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Batch Processing Algorithms for Blind Equalization Using Higher-Order Statistics

Statistical signal processing has been one of the key technologies in the development of wireless communication systems in recent years, especially for broadband multiuser communication systems which severely suffer from intersymbol interference (ISI) and multiple access interference (MAI). This article reviews batch processing algorithms for blind equalization using higher-order statistics for mitigation of the ISI induced by single-input, single-output channels as well as of both the ISI and MAI induced by multiple-input, multiple-output channels. In particular, this article reviews the typical inverse filter criteria (IFC) based algorithm, super-exponential algorithm, and constant modulus algorithm along with their relations, performance, and improvements. Several advanced applications of these algorithms are illustrated, including blind channel estimation, simultaneous estimation of multiple time delays, signal-to-noise ratio (SNR) boost by blind maximum ratio combining, blind beamforming for source separation in multipath, and multiuser detection for direct sequence/code division multiple access (DS/CDMA) systems in multipath.

Introduction

The ever-growing demands of high-speed and high-quality wireless communication services have stimulated research on further promotion of the related technologies, including digital signal processing, antenna, and semiconductor. High-speed wireless communication systems typically require a much larger radio spectrum that may suffer from severe ISI due to the frequency-selective characteristics of the radio channels (including the effects of multipath propagation and limited channel bandwidth). Also, they may suffer from MAI when multiple users share the common radio resources. Mitigation of both the ISI and MAI is accordingly essential to the various types of systems. Familiar examples are single-carrier modulation systems with time-domain equalization, multicarrier modulation systems with frequency-domain equalization, and spread spectrum systems with RAKE reception [88], [103].

With respect to the single-carrier modulation systems, there are three types of equalizers for mitigation of the ISI and MAI: nonblind, semiblind, and blind. The nonblind and semiblind equalizers are designed through both the received and training (or pilot) signals at the expense of system resources (e.g., bandwidth) whereas the blind equalizer is designed with only the received signal. Due to the benefit of resource (bandwidth) saving and no need of training phase, extensive research on blind techniques has been reported, which usually exploit the properties of users' symbol sequences and the received signal such as their statistical properties or constellation properties. Examples of

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exploiting statistical properties of signals include maximum-likelihood (ML) methods, second-order cyclostationary statistics (SOCS) (i.e., cyclic correlations) based methods, and higher-order (≥ 3) statistics (HOS) based methods. The ML methods derive the optimum equalizer according to a presumed probability density function (pdf) of signals, while the SOCS- and HOS-based methods design the blind equalizer using the SOCS and HOS of signals, respectively, as the names indicate. Thus far, there have been extensive studies on these blind equalization methods. Several works ([55], [56], [104], [128], [129], [140], [153] to name a few) have given excellent overviews of these methods with emphasis on the single-user case. This article, on the other hand, provides an in-depth perspective of HOS-based methods in the category of "batch processing" with emphasis on the general framework of dealing with both the single-user and multiuser cases.

The HOS-based methods for blind equalization of single-input, single-output (SISO) channels (i.e., single-user case) can be divided into two classes: implicit and explicit methods [56]. The former, using higher-order moments implicitly, is also known as the Bussgang-type algorithms, which the include the Sato algorithm [110] and the constant modulus algorithm (CMA) (or Godard-2 algorithm) [45], [64], [75], [126], [127] as special cases. In digital communications, the CMA has been a widely applied approach to alleviating the ISI effect induced by telephone, cable, or radio channels [128], [129]. Moreover, its counterparts for blind equalization of multiple-input, multiple-output (MIMO) channels (i.e., multiuser case) have been applied to multiuser detection in DS/CDMA systems, blind beamforming, and source separation in multiple-antenna systems [1], [8], [47], [68], [78,], [92], [93], [95], [119], [120], [142], [137], [147]. On the other hand, the explicit SISO methods using HOS include the IFC-based algorithm [7], [13], [14], [34], [48], [116], [118], [133], [146], the super-exponential algorithm (SEA) [9], [46], [57], [58], [111], [117], [118], and polyspectra-based algorithms [2], [4], [51]-[53], [91]. The IFC-based algorithm and SEA are suitable for seismic exploration as well as blind equalization of communication channels, and their MIMO counterparts have been also applied in multiuser detection, blind beamforming, and source separation [10], [20], [21], [28], [32], [60]-[63], [67]-[69], [76], [81], [100], [101], [130[-[132], [137], [141], [142], [144], [146], [149], [150]. As a result, successful applications of the IFC-based algorithm, SEA, and CMA in blind equalization have motivated further exploration of their relations, performance, improvements, and other applications for both SISO and MIMO cases, which will be reviewed in this article. However, most of the iterative HOS-based blind equalization algorithms suffer from the problem of ill convergence, though the global convergence of some algorithms has been proven under ideal conditions (e.g., noise-free and doubly infinite equalizer) [21], [40], [70], [136]. Recently, research about the convergence behavior of these algorithms under nonideal conditions has been reported [29]-[31], [41], [49], [71], [89], [112], [151], [152], and some globally convergent techniques have also been suggested [97], [125]. Typically, a good initial condition is usually needed to obtain the global optimum solutions. Only the properties of the global solutions of the three basic algorithms (IFC-based algorithm, SEA, and CMA) will be reviewed in this article due to space limitations.

In the next section we present the problems and assumptions for blind equalization of SISO and MIMO channels. We then review the typical ones of the IFC-based algorithm, SEA, and CMA, and their relations, performance, and improvements for the SISO case, followed by the MIMO case. Then, we illustrate several advanced applications of these algorithms, where the MIMO algorithms are applied, as special cases, to blind equalization of single-input, multiple-output (SIMO) channels for some applications.

Table 1. Summary of notations.

Notation	Definition
$*$	Convolution operation of discrete-time (scalar, vector or matrix) signals
$\Vert\cdot\Vert$	Euclidean norm of vectors or matrices
$\mathbf{0}_P$	$P\times 1$ zero vector
$\delta_P[\ell]$	$P\times 1$ vector whose ℓth entry equals unity and the remaining entries equal zero
$E\{\cdot\}$	Expectation operator
$\mathcal{F}_n\{\cdot\}$	Discrete-time Fourier transform with respect to the index n
$\lfloor t \rfloor$	Largest integer no larger than t
Superscript "$*$"	Complex conjugation
Superscript "T"	Transpose of vectors or matrices
Superscript "H"	Complex conjugate transpose (Hermitian) of vectors or matrices
$\mathrm{cum}\{y_1, y_2, \ldots, y_m\}$	mth-order joint cumulant of random variables $y_1, y_2, \ldots, y_m$
$\mathrm{cum}\{y: p, \ldots\}$	$= \mathrm{cum}\{y_1 = y, y_2 = y, \ldots, y_p = y, \ldots\}$
$C_{p,q}\{y\}$	$= \mathrm{cum}\{y: p, y^*: q\}$

SISO and MIMO Blind Equalization Problems

Prior to the presentation of blind equalization problems, let us summarize the notations used throughout the article in Table 1 and briefly review the definitions of HOS in Table 2 for better understanding of the HOS-based methods to be presented.

SISO Blind Equalization

Blind equalization, also known as blind deconvolution, of SISO channels is a signal processing procedure to restore the source signal $u[n]$ from the received signal $x[n]$ given by (see Figure 1)

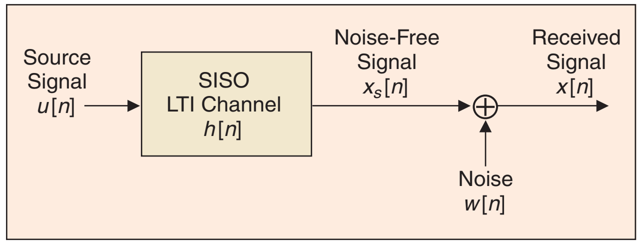
$$x[n]=x_S[n]+w[n] \tag{1}$$

where

$$x_S[n]=h[n]*u[n]=\sum_{i=-\infty}^{\infty}h[i]u[n-i] \tag{2}$$

is the noise-free signal distorted by an unknown linear time-invariant (LTI) SISO channel $h[n]$ and $w[n]$ is the additive noise accounting for sensor noise as well as physical effects not explained by $x_S[n]$. The problem arises not only in digital communications but also in a variety of other engineering and science areas, such as seismic signal processing, speech modeling and synthesis, ultrasonic nondestructive evaluation (NDE), and image restoration.

Since mid 1980s, the problem of SISO blind equalization has been tackled using HOS [6], [7], [9], [11]-[14], [34], [35], [42]-[46], [48], [52], [55], [57], [58], [64], [74], [75], [89], [94], [99], [102], [111], [116], [117], [126], [127], [133]-[135], [138], [146], [153] where $x_S[n]$ is assumed to be non-Gaussian and $w[n]$ Gaussian, by virtue of the following properties of HOS. Higher-order (≥ 3) statistics (cumulants or polyspectra) of the non-Gaussian signal $x[n]$ contain not only the magnitude but also phase information of the unknown channel $h[n]$. Moreover, they are insensitive to Gaussian noise since all HOS (≥ 3) of Gaussian random processes are equal to zero. On the other hand, conventional second-order statistics (SOS) (autocorrelations or power spectra [54], [56], [66], [122]) based methods, e.g., those using the well-known linear prediction error (LPE) filter [79], are blind to the phase of the channel $h[n]$ and therefore cannot be applied to equalization of nonminimum-phase channels. Furthermore, their performance is sensitive to additive noise simply because autocorrelations of the received signal $x[n]$ are the sum of autocorrelations of the noise-free signal $x_S[n]$ and those of the additive



▲ *1. The SISO channel model.*

Table 2. Brief review of the definitions of HOS [27], [84], [90], [91].

The mth-order joint cumulant of random variables $y_1, y_2, \ldots, y_m$ is defined as

$$\text{cum}\{y_1,y_2,\ldots,y_m\}=(-j)^m\left.\frac{\partial^m \ln\Phi(\omega_1,\ldots,\omega_m)}{\partial\omega_1\cdots\partial\omega_m}\right|_{\omega_1=\cdots=\omega_m=0}$$

where

$$\Phi(\omega_1,\ldots,\omega_m)=E\{\exp[j(\omega_1 y_1+\omega_2 y_2+\cdots+\omega_m y_m)]\}$$

is the characteristic function of random variables $y_1, y_2, \ldots, y_m$.

The mth-order cumulant $\text{cum}\{y_1,y_2,\ldots,y_m\}$ is related to the moments of $y_1, y_2, \ldots, y_m$ of orders up to m. For instance, assuming that y_1, y_2, y_3, and y_4 are zero-mean random variables, then

$$\begin{aligned}\text{cum}\{y_1,y_2\}&=E\{y_1y_2\},\\ \text{cum}\{y_1,y_2,y_3\}&=E\{y_1y_2y_3\},\\ \text{cum}\{y_1,y_2,y_3,y_4\}&=E\{y_1y_2y_3y_4\}-E\{y_1y_2\}E\{y_3y_4\}\\ &\quad -E\{y_1y_3\}E\{y_2y_4\}-E\{y_1y_4\}E\{y_2y_3\},\end{aligned}$$

and thus

$$\begin{aligned}C_{1,1}\{y_1\}&=\text{cum}\{y_1,y_1^*\}=E\{|y_1|^2\} \quad \text{(variance)}\\ C_{2,2}\{y_1\}&=\text{cum}\{y_1,y_1,y_1^*,y_1^*\}\\ &=E\{|y_1|^4\}-2\left(E\{|y_1|^2\}\right)^2-\left|E\{y_1^2\}\right|^2 \quad \text{(kurtosis)}.\end{aligned}$$

Remark: In practice, the cumulants of a stationary random process $y[n]$ are usually estimated from the finite measurements $y[0], y[1], \ldots, y[N-1]$. For instance, the cumulants of $y[n]$ (with zero mean) can be obtained by their time-average estimates as

$$\hat{C}_{1,1}\{y[n]\}=\frac{1}{N}\sum_{n=0}^{N-1}|y[n]|^2,$$

$$\hat{C}_{2,2}\{y[n]\}=\frac{1}{N}\sum_{n=0}^{N-1}|y[n]|^4-2\left(\frac{1}{N}\sum_{n=0}^{N-1}|y[n]|^2\right)^2 -\left|\frac{1}{N}\sum_{n=0}^{N-1}y^2[n]\right|^2.$$

Fast real-time batch or adaptive signal processing with low complexity for blind channel estimation and equalization is still a challenging research.

noise $w[n]$. According to the properties of HOS, the HOS-based methods are generally based on the following assumptions for the received signal $x[n]$ modeled by (1) and (2).

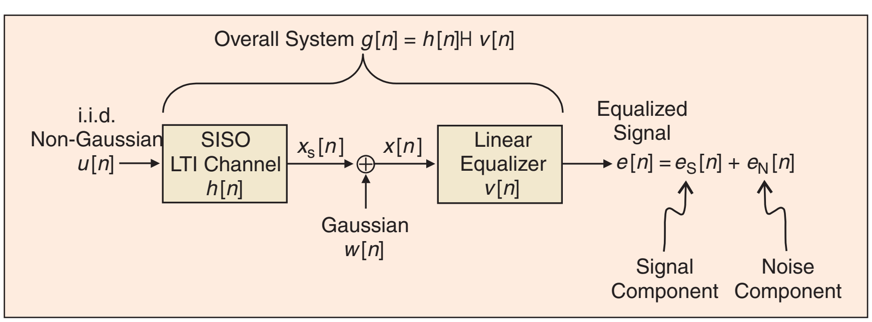
▲ (S-A1) The SISO LTI channel $h[n]$ (or $H(z)$, the transfer function of $h[n]$) is stable.

▲ (S-A2) The source signal $u[n]$ is a zero-mean, independently and identically distributed (i.i.d.) non-Gaussian random process with variance $\sigma_u^2 = E\{|u[n]|^2\}$ and $(p+q)$th-order cumulant $\gamma_{p,q} = C_{p,q}\{u[n]\} \neq 0$.

▲ (S-A3) The noise $w[n]$ is a zero-mean Gaussian random process, which can be colored with correlation function $r_w[l] = E\{w[n]w^*[n-l]\}$.

▲ (S-A4) The source signal $u[n]$ is statistically independent of the noise $w[n]$.

Note that for uncoded digital communication systems with efficiently compressed source signal $u[n]$, each sample in $u[n]$ can be reasonably assumed to be independently taken from a set of constellation points with equal probability [85], [103]. The corresponding probability distribution of $u[n]$ is symmetrically non-Gaussian [satisfying (S-A2)] and, therefore, the $(p+q)$th-order cumulant $\gamma_{p,q} = 0$ for $(p+q)$ odd [84], [90], [91], implying that the HOS-based methods apply only when $(p+q)$ is even. In coded digital communication systems, however, redundancy (memory) in $u[n]$ is introduced by the channel encoder to detect and/or correct symbol errors at the receiving end. Different types of channel encoding schemes result in different impacts on the validity of (S-A2) [80]. For these systems, blind equalization algorithms based on (S-A2) may still apply either with carefully chosen channel encoding scheme [80] or by scrambling (randomizing) the channel encoded data. On the other hand, Dogancay and Kennedy [33] proposed a least squares (LS) blind equalization algorithm based on reformulation of the CMA that is shown to be relatively insensitive to channel input correlation.



▲ *2. Block diagram of SISO linear equalization.*

As shown in Figure 2, let $v[n]$ denote the equalizer to be designed and be a linear finite impulse response (FIR) filter with $v[n] \neq 0$ for $n = L_1, L_1+1, \ldots, L_2$ and length $L = L_2 - L_1 + 1$. The corresponding equalized signal $e[n]$ (the output of $v[n]$) can be expressed as

$$e[n] = v[n] * x[n] = e_S[n] + e_N[n] \tag{3}$$

where

$$e_N[n] = v[n] * w[n] \quad \text{[by (1)]} \tag{4}$$

corresponds to the noise component in $e[n]$ and

$$e_S[n] = v[n] * x_S[n] = g[n] * u[n] \; \text{[by (1) and (2)]} \tag{5}$$

is the corresponding signal component in which

$$g[n] = h[n] * v[n] \tag{6}$$

is the overall system after equalization.

