationary points of $\mathcal{G}_2$ are not local minima. Thus, we have completed the proof that all the stationary points of $\mathcal{G}_2$ are unstable equilibria.

What remains to be proven is that all the stationary points of $\mathcal{G}_1$ are stable local maxima. By the assumptions $C_{1,\,1}\{u_j[n]\} = \sigma_u^2$ and $C_{p,\,q}\{u_j[n]\} = \gamma,\ j = 1, 2, \ldots, K$, it can be easily shown that for all $\mathbf{f}[k] \in \mathcal{G}_1$

$$J_{p,\,q}(\mathbf{f}[k]) = \frac{|C_{p,q}\{u_j[n]\}|}{|C_{1,1}\{u_j[n]\}|^{(p+q)/2}} = \frac{|\gamma|}{\sigma_u^{p+q}} \tag{A.22}$$

(which is a constant). Due to the fact that $0 \leq J_{p,\,q}(\mathbf{f}[k]) \leq B < \infty$ (bounded), there must exist a global optimum $\mathbf{f}[k] \in \mathcal{G}_1$ (also a stable local maximum) with

$$\max\{J_{p,\,q}(\mathbf{f}[k])\} = \frac{|\gamma|}{\sigma_u^{p+q}}. \tag{A.23}$$

If any of $\mathbf{f}[k] \in \mathcal{G}_1$ is a stable or unstable local minimum or an unstable local maximum, there must be a local maximum $\mathbf{f}[k] \in \mathcal{G}_1$ with $J_{p,\,q}(\mathbf{f}[k]) > |\gamma|/\sigma_u^{p+q}$, thus contradicting (A.23). Therefore, all the stationary points of $\mathcal{G}_1$ must be stable local maxima.

The above proof pertains to the stationary points of $J_{p,\,q}(\mathbf{f}[k])$ with respect to the overall channel $\mathbf{f}[k]$. It can be easily shown, following the procedure in [22, App. A], that all the stationary points of $J_{p,\,q}(\boldsymbol{\nu})$ with respect to equalizer coefficients $\boldsymbol{\nu}$ can be described by the stationary points of $J_{p,\,q}(\mathbf{f}[k])$ with respect to the overall channel $\mathbf{f}[k]$. Thus, we have completed the proof. Q.E.D.

Appendix B
Summary of Computation of SINR_1

Assume that $u_1[n]$ is estimated at the ℓth stage of the MSC procedure and that $e_l[n]$, $\hat{\mathbf{h}}_l[n]$, and $\mathbf{v}_l[n]$ are the obtained inverse filter output, channel estimate, and optimum inverse filter, respectively, at the lth stage for $l = 1, \ldots, \ell$. Let

$$\overline{\mathbf{y}}_k(z) = \begin{cases} \mathbf{y}(z), & k = 1 \\ \mathbf{T}_{k-1}(z) \cdot \mathbf{T}_{k-2}(z) \cdots \mathbf{T}_1(z) \cdot \mathbf{y}(z), & k \geq 2 \end{cases} \tag{B.1}$$

where

$$\mathbf{T}_l(z) = \mathbf{I}_P - \hat{\mathbf{h}}_l(z) \cdot \mathbf{v}_l^T(z) \tag{B.2}$$

in which $\mathbf{I}_P$ is a $P \times P$ identity matrix. The inverse filter output $e_\ell(z)$ at the ℓth stage can be easily shown from (37) to be

$$e_\ell(z) = \mathbf{v}_\ell^T(z) \cdot \overline{\mathbf{y}}_\ell(z) = \overline{\mathbf{f}}_\ell^T(z) \cdot \mathbf{u}(z) + \overline{w}_\ell(z) \tag{B.3}$$

where

$$\overline{\mathbf{f}}_\ell^T(z) = \begin{cases} \mathbf{v}_1^T(z) \cdot \mathbf{H}(z), & \ell = 1 \\ \mathbf{v}_\ell^T(z) \cdot \mathbf{T}_{\ell-1}(z) \cdot \mathbf{T}_{\ell-2}(z) \cdots \mathbf{T}_1(z) \cdot \mathbf{H}(z) \\ \qquad \ell \geq 2 \end{cases} \tag{B.4}$$

and

$$\overline{w}_\ell(z) = \begin{cases} \mathbf{v}_1^T(z) \cdot \mathbf{w}(z), & \ell = 1 \\ \mathbf{v}_\ell^T(z) \cdot \mathbf{T}_{\ell-1}(z) \cdot \mathbf{T}_{\ell-2}(z) \cdots \mathbf{T}_1(z) \cdot \mathbf{w}(z) \\ \qquad \ell \geq 2. \end{cases} \tag{B.5}$$

$$\mathrm{SINR}_1 = \frac{C_{1,1}\{u_1[n]\} \cdot |\overline{f}_{\ell,1}[n_0]|^2}{\sum_{k=1}^{K} C_{1,1}\{u_k[n]\} \left(\sum_n |\overline{f}_{\ell,k}[n]|^2\right) + C_{1,1}\{\overline{w}_\ell[n]\} - C_{1,1}\{u_1[n]\} \cdot |\overline{f}_{\ell,1}[n_0]|^2} \tag{B.6}$$

Therefore, SINR_1 can be calculated by (B.6), shown at the top of the page, where $\overline{f}_{\ell,1}[n]$ denotes the first component of $\overline{\mathbf{f}}_\ell[n]$, and $|\overline{f}_{\ell,1}[n_0]|^2 = \max_n\{|\overline{f}_{\ell,1}[n]|^2\}$.

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