Under assumptions (S-A1) through (S-A4) and the condition of linear equalization, the problem of SISO blind equalization is equivalent to finding the coefficients of the equalizer $v[n]$ such that the signal component $e_S[n]$ approximates the source signal $u[n]$ as close as possible (up to a scale factor and a time delay) while maintaining smallest enhancement in the power of the noise component $e_N[n]$. Furthermore, under the following ideal conditions [14], [116], [118], [133], the problem reduces to finding the equalizer $v[n]$ such that the equalized signal $e[n] = \alpha u[n-\tau]$ (perfect equalization) where α is a real/complex constant and τ is an integer:

▲ (S-C1) the inverse system of $h[n]$, denoted $h_I[n]$, (whose transfer function $H_I(z) = 1/H(z)$) is stable.

▲ (S-C2) the SNR associated with the received signal $x[n]$ defined as

$$\text{SNR} = \frac{E\{|x_S[n]|^2\}}{E\{|w[n]|^2\}} \; \text{[see (1)]} \tag{7}$$

equals infinity.

▲ (S-C3) the length of the equalizer $v[n]$ is doubly infinite ($L_1 \to -\infty$ and $L_2 \to \infty$).

The equalization approach in this way is equivalent to finding the equalizer $v[n]$ as an inverse system estimate, i.e., $v[n] = \alpha h_I[n-\tau]$, thereby leading to the name inverse filtering approach. For evaluating how the equalized signal $e[n]$ is close to $\alpha u[n-\tau]$ (i.e., how the resultant overall system $g[n]$ is close to $\alpha\delta[n-\tau]$), a commonly used performance index for the designed equalizer is as follows [118]:

$$\text{ISI}(\eta) = \frac{\|\eta\|^2 - \max_i\{|\eta_i|^2\}}{\max_i\{|\eta_i|^2\}} \tag{8}$$

where $\eta = [\ldots, g[-1], g[0], g[1], \ldots]^T$ is a vector composed of the sequence $g[n], n = \ldots,-1,0,1,\ldots$, and η_i is the ith entry of η. Note that the smaller the value of ISI(η), the more the overall system $g[n]$ approaches a delta function, and ISI$(\alpha\delta_P[\ell]) = 0$ for all $P \geq 1$.

MIMO Blind Equalization

Consider that there are K different source signals $u_1[n]$, $u_2[n]$, ..., $u_K[n]$ simultaneously transmitted through a multipath channel and received by an M-element antenna array in the presence of additive noise. The resultant channel model is depicted in Figure 3. The $M \times 1$ vector received signal can be written as

$$\mathbf{x}[n] = [x_1[n], x_2[n], \ldots, x_M[n]]^T = \mathbf{x}_S[n] + \mathbf{w}[n] \tag{9}$$

where

$$\mathbf{x}_S[n] = \mathbf{H}[n] * \mathbf{u}[n] = \sum_{i=-\infty}^{\infty} \mathbf{H}[i]\mathbf{u}[n-i] \tag{10}$$

is the noise-free signal distorted by the $M \times K$ MIMO LTI channel, $\mathbf{u}[n] = [u_1[n], u_2[n], \ldots, u_K[n]]^T$ is the $K \times 1$ vector source signal, and $\mathbf{w}[n] = [w_1[n], w_2[n], \ldots, w_M[n]]^T$ is the $M \times 1$ vector noise.

It can be seen, from (9) and (10), that in addition to the ISI, the MIMO system also involves MAI, because each component of $\mathbf{x}_S[n]$ is a mixture of all the source signals $u_k[n]$, $k = 1,2,\ldots,K$. Accordingly, blind equalization of the MIMO channel $\mathbf{H}[n]$ is a problem of eliminating both the ISI and MAI, or equivalently, recovering the source signal $\mathbf{u}[n]$ with only the received signal $\mathbf{x}[n]$. The problem arises not only in the above-mentioned multiple-antenna systems but also in multiuser DS/CDMA systems [76], [100], [101], [130], [131], [144]. It reduces to the problem of SIMO blind equalization when there is only one source signal transmitted or when fractionally spaced equalization is employed in single-antenna wireless communication systems [128], [140]. Moreover, it also arises in applications using multiple sensors such as time delay estimation, source separation, and seismic signal processing. It should be noticed that, for the SIMO case ($K = 1$), a lot of SOS-based blind channel equalization algorithms (usually referred to as subspace algorithms), as reported in [86], [96], [121], [123], [128] have been developed based on the equalizability of SIMO channels under certain assumptions. One may refer to [124] for an overview of these techniques as well as the embedded assumptions/restrictions.

In the past decade, blind equalization of MIMO channels using HOS has been extensively reported [21], [32], [46], [60]-[63], [70], [72], [95], [109], [136], [139], [150], which is generally based on the following assumptions for $\mathbf{x}[n]$.

▲ (M-A1) The $M \times K$ LTI channel $\mathbf{H}[n]$ is stable.

▲ (M-A2) The source signal $u_k[n]$, $k \in \{1,2,\ldots,K\}$, is a zero-mean, i.i.d. non-Gaussian random process with variance $\sigma_u^2[k] = E\{|u_k[n]|^2\}$ and $(p+q)$th-order cumulant $\gamma_{p,q}[k] = C_{p,q}\{u_k[n]\} \neq 0$ and is statistically independent of $u_j[n]$ for all $j \neq k$.

▲ (M-A3) The noise $\mathbf{w}[n]$ is a zero-mean, Gaussian vector random process, which can be spatially correlated and temporally colored with covariance matrix $\mathbf{R}_w[l] = E\{\mathbf{w}[n]\mathbf{w}^H[n-l]\}$.

▲ (M-A4) The source signal $\mathbf{u}[n]$ is statistically independent of the noise $\mathbf{w}[n]$.

Note that the assumption of statistical independence between the source signals is reasonable for multiuser communication systems with independent users. On the other hand, extension of MIMO blind equalization methods to the case of temporally colored inputs is also possible; see [65] and [77] for further details.

Let $\mathbf{v}[n] = [v_1[n], v_2[n], \ldots, v_M[n]]^T$ (as shown in Figure 4) denote a multiple-input, single-output (MISO) equalizer to be designed that consists of a bank of linear FIR filters, with $\mathbf{v}[n] \neq \mathbf{0}_M$ ($M \times 1$ zero vector) for $n = L_1, L_1 + 1, \ldots, L_2$ and length $L = L_2 - L_1 + 1$. The equalized signal can be expressed as

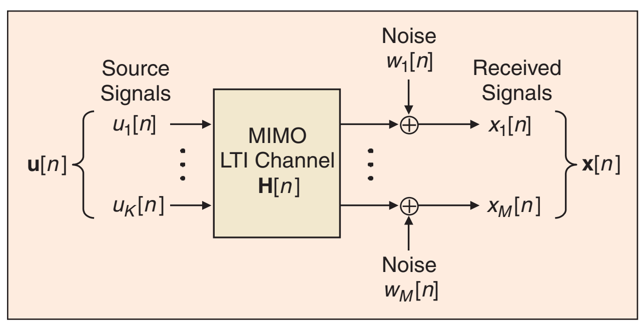
$$e[n] = \mathbf{v}^T[n] * \mathbf{x}[n] = e_S[n] + e_N[n] \tag{11}$$

where

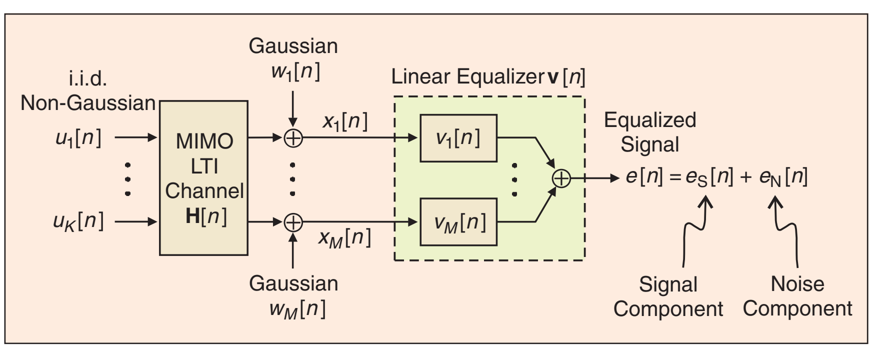
$$e_N[n] = \mathbf{v}^T[n] * \mathbf{w}[n] \text{ [by (9)]} \tag{12}$$

corresponds to the noise component in $e[n]$ and

$$e_S[n] = \mathbf{v}^T[n] * \mathbf{H}[n] * \mathbf{u}[n] = \mathbf{g}^T[n] * \mathbf{u}[n] \text{ [by (9)]} \tag{13}$$



▲ 3. The MIMO channel model.



▲ 4. Block diagram of MISO linear equalization.

is the corresponding signal component in which

$$\mathbf{g}[n] = [g_1[n], g_2[n], \ldots, g_K[n]]^{\mathrm{T}}$$
$$= \mathbf{H}^{\mathrm{T}}[n] \star \mathbf{v}[n] \tag{14}$$

is the overall system after equalization. The signal part given by (13) is depicted in Figure 5 in terms of the overall system $\mathbf{g}[n]$.

Under assumptions (M-A1) through (M-A4) and the condition of linear equalization, the problem of MIMO blind equalization can be partially resolved by adjusting the coefficients of the equalizer $\mathbf{v}[n]$ such that the signal component $e_S[n]$ approximates $u_\ell[n]$ (one of the source signals) with minimum enhancement of the noise component $e_N[n]$. Furthermore, under the following ideal conditions [136]:

▲ (M-C1) $M \geq K$, i.e., there are at least as many outputs as inputs

▲ (M-C2) $\mathcal{H}(z)$ (the transfer function of $\mathbf{H}[n]$) has full column rank for any $|z| = 1$

▲ (M-C3) the SNR associated with the received signal $\mathbf{x}[n]$ defined as

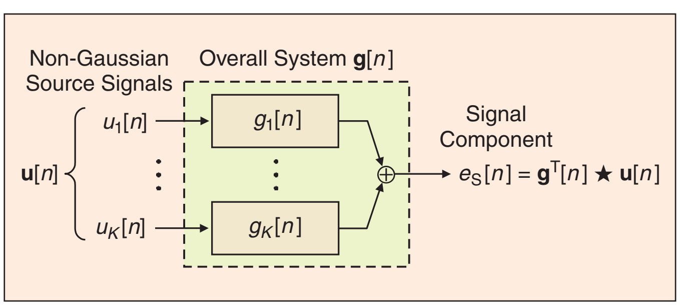
$$\mathrm{SNR} = \frac{E\{\|\mathbf{x}[n] - \mathbf{w}[n]\|^2\}}{E\{\|\mathbf{w}[n]\|^2\}} \text{ [see (9)]} \tag{15}$$

equals infinity

▲ (M-C4) the length of the equalizer $\mathbf{v}[n]$ is doubly infinite ($L_1 \to -\infty$ and $L_2 \to \infty$)

the problem reduces to finding the equalizer $\mathbf{v}[n]$ such that the equalized signal $e[n] = \alpha u_\ell[n-\tau]$ (perfect equalization) where α is a real/complex constant, τ is an integer, and $\ell \in \{1, 2, \ldots, K\}$. Correspondingly, $g_k[n] = \alpha\delta[n-\tau]$. $\delta[k-\ell]$ (the kth component of the resultant overall system $\mathbf{g}[n]$) for all k [see (13)], giving rise to the associated $\mathrm{ISI}(\eta) = 0$ where $\eta = [\ldots, g_1[-1], g_1[0], g_1[1], \ldots, g_K[-1], g_K[0], g_K[1], \ldots]^{\mathrm{T}}$ is a vector composed of all the entries of $\mathbf{g}[n]$, $n = \ldots, -1, 0, 1, \ldots$. Note that the equalizer $\mathbf{v}[n]$ needs to be doubly infinite [see (M-C4)] for perfect equalization of a general $\mathbf{H}[n]$ (could be a square matrix with infinite length). Specifically, if $\mathbf{H}[n]$ has finite impulse response with $M > K$, then a finite-length equalizer may suffice for achieving perfect equalization [137].

Note that only one input signal $u_\ell[n]$ with unknown ℓ can be obtained by $\mathbf{v}[n]$. Estimation of a specific input signal (specific ℓ) needs more prior information about the input or the associated channel depending on applications. Nevertheless, the K input signals can be estimated through the multistage successive cancellation (MSC) procedure [136] (which will be presented later) that may suffer from the problem of error propagation from stage to stage. On the other hand, the K input signals can also be obtained simultaneously using an MIMO linear equalizer of M inputs and K outputs (i.e., a filter bank) [72], [95], [97], [125], without going through the MSC procedure and thus avoiding error propagation effects. However, finding the coefficients of the MIMO equalizer is, in general, computationally demanding and may prohibit practical applications.



▲ 5. Equivalent block diagram of the signal part in Figure 4.

SISO Blind Equalization Algorithms

In this section, let us review the aforementioned typical IFC-based algorithm, SEA, and CMA for blind equalization of SISO channels. Then, their relations and performance under the condition of finite SNR are described, and several aspects of improvements on the convergence rate of these algorithms are presented.

Basic Algorithms

Let $\mathbf{v} = [v[L_1], v[L_1+1], \ldots, v[L_2]]^{\mathrm{T}}$ (associated with the equalizer $v[n]$) denote the $L \times 1$ unknown parameter vector to be determined. The equalized signal $e[n]$ given by (3) can be expressed in vector form as

$$e[n] = \mathbf{v}^{\mathrm{T}}\chi[n] \tag{16}$$

where $\chi[n] = [x[n-L_1], x[n-L_1-1], \ldots, x[n-L_2]]^{\mathrm{T}}$ is an $L \times 1$ vector associated with the received signal $x[n]$.

IFC-Based Algorithm

The IFC-based algorithm, proposed by Wiggins, Donoho, Shalvi and Weinstein, Tugnait, and Chi and Wu [7], [13], [14], [34], [116], [118], [133], [146], finds the optimum $\mathbf{v}$ by maximizing the following class of inverse filter criteria (also known as absolute normalized cumulants) using only two cumulants:

$$J_{p,q}(\mathbf{v}) = \frac{|C_{p,q}\{e[n]\}|}{[C_{1,1}\{e[n]\}]^{(p+q)/2}} \tag{17}$$

where p and q are nonnegative integers and $p+q \geq 3$. Note that (17) represents a general form of equalization criteria using absolute cumulants. For instance, it can be easily shown that the IFC $J_{p,q}$ with $p = q = 2$ is equivalent to the constrained kurtosis-based criterion proposed by Shalvi and Weinstein [116], [118]. In addition, the IFC $J_{p,q}$ with $p+q = 3$ and 4 are usually preferable to those for $p+q > 4$ due to the fact that the larger the cumulant order used, the larger the variance of the designed equalizer and the computational complexity [84], [90], [91].

A remark regarding the IFC is as follows.

▲ (R3.1) Closed-form solution for determining the optimum ν is almost formidable since $J_{p,q}$ is a highly nonlinear function of ν. Accordingly, iterative gradient-type optimization algorithms such as the steepest descent algorithm and the Fletcher-Powell algorithm [5] can be used to find the (local) maximum of $J_{p,q}$ as well as the relevant ν, where an initial condition for ν is needed to initialize the iterative optimization algorithms.

Super-Exponential Algorithm

The SEA, proposed by Shalvi and Weinstein [117], [118], is an iterative algorithm for finding the equalizer $v[n]$. At the ith iteration, it updates the vector ν via the following set of linear equations [36], [117], [118]:

$$\begin{cases} \tilde{\nu} = (\mathbf{R}_{\chi}^{*})^{-1} \cdot \mathbf{d}_{e\chi}^{[i-1]} \\ \nu^{[i]} = \tilde{\nu} / \|\tilde{\nu}\| \end{cases} \tag{18}$$

where

$$\mathbf{R}_{\chi} = E\left\{\chi[n]\chi^{\mathrm{H}}[n]\right\} \tag{19}$$

is the $L \times L$ correlation matrix of $\chi[n]$ and

$$\mathbf{d}_{e\chi}^{[i-1]} = \operatorname{cum}\left\{e^{[i-1]}[n]: p, (e^{[i-1]}[n])^{*}: q-1, \chi^{*}[n]\right\} \tag{20}$$

in which p and q are nonnegative integers and $p+q \geq 3$, and $e^{[i-1]}[n] = (\nu^{[i-1]})^{\mathrm{T}}\chi[n]$ is the equalized signal obtained at the $(i-1)$th iteration. Shalvi and Weinstein [117], [118] showed that under the conditions of (S-C1), (S-C2), and infinite data length, the update equations given by (18) lead the resultant ISI towards zero at a super-exponential (i.e., exponential to the power) rate, provided that the initial ISI is smaller than unity.

Three remarks regarding the SEA are as follows.

▲ (R3.2) The SEA given by (18)-(20) is slightly different from the original one reported in [117] and [118], which also involves the variance and $(p+q)$th-order cumulant of the source signal $u[n]$. Thus, the former is a little bit simpler than the latter, but it results in a designed equalizer with an unknown scale factor.

▲ (R3.3) The major advantages of the SEA are its faster convergence (at a super-exponential rate) and smaller computational load (only solving a set of linear equations at each iteration) over gradient-type algorithms (such as the IFC-based algorithm) [20].

▲ (R3.4) Without an explicit objective function, the iterative SEA may diverge as the data length, N, is insufficient and SNR is finite.

Constant Modulus Algorithm

Godard [45] proposed a family of algorithms, known as Godard-p algorithm, for the design of the optimum equalizer by minimizing the following objective function (see also [55] and [56]):

$$F_p(\nu) = E\left\{(|e[n]|^p - \Upsilon_p)^2\right\} \tag{21}$$

where p is a positive integer and

$$\Upsilon_p = \frac{E\left\{|u[n]|^{2p}\right\}}{E\left\{|u[n]|^{p}\right\}} \tag{22}$$

is referred to as constant modulus (CM). The special case, Godard-2 algorithm $(p=2)$, was developed independently by Treichler and Agee [126] and is well known as the CMA due to the design philosophy that the equalized signal $e[n]$ must be a signal with the same CM property as the desired source signal.

Two remarks regarding the CMA are as follows.

▲ (R3.5) In practice, without the prior knowledge about the statistics of the source signal, Υ_2 is usually replaced by an arbitrary positive number resulting in a scalar ambiguity in the equalizer output. The CMA works only when the kurtosis $\gamma_{2,2}$ of the source signal is negative (known as the sub-Gaussian case) [55], [64].

▲ (R3.6) The CMA needs iterative gradient-type optimization algorithms to obtain the (local) optimum ν because F_2 $(p=2)$ is a highly nonlinear function of ν, although most of the existing CM-based algorithms are sample-by-sample-based adaptive algorithms rather than batch processing algorithms.

Relations and Performance

Relations

Under the ideal conditions (S-C1) through (S-C3), the three basic algorithms have been shown to lead to the same solution $v[n] = \alpha h_1[n-\tau]$, except for a scale factor and a time delay. On the other hand, a number of works [16], [17], [49], [50], [82], [83], [106], [107], [113], [114], [151], [152] have been reported to provide the performance insights of these algorithms and their relations under the practical conditions of finite SNR and finite equalizer length L. Some of the related results are summarized as follows.

▲ (S-F1) The designed equalizer $v[n]$ using the IFC $J_{p,q}$ is equivalent to that using the SEA for the case of real signals, and this also holds true as $p=q\geq 2$ for the case of complex signals.

▲ (S-F2) As the source signal $u[n]$ possesses negative kurtosis $(\gamma_{2,2} < 0)$, the designed equalizer $v[n]$ using the CMA is equivalent to that using the IFC $J_{2,2}$ $(p=q=2)$ and, therefore, also equivalent to that using the SEA with $p=q=2$ according to (S-F1).

One can see from (S-F1) and (S-F2) that the IFC-based algorithm, SEA, and CMA are closely related to each other, implying that under certain conditions they exhibit similar performance and common properties regarding the behavior of the designed equalizers. Thus, for brevity, let us present these common properties via the IFC-based algorithm.

Properties About the Behavior of the Equalizer

For finite SNR, it is highly desirable that the designed blind equalizer $v[n]$ approximates the linear minimum mean square error (MMSE) equalizer, denoted $v_{\mathrm{MMSE}}[n]$ and hence possesses both the ISI reduction and noise reduction capabilities. Notice that the linear MMSE equalizer is a nonblind equalizer that minimizes the mean square error (MSE) $E\{|e[n]-u[n]|^2\}$ and requires training sequences to obtain the optimum solution [54], [56], [66], [122], [145]

$$V_{\mathrm{MMSE}}(\omega)=\mathcal{F}_n\left\{v_{\mathrm{MMSE}}[n]\right\}=\sigma_u^2\cdot\frac{H^*(\omega)}{S_{xx}(\omega)} \tag{23}$$

where $H(\omega)=\mathcal{F}_n\{h[n]\}$ and

$$\begin{aligned}S_{xx}(\omega)&=\mathcal{F}_l\left\{E\left\{x[n]x^*[n-l]\right\}\right\}\\&=\sigma_u^2\cdot|H(\omega)|^2+\mathcal{F}_l\{r_w[l]\}.\end{aligned} \tag{24}$$

The connection between the blind equalizer $v[n]$ (associated with $J_{p,q}$) and the linear MMSE equalizer $v_{\mathrm{MMSE}}[n]$ has been established as follows.

Property S-1

With infinite equalizer length, the optimum equalizer $v[n]$ associated with $J_{p,q}$ is related to the linear MMSE equalizer $v_{\mathrm{MMSE}}[n]$ via [38]

$$V(\omega)=\mathcal{F}_n\{v[n]\}=V_{\mathrm{MMSE}}(\omega)\cdot Q(\omega) \tag{25}$$

where

$$Q(\omega)=\frac{1}{\sigma_u^2}\left\{\alpha_{p,q}\gamma_{p,q}\cdot\widetilde{G}_{p,q}(\omega)+\alpha_{q,p}\gamma_{q,p}\cdot\widetilde{G}_{q,p}(\omega)\right\} \tag{26}$$

in which

$$\alpha_{p,q}=\frac{q}{p+q}\cdot\frac{C_{1,1}\{e[n]\}}{C_{p,q}\{e[n]\}}, \tag{27}$$

$$\widetilde{G}_{p,q}(\omega)=\mathcal{F}_n\left\{\widetilde{g}_{p,q}[n]\right\} \tag{28}$$

and

$$\widetilde{g}_{p,q}[n]=(g[n])^p\left(g^*[n]\right)^{q-1}. \tag{29}$$

Further observations on the relationship between $v[n]$ and $v_{\mathrm{MMSE}}[n]$ are inferred as follows.

Property S-2

With infinite equalizer length, the optimum equalizer $v[n]$ associated with $J_{p,q}$ approaches the linear MMSE equalizer $v_{\mathrm{MMSE}}[n]$ up to a scale factor and a time delay as SNR or the cumulant order $(p+q)$ increases. The former also approaches the latter as the channel has wider bandwidth for the white noise case [38].

This property implies that the equalizer $v[n]$ is equivalent to the linear MMSE equalizer $v_{\mathrm{MMSE}}[n]$ as 1) SNR $=\infty$ or $p+q=\infty$ or as 2) the channel is an allpass system when the noise is white. Moreover, a property regarding the phase response of $v[n]$ is as follows.

Property S-3

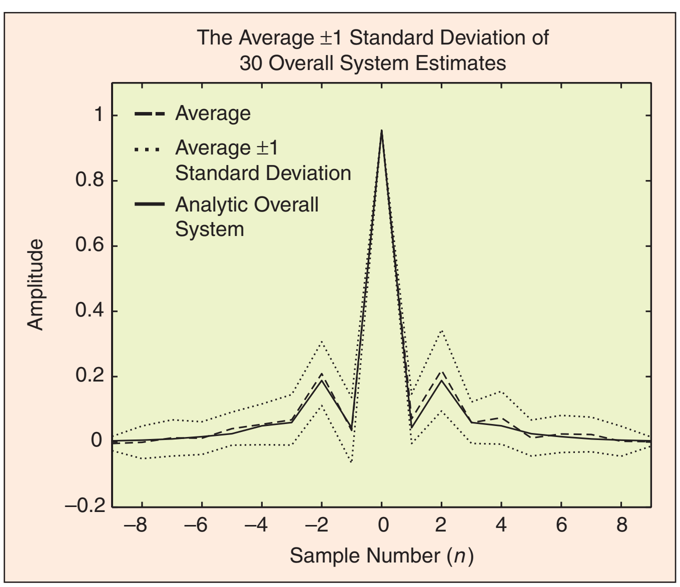
With infinite equalizer length, the phase response, $\arg[V(\omega)]$, of the optimum equalizer $v[n]$ associated with $J_{p,q}$ is given by

$$\arg[V(\omega)]=-\arg[H(\omega)]-\omega\tau+\xi,\quad -\pi\le\omega<\pi \tag{30}$$

where τ and ξ are real constants [38].

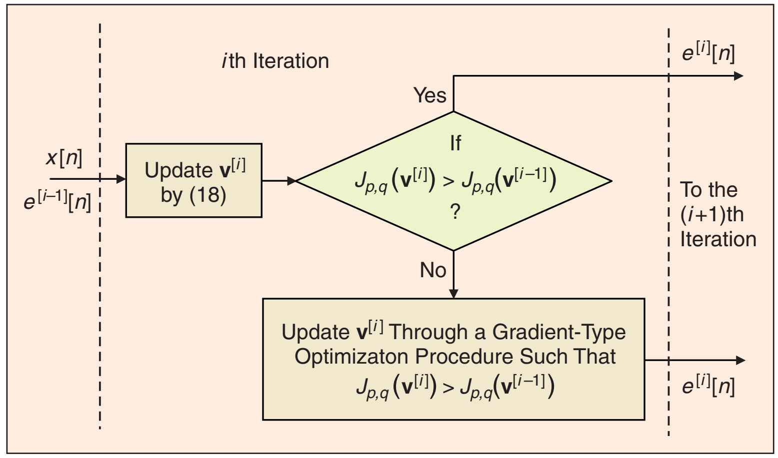
This property states that the equalizer $v[n]$ completely cancels (equalizes) the channel-induced phase distortion (except for a time delay τ and a constant phase shift ξ) and thus performs as a perfect phase equalizer.

Note that properties S-1 through S-3 of the optimum equalizer $v[n]$ associated with $J_{p,q}$ are valid for an infinite equalizer length. For any finite equalizer length L, these properties can still provide some performance insights of this equalizer. Moreover, some characteristics of $v[n]$ for finite L can be found in [107]. By the connection established in Property S-1, Feng and Chi [37]-[39] proposed a computationally efficient FFT-based iterative algorithm to obtain the theoretical optimum $v[n]$ associated with $J_{p,q}$ from the linear MMSE equalizer $V_{\mathrm{MMSE}}(\omega)$ given by (23) and (24). The obtained theoretical $v[n]$ (or the theoretical $g[n]$) provides a good prediction to the designed equalizer $v[n]$ (the overall system $g[n]$), as demonstrated in Figure 6, which shows the average and the average ± 1 standard deviation of 30 overall system estimates $\hat{g}[n]$s obtained from 30 sets of synthetic data using the IFC



▲ *6. Simulation results for verification of the calculated theoretical overall system g[n] [38].*

$J_{p,q}$ with $p+q=3$, where SNR = 0 dB, data length $N=2{,}048$, and the equalizer length $L=17$ ($L_1=0$ and $L_2=16$). One can see, from Figure 6, that the estimate $\hat{g}[n]$ approximates the theoretical $g[n]$ well in spite of the low SNR (0 dB). Moreover, $\hat{g}[n]$ is approximately zero-phase (symmetric), which is consistent with Property S-3.



▲ 7. *Signal processing procedure of the hybrid IFC algorithm.*

Algorithm Improvements

The convergence rate of the basic algorithms may be further improved using hybrid framework, whitening preprocessing, and better initial conditions, as described below.

Hybrid Framework

Recall that the computationally efficient SEA [see (R3.3)] is closely related to the IFC-based algorithm [see (S-F1)]. This suggests that the optimum ν associated with $J_{p,q}$ can be obtained using the following hybrid framework of the IFC-based algorithm and SEA [16].

Hybrid IFC Algorithm (see Figure 7)

At the ith iteration, the parameter vector $\nu^{[i]}$ is obtained through the following two steps:

▲ (T1) Obtain $\nu^{[i]}$ using the SEA update equations given by (18)-(20).

▲ (T2) If $J_{p,q}(\nu^{[i]}) > J_{p,q}(\nu^{[i-1]})$, go to the next iteration; otherwise, update $\nu^{[i]}$ through a gradient-type optimization algorithm such that $J_{p,q}(\nu^{[i]}) > J_{p,q}(\nu^{[i-1]})$.

Two remarks regarding the hybrid IFC algorithm are as follows.

▲ (R3.7) According to (S-F1), the hybrid IFC algorithm is always applicable for the case of real signals and only applicable for the case of complex signals as $p=q\geq 2$.

▲ (R3.8) In comparison with gradient-type algorithms for finding the maximum of $J_{p,q}$, the hybrid IFC algorithm exhibits fast convergence and significant computational saving by taking advantages of the SEA in (T1). Furthermore, the hybrid IFC algorithm does not suffer from the divergence problem of the SEA [see (R3.4)] due to the guaranteed convergence of (T2).

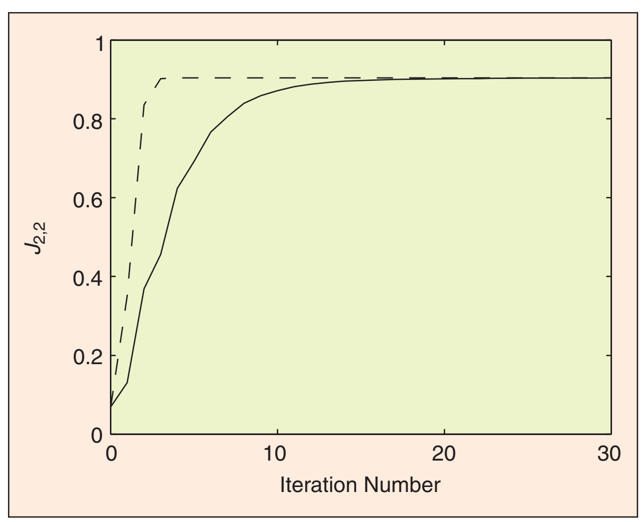
Figure 8 demonstrates the efficacy of the hybrid IFC algorithm, where 30 independent runs using the IFC-based algorithm (solid line) and the hybrid IFC algorithm (dashed line) were performed to obtain the averaged values of 30 $J_{2,2}(\nu)$s for SNR = 20 dB and data length $N=4{,}096$. It can be seen that the step (T1) of the hybrid IFC algorithm significantly improves the convergence rate of the IFC-based algorithm, as stated in (R3.8).

Whitening Preprocessing

It is well known [54], [56], [66], [79], [122] that when the signal of interest is processed by a whitening filter such as the forward LPE filter of sufficient length, the filter output is basically an amplitude equalized signal. This fact may be useful to all the existent blind equalization algorithms. As illustrated in Figure 9, the amplitude equalized signal $y[n]$ simplifies the equalization task of the equalizer $v[n]$ because basically only the channel phase distortion remains to be compensated by $v[n]$, thereby speeding up the convergence of the blind equalization algorithms. Note that as the channel is minimum phase, the LPE filter, which itself is also minimum phase, is sufficient for blind channel equalization.

Improved Initial Condition

Required by iterative blind equalization algorithms, the initial condition for $v[n]$ is usually chosen as $v^{[0]}[n]=\delta[n-L_C]$ for the sake of simplicity where $L_C=\lfloor (L_1+L_2)/2 \rfloor$. However, a better initial condition may be needed to further improve the convergence rate as well as reliability of the blind equalization algorithms in practical conditions. Next, let us present a low complexity method for providing a better initial condition [36].



▲ 8. *Averaged values of 30 $J_{2,2}(\nu)$s corresponding to the IFC-based algorithm (solid line) and the hybrid IFC algorithm (dashed line).*

Procedure for Obtaining Improved Initial Condition:

▲ (S1) Preprocess the data $x[n]$ by a (causal) forward LPE filter $a[n]$ of length $L_a < L$ and obtain the forward prediction error (the filter output) $y[n]$.

▲ (S2) Further process the forward prediction error $y[n]$ by an FIR filter $b[n]$ where $b[n] \neq 0$ for $n = L_1, L_1+1, \ldots, L_2 - L_a + 1$. Obtain the parameter vector $\mathbf{b} = [b[L_1], b[L_1+1], \ldots, b[L_2 - L_a + 1]]^{\mathrm{T}}$ via

$$\begin{cases} \mathbf{d}_y = \mathrm{cum}\left\{ y\left[n - \widetilde{L}\right]: p, y^*\left[n - \widetilde{L}\right]: q-1, \mathbf{y}^*[n] \right\} \\ \mathbf{b} = \mathbf{d}_y / \|\mathbf{d}_y\| \end{cases} \tag{31}$$

where

$$\mathbf{y}[n] = \left[y[n - L_1], y[n - L_1 - 1], \ldots, y[n - L_2 + L_a - 1] \right]^{\mathrm{T}}$$

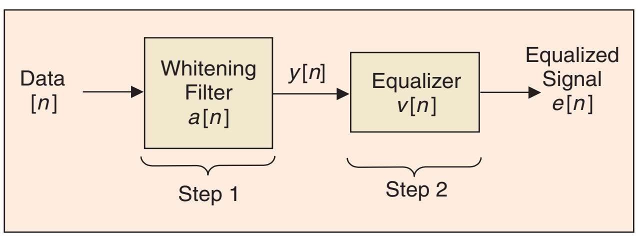
and $\widetilde{L} = \lfloor (L_1 + L_2 - L_a + 1)/2 \rfloor$.

▲ (S3) Obtain the initial condition $v^{[0]}[n] = a[n] * b[n]$.

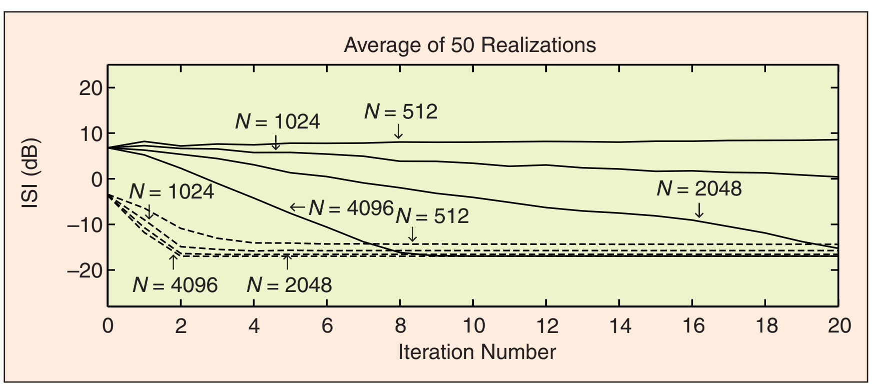
A remark regarding this procedure is as follows.

▲ (R3.9) The second step (S2) is a simplified version of the SEA given by (18)-(20) where the correlation matrix of $y[n]$ is replaced by an identity matrix (because $y[n]$ approximates an amplitude equalized signal) with $b^{[0]}[n] = \delta[n - \widetilde{L}]$ (i.e., $e^{[0]}[n] = b^{[0]}[n] * y[n] = y[n - \widetilde{L}]$). In other words, the improved initial condition is obtained through whitening preprocessing followed by one iteration of the SEA with reduced computational complexity.

Figure 10 demonstrates the efficacy of the improved initial condition, where SNR = 20 dB, $L_a = 5$, $L_1 = 0$, $L_2 = 20$, and $L_C = 10$. From this figure, one can see that the initial values of ISI(η) associated with the improved initial condition $v^{[0]}[n] = a[n] * b[n]$ (dashed lines) are about 10 dB below those associated with the initial condition $v^{[0]}[n] = \delta[n-10]$ (solid lines). Moreover, one can observe that the SEA using the improved initial condition works well for all data length N with much faster convergence speed than that using the initial condition $v^{[0]}[n] = \delta[n-10]$ and the latter converges slower for smaller N (1,024, 2,048, and 4,096) and diverges for $N = 512$ [as mentioned in (R3.4)]. These results indicate that the improved initial condition significantly improves the convergence and reliability of the SEA.



▲ *9. Blind equalization with whitening preprocessing.*



▲ *10. Averaged values of 50 ISI(η)s (in dedcibels) for the SEA using the initial condition $v^{[0]}[n] = \delta[n-10]$ (solid lines) and the improved initial condition (dashed lines) [36], [37].*

MIMO Blind Equalization Algorithms

This section reviews the typical IFC-based algorithm, SEA, and CMA for blind equalization of MIMO channels, together with their relations, performance, and convergence improvements. To distinguish the MIMO case from the SISO case, let us refer to the three algorithms as the M-IFC-based algorithm, M-SEA, and M-CMA.

Basic Algorithms

Estimation of One Source Signal

Let $\mathrm{v}_m = [v_m[L_1], v_m[L_1+1], \ldots, v_m[L_2]]^{\mathrm{T}}$, and $\mathrm{v} = [\mathrm{v}_1^{\mathrm{T}}, \mathrm{v}_2^{\mathrm{T}}, \ldots, \mathrm{v}_M^{\mathrm{T}}]^{\mathrm{T}}$ denote the $ML \times 1$ unknown parameter vector (associated with the equalizer $\mathbf{v}[n]$) to be determined. The equalized signal $e[n]$ given by (11), the output of the MISO equalizer $\mathbf{v}[n]$, can be expressed as

$$e[n] = \mathrm{v}^{\mathrm{T}} \chi[n] \tag{32}$$

where $\chi[n] = [\chi_1^{\mathrm{T}}[n], \chi_2^{\mathrm{T}}[n], \ldots, \chi_M^{\mathrm{T}}[n]]^{\mathrm{T}}$ is an $ML \times 1$ vector associated with the received signal $\mathbf{x}[n]$ in which

$$\chi_m[n] = \left[x_m[n - L_1], x_m[n - L_1 - 1], \ldots, x_m[n - L_2] \right]^{\mathrm{T}}.$$

Comparing the formulation (32) with (16) reveals that finding the vector v would be similar to the SISO case. In fact, the vector v can be obtained using the IFC-based algorithm given by (17) [21], [136] and the SEA given by (18)-(20) [63], [150] without any modifications. However, for the CMA given by (21) and (22) [127], [137], the constant modulus Υ_2 must be modified as

$$\Upsilon_2[k] = \frac{E\left\{|u_k[n]|^4\right\}}{E\left\{|u_k[n]|^2\right\}}. \tag{33}$$

It has been shown that under the conditions (M-C1) through (M-C4), the three algorithms all lead to the equalized signal

$$e[n] = \alpha u_\ell[n - \tau] \tag{34}$$

where α is a real/complex constant, τ is an integer, and $\ell \in \{1, 2, \ldots, K\}$ is an unknown index.

Some remarks regarding the basic algorithms are as follows.

▲ (R4.1) Similar to the SISO case [see (R3.3) and (R3.4)], the M-SEA is computationally efficient with faster convergence than the M-IFC-based algorithm but may diverge for finite data length N and finite SNR.

▲ (R4.2) The M-CMA can only recover those input signals with negative kurtosis. Even if $\Upsilon_2[k]$ (corresponding to an interested input signal $u_k[n]$) is given for the M-CMA, the algorithm may still recover a different input signal as given by (34). Therefore, $\Upsilon_2[k]$ is usually chosen as an arbitrary positive number when using the M-CMA. Recently, Schniter and Johnson [112] have reported the sufficient conditions under which the M-CMA will locally converge to an equalizer associated with the desired input source signal with a particular delay.

▲ (R4.3) Let $\mathbf{h}_k[n]$ (an $M \times 1$ vector) be the kth column of the $M \times K$ channel impulse response matrix $\mathbf{H}[n]$ and express the received signal $\mathbf{x}[n]$ given by (9) as

$$\mathbf{x}[n] = [\mathbf{h}_1[n], \mathbf{h}_2[n], \ldots, \mathbf{h}_K[n]] * \mathbf{u}[n] + \mathbf{w}[n] \\ = \sum_{k=1}^{K} \mathbf{h}_k[n] * u_k[n] + \mathbf{w}[n]. \tag{35}$$

With the source signal estimate $\hat{u}_\ell[n] = e[n]$ by (34) (up to an unknown scale factor and an unknown time delay), the channel $\mathbf{h}_\ell[i]$ can be estimated as [136]

$$\hat{\mathbf{h}}_\ell[i] = \frac{E\left\{\mathbf{x}[n+i]\hat{u}_\ell^*[n]\right\}}{E\left\{|\hat{u}_\ell[n]|^2\right\}} \tag{36}$$

where $\ell \in \{1, 2, \ldots, K\}$.

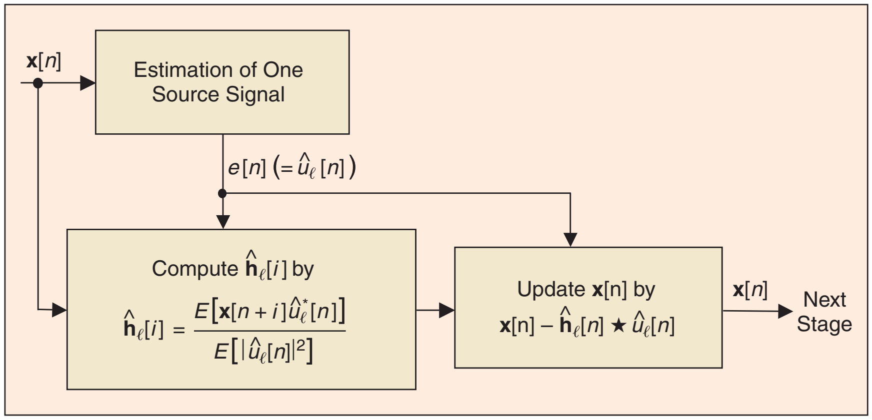
In addition to the reviewed three MIMO blind equalization algorithms (M-IFC, M-SEA, and M-CMA), we note that there exist some other related methods such as the contrast criteria proposed by Comon [24] for blind source separation from noisy multichannel observations. One can refer to [24]-[26] for the details.

Estimation of All Source Signals

Estimates $\hat{u}_1[n], \hat{u}_2[n], \ldots, \hat{u}_K[n]$ of all the source signals can be obtained by the M-IFC, M-SEA, or M-CMA through an MSC procedure [136] (possibly in a nonsequential order) that includes the following two steps at each stage.

MSC Procedure (see Figure 11)

▲ (T1) Find a source signal estimate, said $\hat{u}_\ell[n]$ (where ℓ is unknown), by the M-IFC, M-SEA, or M-CMA, and obtain the associated channel estimate $\hat{\mathbf{h}}_\ell[n]$ by (36).



▲ *11. Signal processing procedure in each stage of the MSC procedure.*

▲ (T2) Update $\mathbf{x}[n]$ by $\mathbf{x}[n] - \hat{\mathbf{h}}_\ell[n] * \hat{u}_\ell[n]$, namely, cancel the contribution of $\hat{u}_\ell[n]$ from $\mathbf{x}[n]$.

Relations and Performance

Relations

Under the conditions of finite SNR and finite equalizer length, the relation between the M-IFC-based algorithm and M-SEA, as summarized below, was reported by Chi and Chen [10], [20], while their relations to the M-CMA still need to be explored.

▲ (M-F1) The designed equalizer $\mathbf{v}[n]$ using the M-IFC is equivalent to that using the M-SEA for the case of real signals, and this also holds true for the case of complex signals as $p = q \geq 2$.

This fact reveals that the M-IFC is closely related to the M-SEA, and the designed equalizers by them thus exhibit similar performance and behaviors. Next, let us present some properties of the M-IFC that are therefore shared by the M-SEA.

Properties About the Behavior of the Equalizer

Let $\mathbf{D}_{p,q}$ be a $K \times K$ diagonal matrix defined as

$$\mathbf{D}_{p,q} = \mathrm{diag}\left\{\gamma_{p,q}[1], \gamma_{p,q}[2], \ldots, \gamma_{p,q}[K]\right\} \tag{37}$$

where $\mathbf{D}_{1,1} = \mathrm{diag}\{\sigma_u^2[1], \sigma_u^2[2], \ldots, \sigma_u^2[K]\}$. It is known that the nonblind MIMO linear MMSE equalizer, denoted $\mathcal{V}_{\mathrm{MMSE}}(\omega)$ ($K \times M$ matrix), can be derived by orthogonality principle [122] as

$$\mathcal{V}_{\mathrm{MMSE}}^{\mathrm{T}}(\omega) = \left[\mathbf{V}_{\mathrm{MMSE}}^{[1]}(\omega), \mathbf{V}_{\mathrm{MMSE}}^{[2]}(\omega), \ldots, \mathbf{V}_{\mathrm{MMSE}}^{[K]}(\omega)\right] \\ = \left[\mathcal{S}_{xx}^{\mathrm{T}}(\omega)\right]^{-1} \cdot \mathcal{H}^*(\omega) \cdot \mathbf{D}_{1,1} \tag{38}$$

where $\mathbf{V}_{\mathrm{MMSE}}^{[k]}(\omega) = \mathcal{F}_n\{\mathbf{v}_{\mathrm{MMSE}}^{[k]}[n]\}$ is an $M \times 1$ MISO linear MMSE equalizer associated with $u_k[n]$, $\mathcal{H}(\omega) = \mathcal{F}_n\{\mathbf{H}[n]\}$, and

$$\mathcal{S}_{xx}(\omega) = \mathcal{F}_l\left\{E\left\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-l]\right\}\right\} \\ = \mathcal{H}(\omega) \cdot \mathbf{D}_{1,1} \cdot \mathcal{H}^{\mathrm{H}}(\omega) + \mathcal{F}_l\{\mathbf{R}_w[l]\} \tag{39}$$

is the cross-spectral matrix of $\mathbf{x}[n]$.

Two properties of the equalizer $\mathbf{v}[n]$ associated with the M-IFC-based algorithm for any SNR are summarized as follows [10], [20].

Property M-1

With infinite equalizer length, the equalizer $\mathbf{v}[n]$ associated with $J_{p,q}$ is related to the MIMO linear MMSE equalizer $\mathcal{V}_{\text{MMSE}}(\omega)$ via

$$\mathbf{V}(\omega) = \mathcal{V}_{\text{MMSE}}^{\text{T}}(\omega) \cdot \mathbf{Q}(\omega) \tag{40}$$

where

$$\mathbf{Q}(\omega) = \mathbf{D}_{1,1}^{-1}\left\{\alpha_{p,q}\mathbf{D}_{p,q}\widetilde{\mathbf{G}}_{p,q}(\omega) + \alpha_{q,p}\mathbf{D}_{q,p}\widetilde{\mathbf{G}}_{q,p}(\omega)\right\} \tag{41}$$

in which $\alpha_{p,q}$ is given by (27),

$$\widetilde{\mathbf{G}}_{p,q}(\omega) = \left[\mathcal{F}_n\left\{\widetilde{g}_{p,q}[n;1]\right\}, \mathcal{F}_n\left\{\widetilde{g}_{p,q}[n;2]\right\}, \ldots, \mathcal{F}_n\left\{\widetilde{g}_{p,q}[n;K]\right\}\right]^{\text{T}} \tag{42}$$

and

$$\widetilde{g}_{p,q}[n;k] = (g_k[n])^p \left(g_k^*[n]\right)^{q-1}. \tag{43}$$

Property M-2

As the noise $w_m[n]$ (the mth component of $\mathbf{w}[n]$) is uncorrelated with $w_j[n]$ for all $m \neq j$, each component of the optimum overall system $\mathbf{g}[n]$ associated with $J_{p,q}$ is linear phase for infinite equalizer length, i.e.,

$$\arg[G_k(\omega)] = \omega\tau_k + \xi_k, \quad -\pi \leq \omega < \pi \tag{44}$$

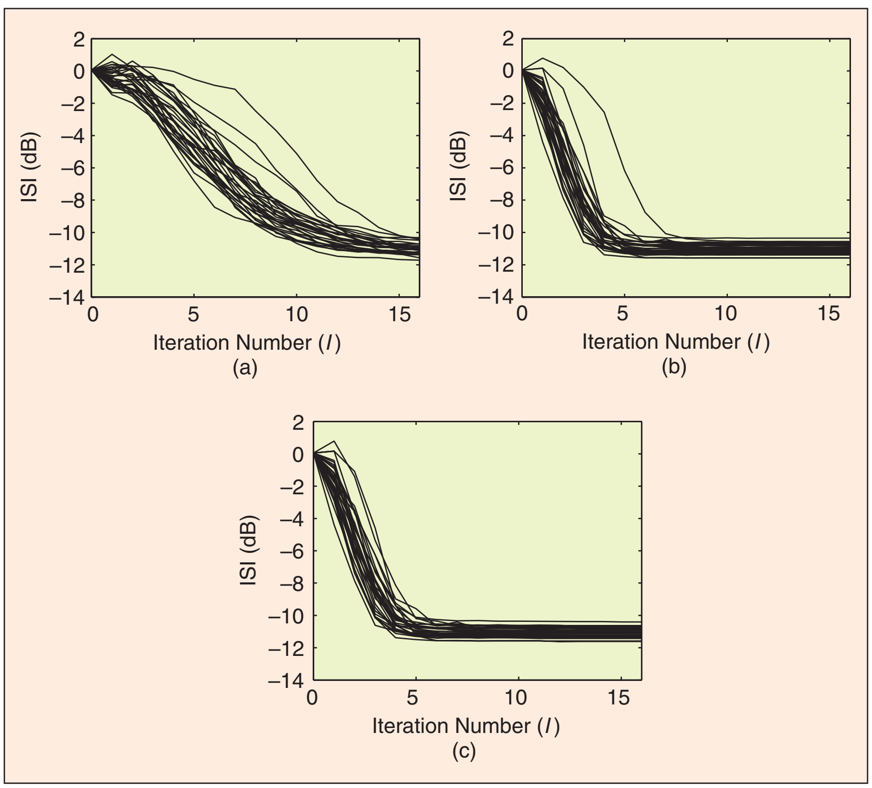
where $G_k(\omega) = \mathcal{F}_n\{g_k[n]\}$, and τ_k and ξ_k are real constants.

As in the SISO case, properties M-1 and M-2 of the equalizer $\mathbf{v}[n]$ associated with $J_{p,q}$ are valid for infinite equalizer length, and they can still provide some performance insights of this equalizer for finite L. Some characteristics of $\mathbf{v}[n]$ for finite L can be found in [108] and [115]. According to the connection established in property M-1, the theoretical optimum $\mathbf{v}[n]$ associated with $J_{p,q}$ can also be efficiently obtained from $\mathcal{V}_{\text{MMSE}}(\omega)$ given by (38) and (39) using FFT-based iterative algorithm, as reported in [10] and [20].

Hybrid IFC Algorithm

The fact of (M-F1) implies that the hybrid IFC algorithm is still applicable for the MIMO case, which is referred to as the M-hybrid IFC algorithm [20] to distinguish between the SISO case and the MIMO case. In other words, the optimum $\mathbf{v}$ associated with the M-IFC can be obtained by virtue of the M-SEA in the first step (T1) when $\mathbf{x}[n]$ is real or when $\mathbf{x}[n]$ is complex and $p = q \geq 2$. Moreover, the M-hybrid IFC algorithm can also be combined with the MSC procedure for the estimation of all the source signals.

Figure 12 demonstrates the efficacy of the M-hybrid IFC algorithm, where a two-input, two-output channel was considered. The data $\mathbf{x}[n]$ were synthesized for data length $N = 900$ and SNR [defined by (15)] equal to 15 dB. Thirty independent runs were performed using the M-IFC-based algorithm, M-SEA, and M-Hybrid IFC algorithm (all with $p = q = 2$) to obtain the equalizer $\mathbf{v}[n]$ and the corresponding 30 ISI($\boldsymbol{\eta}$)s. Figure 12 exhibits that the M-IFC-based algorithm, M-SEA, and M-hybrid IFC algorithm result in similar ISI reduction after convergence, while the M-hybrid IFC algorithm converges faster than the other two algorithms.



▲ 12. *Thirty ISI($\boldsymbol{\eta}$)s versus iteration number, I, for equalization of a two-input, two-output channel using (a) the M-IFC-based algorithm, (b) M-SEA, and (c) M-Hybrid IFC algorithm all with $p = q = 2$ [10], [20].*

Applications of Blind Equalization Algorithms

Numerous applications of the aforementioned blind equalization algorithms have been reported such as channel estimation, time delay estimation (TDE), SNR boost by maximum ratio combining, beamforming for source separation in multipath, and multiuser detection for DS/CDMA systems, as illustrated below.

Blind Channel Estimation

Recalling the signal model given by (1) [or (9)], blind channel estimation (BCE) is a fundamental problem in communications that estimates the unknown channel $h[n]$ (or $\mathbf{H}[n]$) with only the channel output measurements $x[n]$ (or $\mathbf{x}[n]$). Applying the aforementioned blind equalization algorithms to BCE can be performed through the following two steps.

BCE Procedure:

▲ (T1) Process the given data $\chi[n]$ (associated with $x[n]$ or $\mathbf{x}[n]$) by the equalizer ν (associated with $v[n]$ or $\mathbf{v}[n]$) using the aforementioned blind equalization algorithms.

▲ (T2) Obtain the channel estimate through the equalized signal $e[n] = \nu^T \chi[n]$ or the equalizer ν.

Different signal processing in (T2) leads to different channel estimates, as described as follows.

SISO Channels

Assume that the data $x[n]$ are generated from (1) and (2) under assumptions (S-A1) through (S-A4). With the source signal estimate $\hat{u}[n] = e[n]$ (obtained by (T1) of the BCE procedure), the channel $h[n]$ can be simply estimated via

$$\hat{h}[i] = \frac{E\left\{x[n+i]\hat{u}^*[n]\right\}}{E\left\{|\hat{u}[n]|^2\right\}}. \tag{45}$$

When SNR $= \infty$, the equalized signal $e[n] = \alpha u[n-\tau]$, giving rise to the channel estimate $\hat{h}[n] = h[n]$ (except for a scale factor and a time delay). When SNR is finite, however, the resultant $\hat{h}[n]$ has bias due to the noise in $x[n]$ and the estimation error $e_N[n]$ in the input signal estimate $\hat{u}[n]$.

To provide noise-insensitive channel estimate $\hat{h}[n]$, Chi and Feng [37], [15] proposed an iterative FFT-based algorithm for (T2) of the BCE procedure that is based on the properties of the designed equalizer presented earlier. From (23) and (25), it follows that

$$|H(\omega)| \propto S_{xx}(\omega) \cdot \frac{|V(\omega)|}{|Q(\omega)|}, \tag{46}$$

which can be used to provide the magnitude response estimate for $h[n]$. Since $Q(\omega)$ is a nonlinear function of the overall system $g[n]$ and thus the channel $h[n]$, Chi and Feng's algorithm iteratively updates the estimate $|\hat{H}(\omega)|$ using (46) with a chosen initial condition for $|\hat{H}(\omega)|$ (e.g., $|\hat{H}(\omega)| = 1\ \forall\omega$), where the power spectrum $S_{xx}(\omega)$ can be estimated using conventional power spectrum estimation methods [54], [56], [66], [122]. The phase response of $h[n]$, on the other hand, is estimated via $\arg[\hat{H}(\omega)] = -\arg[\hat{V}(\omega)]$ due to Property S-3. As a result, the obtained estimate $\hat{h}[n]$ is equivalent to $h[n]$ (up to a scale factor and a time delay). A remark regarding Chi and Feng's iterative FFT-based algorithm is as follows.

The convergence rate of the basic algorithms may be further improved using hybrid framework, whitening preprocessing, and better initial conditions.

▲ (R5.1) Chi and Feng's algorithm is never limited by the length of $\hat{h}[n]$ as long as the FFT length used is chosen sufficiently large such that aliasing effects on the resultant $\hat{h}[n]$ are negligible.

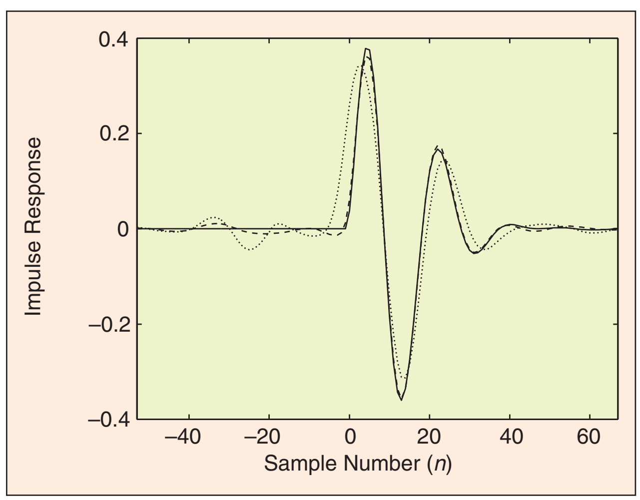
Figure 13 shows some simulation results for estimation of an ARMA(5,3) narrowband channel using (45) and Chi and Feng's FFT-based algorithm where SNR = 10 dB and data length $N = 8000$. This figure reveals that the channel estimate $\hat{h}[n]$ obtained by Chi and Feng's algorithm (dashed line) is much closer to $h[n]$ (solid line) than using (45) (dotted line) for this case.

SIMO Channels

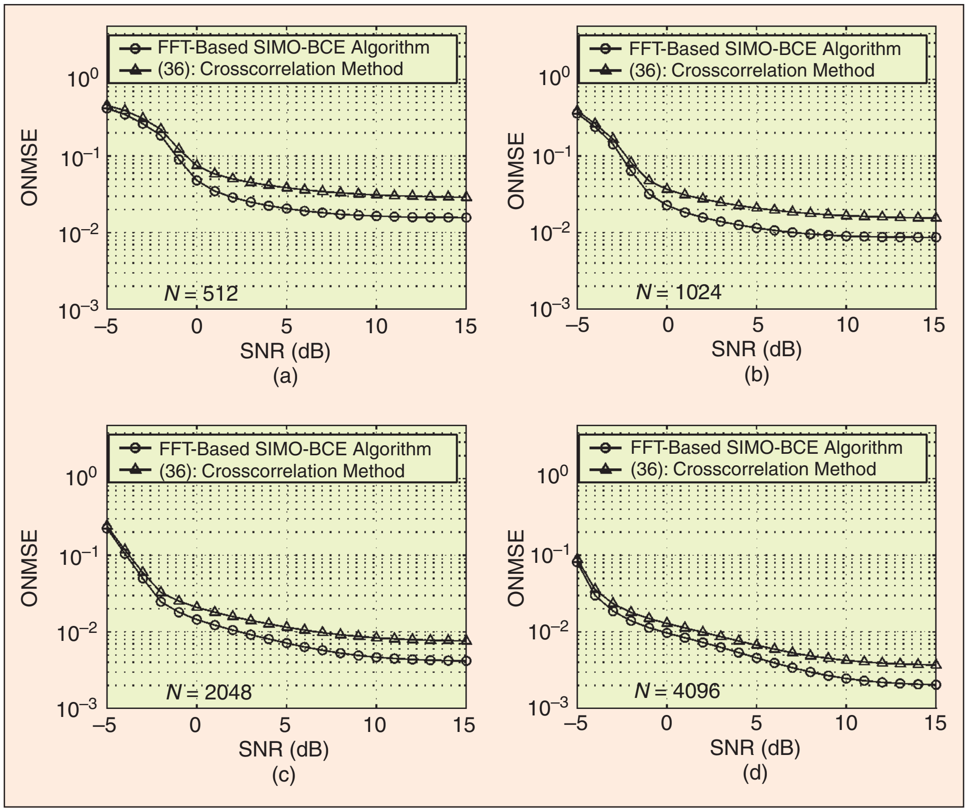
Consider that the data $\mathbf{x}[n]$ are generated from (9) with assumptions (M-A1) through (M-A4). As the number of source signals is equal to one (i.e., $K = 1$), the data $\mathbf{x}[n]$ reduces to the SIMO model as follows:

$$\mathbf{x}[n] = \sum_{i=-\infty}^{\infty} \mathbf{h}_1[i]u_1[n-i] + \mathbf{w}[n] \quad \text{[see (35)]}. \tag{47}$$

With the source signal estimate $\hat{u}_1[n] = e[n]$ (obtained by (T1) of the BCE procedure), the channel $\mathbf{h}_1[n]$ (al-



▲ *13. Simulation results of SISO blind channel estimation for SNR = 10 dB and N = 8,000. The average of 30 channel estimates using (46) (dotted line) and that using Chi and Feng's FFT-based algorithm (dashed line) together with the true channel impulse response (solid line).*



▲ 14. Simulation results for blind 3×1 channel estimation. Plots of averaged ONMSEs associated with the FFT-based SIMO-BCE algorithm and the crosscorrelation method using (36), respectively, versus SNR for data length (a) $N=512$, (b) $N=1{,}024$, (c) $N=2{,}048$, and (d) $N=4{,}096$.

lowed to have common subchannel zeros and with no need of the channel order information) can be simply estimated via (36) with $\ell=1$. As the SNR is finite, the resultant $\hat{\mathbf{h}}_1[n]$ has bias as mentioned in the SISO case above.

Chi et al. [22] further proposed a noise-insensitive approach for (T2) of the BCE procedure that is based on the properties of the designed equalizer presented previously. Letting $p=q$ and $K=1$ in (38) and (40) results in

$$\mathcal{H}^*(\omega)=\left(\mathcal{F}_n\{\mathbf{h}_1[n]\}\right)^*=\alpha\cdot\frac{\mathcal{S}_{xx}^{\mathrm{T}}(\omega)\mathbf{V}(\omega)}{\mathcal{F}_n\{\widetilde{\mathcal{G}}_{p,p}[n;1]\}} \tag{48}$$

where $\alpha>0$ is a constant. Based on this relationship, Chi et al. [22] proposed an SIMO-BCE criterion as well as an iterative FFT-based SIMO-BCE algorithm for estimating $\mathbf{h}_1[n]$ from the obtained M-IFC equalizer $\mathbf{v}[n]$ and the cross-spectral matrix $\mathcal{S}_{xx}(\omega)$ of $\mathbf{x}[n]$.

Some simulation results for blind 3×1 channel estimation are shown in Figure 14. This figure displays the averaged overall normalized mean square error (ONMSE) (defined as the averaged mean square error of $\hat{\mathbf{h}}_1[n]$ over all subchannels) of 100 independent runs using (36) and the FFT-based SIMO-BCE algorithm using $J_{2,2}$, respectively, for different data lengths and SNRs [defined by (15)]. One can see in Figure 14 that the FFT-based SIMO-BCE algorithm performs better than that using (36).

MIMO Channels

Given $\mathbf{x}[n]$ modeled by (9) with assumptions (M-A1) through (M-A4), the estimate of the MIMO channel $\mathbf{H}[n]$ can be obtained using the blind equalization algorithms and the MSC procedure [136]. As mentioned in (T1) of the MSC procedure (see Figure 11), one column of $\mathbf{H}[n]$ can be estimated by (36) in each stage. Therefore, the whole MIMO system estimate can be obtained after K stages of the MSC procedure. Due to the same reason as the SIMO case, the estimate of each column of the MIMO system has bias for finite SNR. In addition, the estimate of a column of $\mathbf{H}[n]$ obtained at the kth stage may suffer from severer estimation error for larger k due to the error propagation problem of the MSC procedure. On the other hand, possible improvements by the MIMO extension of Chi et al.'s FFT-based SIMO-BCE algorithm are currently under study.

Simultaneous Estimation of Multiple Time Delays

The estimation of time delay(s) between received signals at two (or more) sensor locations is crucial in many signal processing areas such as direction of arrival and range estimation in multisensor arrays, sonar, radar, biomedicine, and geophysics [59], [133], [73], [23]. Consider that a single source signal $x_S[n]$ modeled by (2) is transmitted and received by M (≥ 2) spatially separated sensors. The received signal vector $\mathbf{x}[n]$ can be modeled as

$$\begin{aligned}\mathbf{x}[n]&=\mathbf{x}_S[n]+\mathbf{w}[n]\\&=[x_S[n],a_2x_S[n-d_2],\dots,a_Mx_S[n-d_M]]^{\mathrm{T}}+\mathbf{w}[n]\end{aligned} \tag{49}$$

where a_m and d_m, $m=2,3,\dots,M$, are (real or complex) gains and integer time delays, respectively, and $\mathbf{w}[n]$ is a zero-mean Gaussian noise. The goal here is to estimate all the $(M-1)$ time delays $\{d_2,d_3,\dots,d_M\}$ simultaneously from the measurements $\mathbf{x}[n]$. Unlike the conventional TDE approaches which estimate a single time delay with the two associated sensor measurements at a time, approaches processing all the measurements simultaneously are insensitive to the distribution of SNRs of sensors.

Substituting (2) into (49) gives rise to the following SIMO model:

$$\mathbf{x}[n] = \sum_{i=-\infty}^{\infty} \mathbf{h}[i]u[n-i] + \mathbf{w}[n] \tag{50}$$

where $u[n]$ is zero-mean, i.i.d. non-Gaussian and

$$\mathbf{h}[n] = [h[n], a_2 h[n-d_2], \ldots, a_M h[n-d_M]]^{\mathrm{T}}. \tag{51}$$

Note that all the M sub-channels of the SIMO system $\mathbf{h}[n]$ given by (51) have the same zeros. The time delays $\{d_2, d_3, \ldots, d_M\}$ can be extracted from $\mathbf{h}[n]$, which can be estimated from the measurements $\mathbf{x}[n]$ ahead of time by means of SIMO BCE algorithms. On the other hand, in light of the specific form of $\mathbf{h}[n]$ given by (51) and the fact that the phase information of $\mathbf{h}[n]$ is sufficient for retrieving the multiple time delays, the estimation of $\{d_2, d_3, \ldots, d_M\}$ can be quite efficient as illuminated below.

According to the relation given by (48), let

$$\psi(\omega) = [\psi_1(\omega), \psi_2(\omega), \ldots, \psi_M(\omega)]^{\mathrm{T}} = -\arg\left\{\mathcal{S}_{xx}^{\mathrm{T}}(\omega)\mathbf{V}(\omega)\right\} \tag{52}$$

where $\mathbf{V}(\omega)$ is the designed MISO equalizer associated with the measurements $\mathbf{x}[n]$ and $\mathcal{S}_{xx}(\omega)$ is the cross-spectral matrix. Note that $\mathbf{V}(\omega)$ and $\mathcal{S}_{xx}(\omega)$ can be obtained using the MIMO blind equalization algorithms and the multichannel spectral estimator [66], respectively. By letting

$$B(\omega) = \left[1, e^{j(\psi_2(\omega)-\psi_1(\omega))}, \ldots, e^{j(\psi_M(\omega)-\psi_1(\omega))}\right]^{\mathrm{T}} \tag{53}$$

The analytic results for these algorithms provide a perspective on the behavior of the designed blind equalizers as well as the relation to the nonblind linear MMSE equalizer.

and

$$\mathbf{b}[n] = [b_1[n], b_2[n], \ldots, b_M[n]]^{\mathrm{T}} = \mathcal{F}_n^{-1}\{B(\omega)\}, \tag{54}$$

Chi et al. [22] showed that

$$|b_m[n]| = \delta[n-d_m], \quad m = 2, 3, \ldots, M \tag{55}$$

and therefore the $(M-1)$ time delays can be simultaneously estimated as

$$\hat{d}_m = \arg\max_n \{|b_m[n]|\}, \quad m = 2, 3, \ldots, M. \tag{56}$$

Some simulation results of Chi et al.'s TDE algorithm for estimating two time delays d_2 and d_3 among three separate sensors are shown in Table 3, where the noise $\mathbf{w}[n]$ was assumed to be spatially correlated and temporally colored Gaussian. Table 3 lists the means, standard deviations (STDs), and root mean square errors (RMSEs) of 100 estimates of d_2 and d_3. One can see, from this table, that Chi et al.'s TDE algorithm performs well even when the SNR is as low as –5 dB.

Table 3. Means, standard deviations (STDs), and root mean square errors (RMSEs) of the estimated time delays $\hat{d}_2$ and $\hat{d}_3$ using Chi et al.'s TDE algorithm [22].

True time delays: $d_2 = 2$ and $d_3 = 11$

Data length N	$\hat{d}_m$	SNR = –5 dB			SNR = 0 dB		
		Mean	STD	RMSE	Mean	STD	RMSE
1,024	$\hat{d}_2$	2.0400	0.4000	0.4000	2.0000	0.0000	0.0000
	$\hat{d}_3$	10.9600	0.5109	0.5099	11.0000	0.0000	0.0000
2,048	$\hat{d}_2$	2.0000	0.0000	0.0000	2.0000	0.0000	0.0000
	$\hat{d}_3$	11.0000	0.0000	0.0000	11.0000	0.0000	0.0000
4,096	$\hat{d}_2$	2.0000	0.0000	0.0000	2.0000	0.0000	0.0000
	$\hat{d}_3$	11.0000	0.0000	0.0000	11.0000	0.0000	0.0000

SNR Boost by Blind Maximum Ratio Combining (BMRC)

Consider the following SIMO model:

$$\mathbf{x}[n] = \boldsymbol{a} \cdot x_S[n] + \mathbf{w}[n] \tag{57}$$

where $x_S[n]$ is the desired signal, $\boldsymbol{a}$ is an $M \times 1$ unknown column vector, and $\mathbf{w}[n]$ is the $M \times 1$ vector noise. The data $\{x_1[n], x_2[n], \ldots, x_M[n]\}$ (components of $\mathbf{x}[n]$) can be thought of as the received signals from M diversities (such as multiple paths or multiple sensors or antennas). Then, the problem is to linearly combine these signals into an estimate $x_S[n]$ (up to a scale factor) with maximum SNR.

Let us assume that the desired signal $x_S[n]$ is zero-mean non-Gaussian and the noise $\mathbf{w}[n]$ is zero-mean Gaussian, and they are statistically independent of each other. The data $\mathbf{x}[n]$ is processed by an MISO equalizer $\mathbf{v}[n]$ of length $L = 1$ ($L_1 = L_2 = 0$), which also performs as a linear combiner (or a beamformer in array signal processing) producing the combiner output

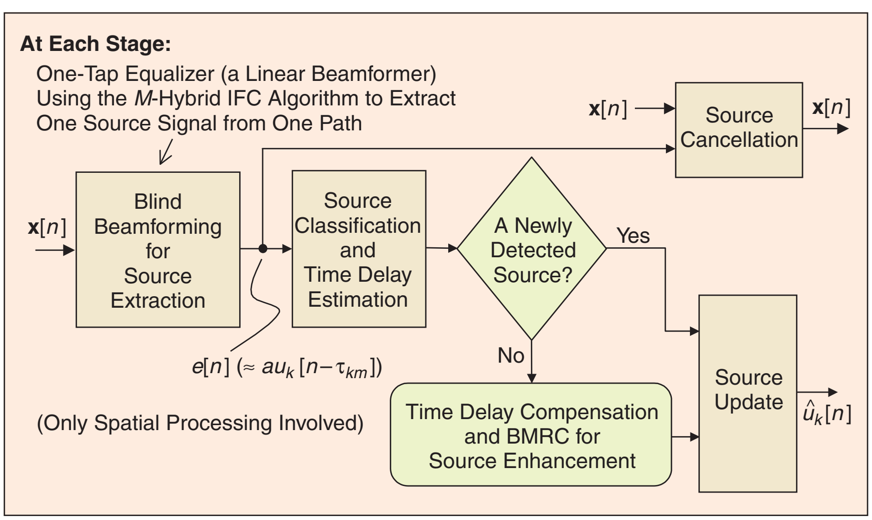
$$e[n] = \mathbf{v}^T[0]\mathbf{x}[n] = \left(\mathbf{v}^T[0]\boldsymbol{a}\right)x_S[n] + \mathbf{v}^T[0]\mathbf{w}[n]. \tag{58}$$

It has been shown [10], [18], [19], [32] that the optimum combiner $\mathbf{v}[0]$ obtained by the M-IFC or the M-hybrid IFC algorithm is given by

$$\mathbf{v}[0] = \lambda \cdot \left(\mathbf{R}_w^*[0]\right)^{-1} \boldsymbol{a}^*, \quad \lambda \neq 0 \tag{59}$$

that is identical to (up to a scale factor) the nonblind linear MMSE estimator of $x_S[n]$ with maximum SNR in $e[n]$ as follows:

$$\begin{aligned} \mathrm{SNR}_{\max} &= \frac{E\left\{|(\mathbf{v}^T[0]\boldsymbol{a})x_S[n]|^2\right\}}{E\left\{|\mathbf{v}^T[0]\mathbf{w}[n]|^2\right\}} \\ &= E\left\{|x_S[n]|^2\right\} \cdot \boldsymbol{a}^H (\mathbf{R}_w[0])^{-1} \boldsymbol{a}. \end{aligned} \tag{60}$$



▲ 15. *Signal processing procedure of each stage of the MSS algorithm.*

In other words, the one-tap equalizer $\mathbf{v}[0]$, with only spatial processing involved, performs maximum ratio combining of the signals $\{x_1[n], x_2[n], \ldots, x_M[n]\}$ with no need of any information about $\boldsymbol{a}$.

Blind Beamforming for Source Separation in Multipath

Blind beamforming is an array signal processing problem concerned with automatically shaping the antenna beam pattern to receive signals of interest without need of prior information of the array directional characteristics. Consider the case of K independent source signals arriving as planewaves at an M-element antenna array in the presence of multipath and additive noise. The received $M \times 1$ signal vector is given by

$$\mathbf{x}[n] = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \alpha_{km} \mathbf{a}(\theta_{km}) u_k[n - \tau_{km}] + \mathbf{w}[n] \tag{61}$$

where α_{km}, θ_{km}, τ_{km}, and M_k are, respectively, the fading factor, direction of arrival, and propagation delay of the mth path and the number of paths associated with the source signal $u_k[n]$, $\mathbf{a}(\theta)$ is the $M \times 1$ steering vector, and $\mathbf{w}[n]$ is a zero-mean Gaussian noise vector. The received signal model $\mathbf{x}[n]$ given by (61) can be also expressed as

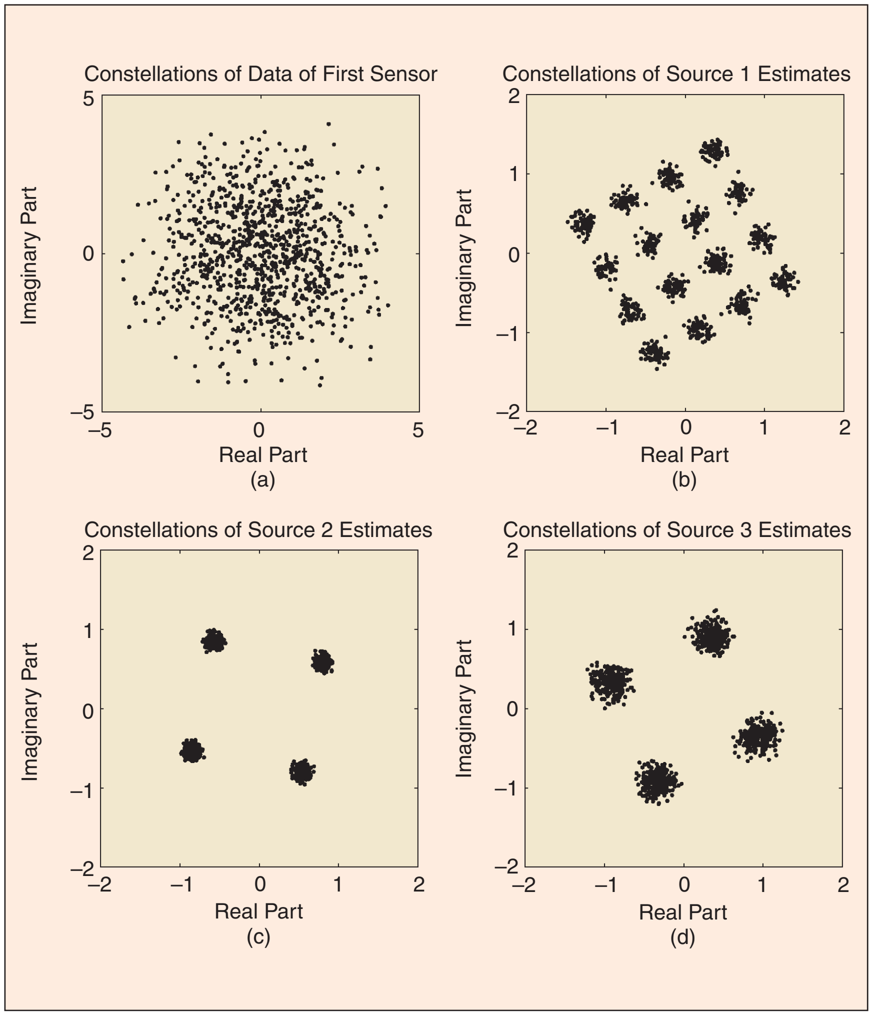
$$\mathbf{x}[n] = \mathbf{A}\mathbf{s}[n] + \mathbf{w}[n] \tag{62}$$

where $\mathbf{s}[n]$ is an $\mathcal{M} \times 1$ ($\mathcal{M} = M_1 + M_2 + \cdots + M_K$) column vector comprising all the source signals from different paths $u_k[n - \tau_{km}]$ and $\mathbf{A}$ is an $M \times \mathcal{M}$ matrix formed by α_{km} and $\mathbf{a}(\theta_{km})$. Note that the MIMO model given by (62) is a special case of the MIMO model given by (9) ($\mathbf{H}[n] = \mathbf{A}\delta[n]$) and the SIMO model given by (57) is also a special case of the former. Next, let us present how to extract the K source signals $u_k[n]$ with a given set of measurements $\mathbf{x}[n]$, $n = 0, 1, \ldots, N-1$.

Under the assumptions that $M \geq \mathcal{M}$, $\mathbf{A}$ has full column rank, and $u_k[n]$, $k = 1, 2, \ldots, K$, are zero-mean, i.i.d., non-Gaussian, Chi and Chen [19] proposed a multistage source separation (MSS) algorithm whose signal processing procedure at each stage is shown in Figure 15. Let $S_k = \{u_k[n - \tau_{km}], m = 1, 2, \ldots, M_k\}$ and assume that $\hat{u}_k[n]$ is the obtained estimate at the $(\ell - 1)$th stage. At the ℓth stage, a source signal from one path, denoted by $e[n] \approx \alpha u_k[n - \tau_{km}]$, is extracted using the M-hybrid IFC algorithm (with equalizer length $L = 1$, i.e., only spatial processing is involved) followed by classification of $e[n]$ to the associated S_k (i.e., $e[n] \in S_k$) and the time delay estimation of τ_{km} by crosscorrelation of $e[n]$ and $\hat{u}_k[n]$, and then BMRC of

$\hat{u}_k[n]$ and $e[n+\tau_{km}]$ is performed to update $\hat{u}_k[n]$. Meanwhile, cancellation of the contribution of the extracted source signal $e[n]$ from the received signal $\mathbf{x}[n]$ is performed (see (T2) of the MSC procedure).

Some simulation results for separating three sources [source 1 is 16-QAM signal, the other two sources are 4-QAM signals and the numbers of paths $(M_1, M_2, M_3) = (2,3,1)$] using Chi and Chen's MSS algorithm with a ten-element sensor array for $N = 1{,}024$ and SNR = 20 dB are shown in Figure 16. One can see from Figure 16 that all the three sources were separated successfully, and that sources 1 and 2 (with multipath diversity) are estimated more accurately than source 3 (without multipath diversity).



▲ *16. (a) The constellations of the synthetic data $x_1[n]$ received by the first sensor of the ten-element sensor array; (b), (c), (d) constellations of the source estimates $\hat{u}_1[n]$, $\hat{u}_2[n]$, and $\hat{u}_3[n]$, respectively, obtained by Chi and Chen's MSS algorithm [19].*

Blind Multiuser Detection for DS/CDMA Systems in Multipath

Consider a K-user asynchronous DS/CDMA communication system in the presence of multipath where the source signals associated with the active users arrive as planewaves at a $\widetilde{J}$-element antenna array. The received baseband continuous-time $\widetilde{J} \times 1$ signal vector $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_{\widetilde{J}}(t)]^{\mathrm{T}}$ can be expressed as [144]

$$\mathbf{x}(t) = \sum_{k=1}^{K} \sum_{n=-\infty}^{\infty} \sum_{m=1}^{M_k} \mathbf{a}(\theta_{km}) A_{km} u_k[n] s_k(t - nT - \tau_{km}) + \mathbf{w}(t) \tag{63}$$

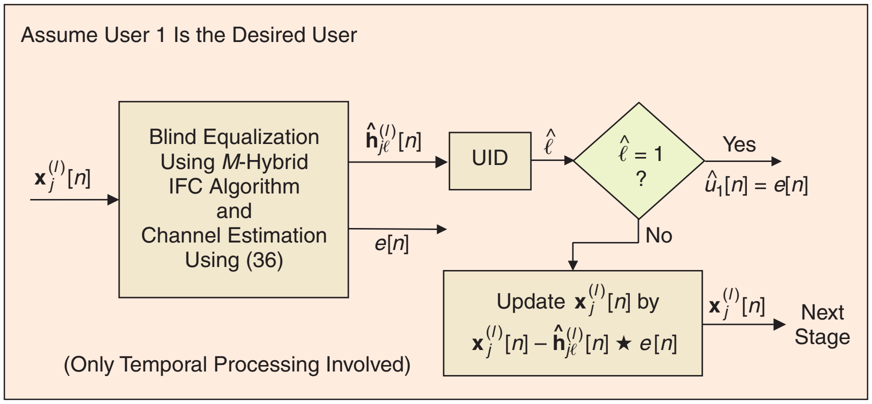
where θ_{km}, A_{km}, τ_{km}, $u_k[n]$, and M_k are, respectively, the direction of arrival, amplitude, and propagation delay of the mth path, symbol sequence, and number of propagation paths associated with user k, $\mathbf{a}(\theta)$ is the $\widetilde{J} \times 1$ steering vector, $\mathbf{w}(t) = [w_1(t), w_2(t), \ldots, w_{\widetilde{J}}(t)]^{\mathrm{T}}$ is a zero-mean white Gaussian noise vector, T is the symbol period, and $s_k(t)$ is the signature waveform of unit energy associated with user k. In particular,

$$s_k(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{\mathcal{P}-1} c_k[n] p(t - nT_c) \tag{64}$$

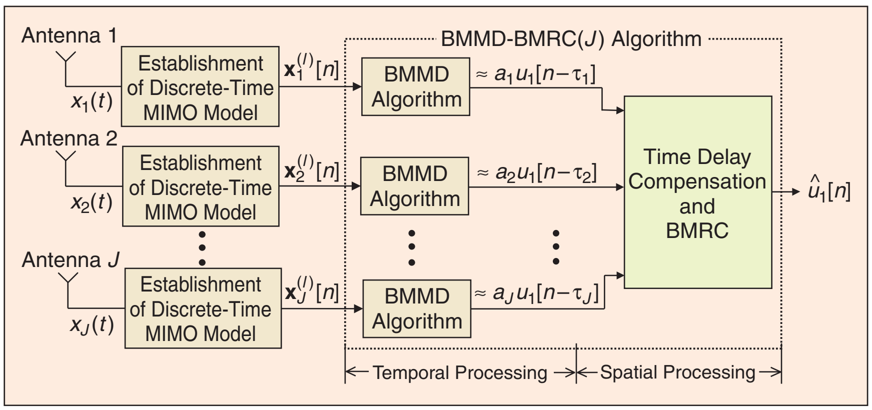
where $\mathcal{P}$ is the processing gain, $T_c = T/\mathcal{P}$ is the chip period, $c_k[n]$, $n = 0,1,\ldots,\mathcal{P}-1$ is a binary pseudo random sequence of {+1,–1}, and $p(t)$ is the chip waveform [e.g., rectangular chip pulse of magnitude equal to unity within the interval $t \in [0, T_c)$].

The objective of the blind multiuser detection is either to estimate the symbol sequence of the desired user (e.g., $u_1[n]$) or to estimate all the symbol sequences $(u_1[n], u_2[n], \ldots, u_K[n])$ with only the received signal $\mathbf{x}(t)$. Recently, there have been many methods reported for multiuser detection of the DS/CDMA system either by temporal processing [18], [20], [21], [68], [130]-[132], [141], [142], [147] or by space-time processing [87], [98], [105]. Due to the space limit, we will only introduce one of these methods to exhibit how the M-hybrid IFC algorithm can be applied to multiuser detection in wireless communications.

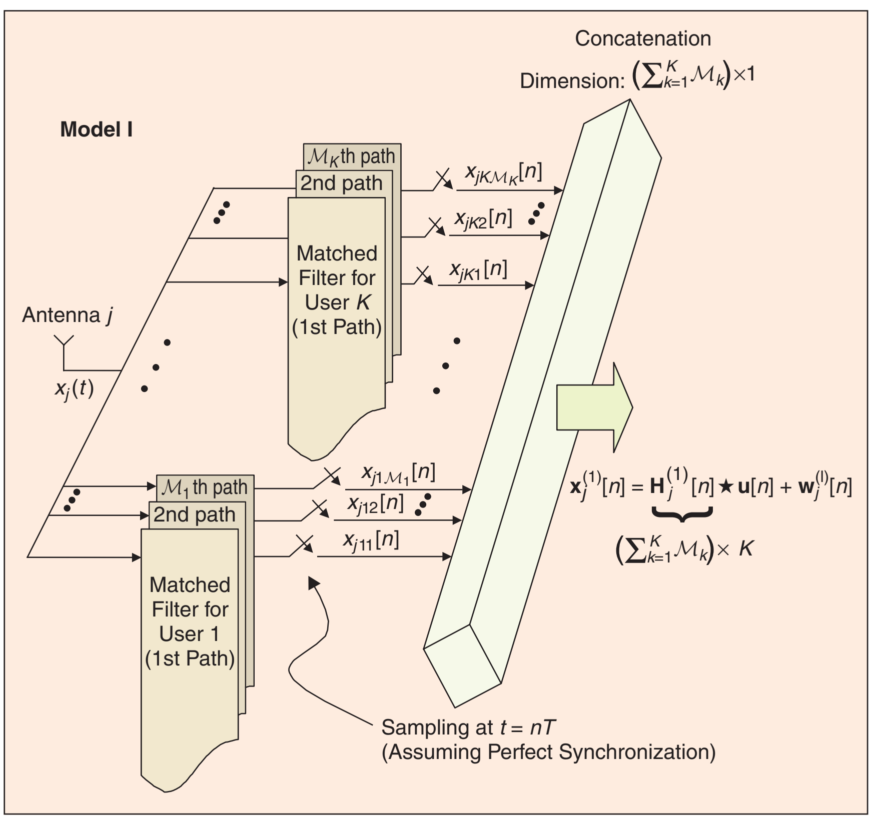
The continuous-time model $\mathbf{x}(t)$ needs to be transformed into an equivalent discrete-time MIMO model first for applying the aforementioned MIMO blind equalization algorithms to blind multiuser detection. As will be illustrated below, two discrete-time MIMO models associated with antenna j ($\mathbf{x}_j^{(1)}[n]$ and $\mathbf{x}_j^{(2)}[n]$) can be formed through signature waveform matched filtering of $\mathbf{x}(t)$, and one ($\mathbf{x}_j^{(3)}[n]$) through chip waveform matched filtering of $\mathbf{x}(t)$. With the established discrete-time MIMO models, Chi and Chen [18], [20], [21] proposed a blind multistage



▲ 17. *Signal processing procedure of each stage of Chi and Chen's BMMD algorithm using one antenna.*



▲ 18. *The proposed BMMD-BMRC(J) algorithm using J antennas.*



▲ 19. *Model I: establishment of* $\mathbf{x}_j^{(1)}[n]$ *from* $x_j(t)$.

multiuser detection (BMMD) algorithm [18], [20], [21] shown in Figure 17 for estimation of the desired symbol sequence (e.g., $u_1[n]$ in Figure 17). Specifically, at each stage, the BMMD algorithm includes blind equalization using the M-hybrid IFC algorithm to extract one user's symbol sequence and estimate the corresponding channel, user identification (UID) with the estimated channel, and signal cancellation. The UID associated with $\mathbf{x}_j^{(3)}[n]$ is based on a detection criterion involving cross correlation of the estimated channel with the active users' spreading codes, whereas the UID associated with $\mathbf{x}_j^{(1)}[n]$ and $\mathbf{x}_j^{(2)}[n]$ exploits some inherent characteristics of the discrete-time MIMO channels, respectively. Furthermore, when J ($\le \tilde{J}$) antennas are used, a straightforward algorithm, called a BMMD-BMRC(J) algorithm, is depicted in Figure 18 which comprises J parallel signal processing procedure of Chi and Chen's BMMD algorithm (with only temporal processing involved) followed by appropriate compensation of relative time delays, and a BMRC procedure over J antennas (with only spatial processing involved) for obtaining the maximum SNR estimate of the desired user's symbol sequence. Note that the BMMD-BMRC(1) algorithm is exactly the same as Chi and Chen's BMMD algorithm. Next, let us present the formulation of the three discrete-time MIMO models and the associated simulation results with some notations defined as follows:

▲ $x_{jkm}[n]$: signature waveform matched filter output (sampled at symbol rate) synchronized with the mth path of the kth user at antenna j (see Figure 19)

▲ $x_j[n]$: chip waveform matched filter output (sampled at chip rate) at antenna j (see Figure 22)

▲ $\mathbf{x}_{jk}[n]$: $[x_{jk1}[n], x_{jk2}[n], \ldots, x_{jk\mathcal{M}_k}[n]]^T$ $(1 \le \mathcal{M}_k \le M_k)$.

Signature Waveform Matched Filtering Based MIMO Models [10], [18], [20]

By concatenation of $\mathbf{x}_{jk}[n]$, $k = 1, 2, \ldots, K$ each comprising $\mathcal{M}_k$ matched filter outputs as shown in

Figure 19, a discrete-time MIMO model (Model I) can be established as

$$\begin{aligned}\mathbf{x}_j^{(1)}[n] &= \left[\mathbf{x}_{j1}^{\mathrm{T}}[n], \mathbf{x}_{j2}^{\mathrm{T}}[n], \ldots, \mathbf{x}_{jK}^{\mathrm{T}}[n]\right]^{\mathrm{T}} \\ &= \mathbf{H}_j^{(1)}[n] \star \mathbf{u}[n] + \mathbf{w}_j^{(1)}[n]\end{aligned} \tag{65}$$

where $\mathbf{H}_j^{(1)}[n] = [\mathbf{h}_{j1}^{(1)}[n], \mathbf{h}_{j2}^{(1)}[n], \ldots, \mathbf{h}_{jK}^{(1)}[n]]$ is a $(\sum_{k=1}^{K} \mathcal{M}_k) \times K$ system, and $\mathbf{w}_j^{(1)}[n]$ is a zero-mean spatially correlated and temporally colored Gaussian noise vector due to $\mathbf{w}(t)$.

Another MIMO model can be established by concatenation of the signals obtained by BMRC over each $\mathbf{x}_{jk}[n], k = 1, 2, \ldots, K$ as shown in Figure 20. The BMRC method using the M-hybrid IFC algorithm presented earlier can be employed to combine the signals from the $\mathcal{M}_k$ paths for each user as follows

$$\bar{x}_{jk}[n] = \mathbf{v}_{jk}^{\mathrm{T}} \mathbf{x}_{jk}[n], k = 1, 2, \ldots, K \tag{66}$$

where $\mathbf{v}_{jk}$ is the weight vector of the combiner. Then one can form the following MIMO model (Model II):

$$\begin{aligned}\mathbf{x}_j^{(2)}[n] &= [\bar{x}_{j1}[n], \bar{x}_{j2}[n], \ldots, \bar{x}_{jK}[n]]^{\mathrm{T}} \\ &= \mathbf{H}_j^{(2)}[n] \star \mathbf{u}[n] + \mathbf{w}_j^{(2)}[n]\end{aligned} \tag{67}$$

where $\mathbf{H}_j^{(2)}[n] = [\mathbf{h}_{j1}^{(2)}[n], \mathbf{h}_{j2}^{(2)}[n], \ldots, \mathbf{h}_{jK}^{(2)}[n]]$ is a $K \times K$ system and $\mathbf{w}_j^{(2)}[n]$ is a zero-mean spatially correlated and temporally colored Gaussian noise vector. Note that Model II can be thought of as a dimension reduced version of Model I and that $\mathbf{x}_j^{(2)}[n] = \mathbf{x}_j^{(1)}[n]$ when $\mathcal{M}_k = 1 \forall k$.

Next, let us show some simulation results using the proposed BMMD-BMRC(J) algorithm with $\mathbf{x}_j^{(1)}[n]$ (Model I) and $\mathbf{x}_j^{(2)}[n]$ (Model II) for a case of a five-user ($K = 5$) asynchronous DS/CDMA system with three paths for each user ($M_k = 3 \forall k$). There were 30 independent runs for data length $N = 2{,}000$ performed with Gold codes of length $\mathcal{P} = 31$ for users' spreading codes $c_k[n]$. The symbol sequences $u_k[n]$, $k = 1, 2, \ldots, 5$ were synthesized as equally probable binary random sequences whose amplitudes were adjusted so that the signal energies in the discrete-time signal $[(\mathbf{x}_1^{(1)}[n])^{\mathrm{T}}, (\mathbf{x}_2^{(1)}[n])^{\mathrm{T}}, \ldots, (\mathbf{x}_{\tilde{J}}^{(1)}[n])^{\mathrm{T}}]^{\mathrm{T}}$ satisfying

$$\mathcal{E}_k = E\left\{\|\mathbf{h}_k^{(1)}[n] \star u_k[n]\|^2\right\} = \mathcal{E}, \quad k = 2, \ldots, 5 \tag{68}$$

where $\mathbf{h}_k^{(1)}[n]$ denotes the kth column of

$$\mathbf{H}^{(1)}[n] = \left[\left(\mathbf{H}_1^{(1)}[n]\right)^{\mathrm{T}}, \left(\mathbf{H}_2^{(1)}[n]\right)^{\mathrm{T}}, \ldots, \left(\mathbf{H}_{\tilde{J}}^{(1)}[n]\right)^{\mathrm{T}}\right]^{\mathrm{T}}.$$

The averaged output signal-to-interference-plus-noise ratios (SINRs) of user 1 (the weak user) are shown in Figure 21 associated with Models I and II for $J = \tilde{J} = 1$ [18], and Figure 22 associated with Model II for $\tilde{J} = 4$ and $J = 1$, 2, and 4. One can observe, from Figure 21, that the performance of the BMMD-BMRC(1) algorithm (i.e., Chi and Chen's BMMD algorithm) is quite close to that of the nonblind linear MMSE equalizer and better for larger $\mathcal{M}_k$ (i.e., more multipath diversity). The similar performance for the near-far ratio (NFR), defined as NFR$= \mathcal{E} / \mathcal{E}_1$, equal to 0 dB and 9 dB also implies that the BMMD-BMRC(1) algorithm is near-far resistant. One can also observe, from Figure 22, that the performance of the BMMD-BMRC(J) algorithm is also close to that of the nonblind linear MMSE equalizer, and that the output SINR of user 1 is higher for larger J (i.e., more antennas or spatial diversity used).

Chip Waveform Matched Filtering Based MIMO Model [10], [21]

As shown in Figure 23, a discrete-time MIMO model (Model III) by polyphase decomposition [101], [130]-[132] of $x_j[n]$ is expressed as

$$\begin{aligned}\mathbf{x}_j^{(3)}[n] &= [x_j[n\mathcal{P}], x_j[n\mathcal{P}+1], \ldots, x_j[n\mathcal{P}+\mathcal{P}-1]]^{\mathrm{T}} \\ &= \mathbf{H}_j^{(3)}[n] \star \mathbf{u}[n] + \mathbf{w}_j^{(3)}[n]\end{aligned} \tag{69}$$

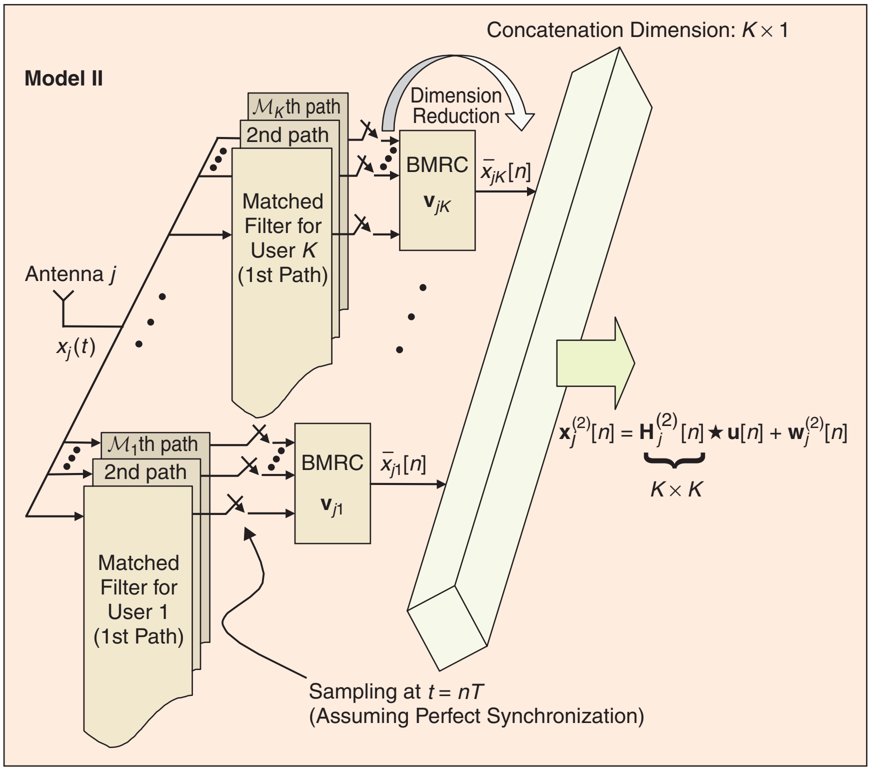
where $\mathbf{H}_j^{(3)}[n] = [\mathbf{h}_{j1}^{(3)}[n], \mathbf{h}_{j2}^{(3)}[n], \ldots, \mathbf{h}_{jK}^{(3)}[n]]$ is a $\mathcal{P} \times K$ system and $\mathbf{w}_j^{(3)}[n]$ is a white Gaussian noise vector. In comparison with Models I and II based on signature waveform matched filtering, the formulation of Model III dose not require synchronization with user's paths but may result in lower SNR for the desired user in the obtained discrete-time signal $\mathbf{x}_j^{(3)}[n]$.

Let us present some simulation results using the proposed BMMD-BMRC(J) algorithm associated with Model III for a case of a six-user ($K = 6$) asynchronous DS/CDMA system with three paths for each user ($M_k = 3$ $\forall k$). Thirty independent runs for data length $N = 2{,}500$ were performed with Gold codes of length $\mathcal{P} = 31$ for users' spreading codes. The symbol sequences $u_k[n], k = 1, 2, \ldots, 6$ were assumed to be equally probable binary random sequences of $\{+1, -1\}$ with

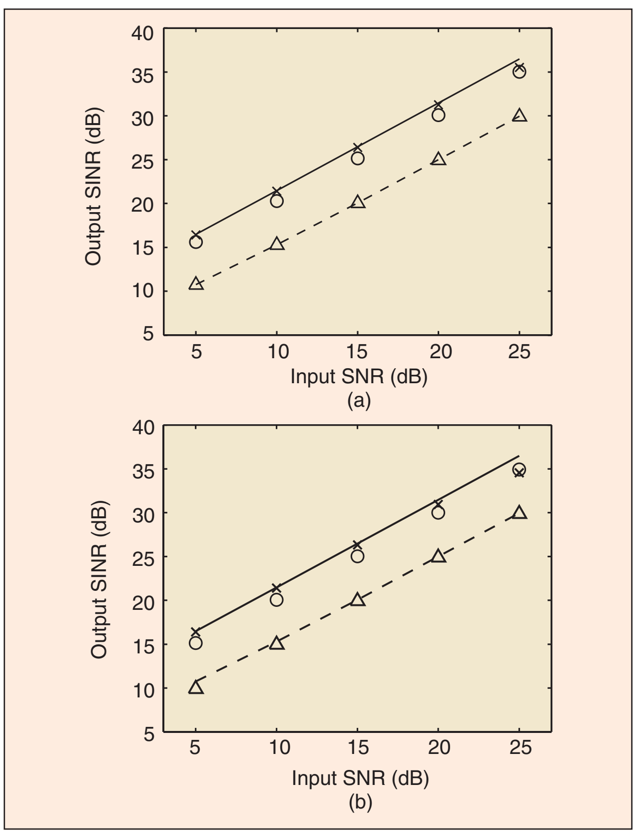
$$\mathcal{E}_k = E\left\{\|\mathbf{h}_k^{(3)}[n] \star u_k[n]\|^2\right\} = \mathcal{E}, \quad k = 2, \ldots, 6 \tag{70}$$

where $\mathbf{h}_k^{(3)}[n]$ denotes the kth column of $\mathbf{H}^{(3)}[n] = [(\mathbf{H}_1^{(3)}[n])^{\mathrm{T}}, (\mathbf{H}_2^{(3)}[n])^{\mathrm{T}}, \ldots, (\mathbf{H}_{\tilde{J}}^{(3)}[n])^{\mathrm{T}}]^{\mathrm{T}}$.

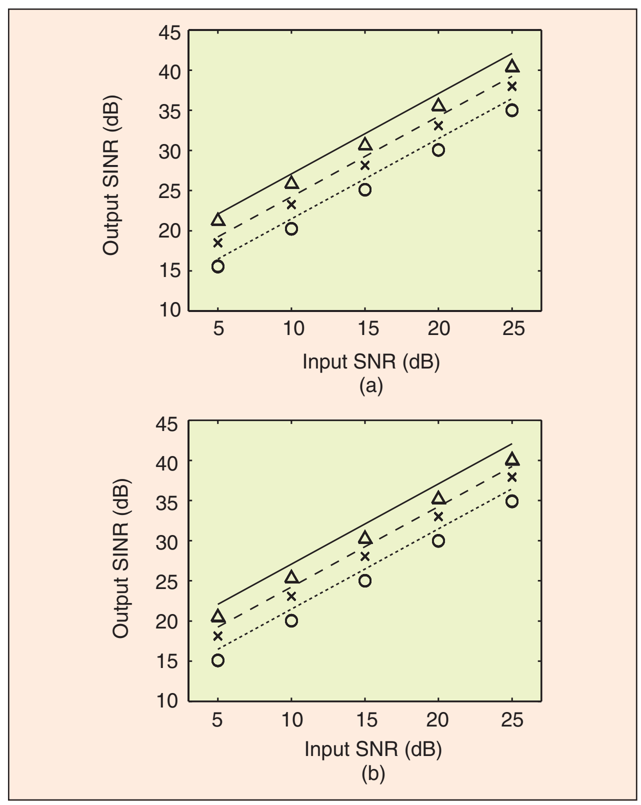
The output SINRs for $J = \tilde{J} = 1$ are shown in Figure 24 together with those obtained using the blind minimum variance (MV) receiver [21], [132], and those for $\tilde{J} = 4$ and $J = 1$, 2, and 4 are shown in Figure 25. One can observe, from Figure 24, that the BMMD-BMRC(1) algorithm performs much better than the MV receiver for NFR$= 0$ dB with larger performance difference for higher input SNR, whereas for NFR $= 10$ dB the former outperforms the latter only by about $1 \sim 3$ dB. Moreover, the performance of the BMMD-BMRC(1) algorithm is close to that of the nonblind linear MMSE equalizer for NFR $= 0$ dB (i.e., the near-far problem is not existent), but the latter is superior to the former for NFR $= 10$ dB (high NFR). On the other hand, from Figure 25, one can see that the performance of the BMMD-BMRC(J) algorithm as well



▲ 20. *Model II: establishment of* $\mathbf{x}_j^{(2)}[n]$ *from* $x_j(t)$.



▲ 21. *Averaged output SINR of user 1 for (a) NFR = 0 dB and (b) NFR = 9 dB, respectively, associated with the nonblind linear MMSE equalizer for* $\mathcal{M}_k = 1\ \forall k$ *(dashed lines) and* $\mathcal{M}_k = 3\ \forall k$ *(solid lines), the BMMD-BMRC(1) algorithm with Model I used for* $\mathcal{M}_k = 1\ \forall k$ *(△) and* $\mathcal{M}_k = 3\ \forall k$ *(×) and the BMMD-BMRC(1) algorithm with Model II used for* $\mathcal{M}_k = 3\ \forall k$ *(○) [18].*



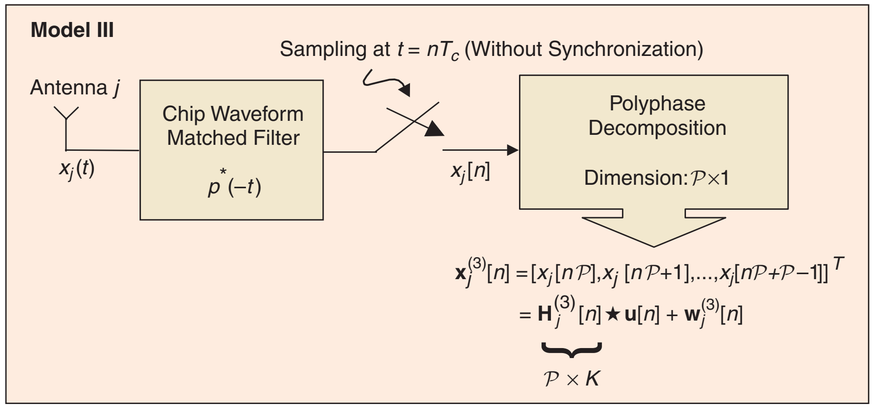
▲ 22. *Averaged output SINR of user 1 for (a) NFR = 0 dB and (b) NFR = 9 dB, respectively, associated with the BMMD-BMRC(J) algorithm with Model II used for single antenna (○), two antennas (×), and four antennas (△), and the nonblind linear MMSE equalizer for single antenna (dotted lines), two antennas (dashed lines), and four antennas (solid lines), where* $\mathcal{M}_k = 3\ \forall k$ *was used for all the results.*

as the nonblind linear MMSE equalizer can be significantly improved by using more antennas, even though the latter is superior to the former for high NFR. The performance degradation of the BMMD-BMRC(J) algorithm for high NFR (see Figures 24(b) and 25(b)) results from the error propagation in the MSC procedure because of more stages in the MSC procedure performed in the BMMD algorithm. Specifically, we found that antenna 2 performs significantly worse than the other three antennas for high NFR, leading to less contribution to performance improvement as shown in Figure 25(b).

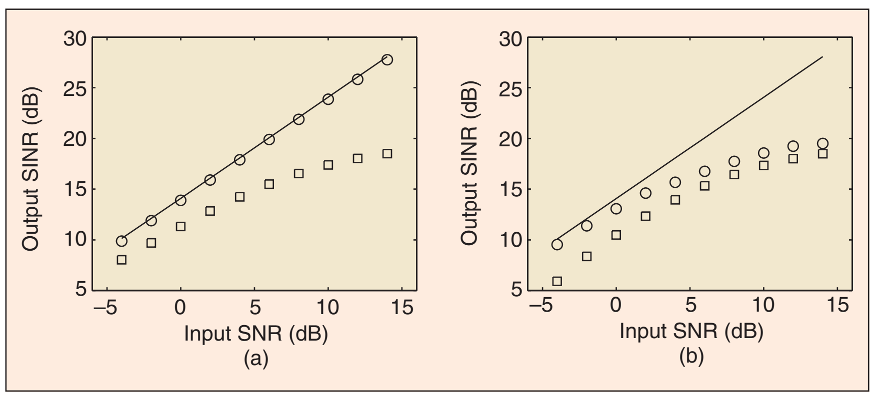
Conclusions

We have reviewed several widely known HOS-based blind equalization algorithms, i.e., the IFC-based algorithm, SEA, and CMA, along with their relations and performance for both the SISO and MIMO cases, which can be mapped to the single-user and multiuser (wired or wireless) communication systems, respectively. The analytic results for these algorithms provide a perspective on the behavior of the designed blind equalizers as well as the relation to the nonblind linear MMSE equalizer. Some possible improvements and applications of these blind equalization algorithms (including BCE, TDE, BMRC, blind beamforming for source separation in multipath, and blind multiuser detection of the DS/CDMA systems) were presented. Note that a blind multiuser detection algorithm with multiple antennas, also known as a blind space-time multiuser detection algorithm, reduces to a blind beamforming algorithm (needed by smart antennas) when the lengths of both the associated discrete-time MIMO channel and the designed MISO equalizer are equal to one. Also note that the reviewed blind equalization algorithms are all formulated in batch processing (block processing) forms that could be implemented by real-time DSP processors with sufficient computation power or field-programmable gate arrays. This may be a foreseeable reality in the near future, particularly for the software-defined radio with a programmable module of implementing blind multiuser detection algorithms.

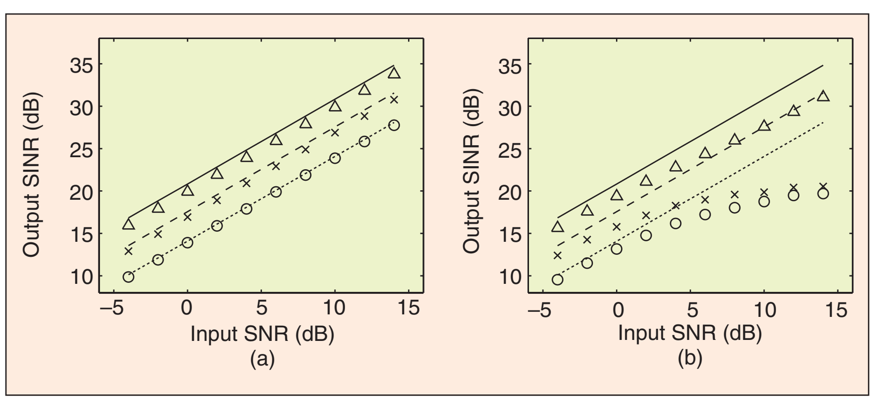
Notice, however, that all the reviewed algorithms for the SISO case are based on the linear time-invariance assumption of the channels and the assumption that both the channel and its inverse system are stable [see (S-C1)], namely, the channel has no zeros on the unit circle. In practical applications, the locations of the channel's zeros are almost out of the designer's control. Accordingly, Feng and Chi [39] have shown that the IFC-based algorithm [and therefore the SEA according to (S-F1)] is still applicable



▲ 23. *Model III: Establishment of* $\mathbf{x}_j^{(3)}[n]$ *from* $x_j(t)$.



▲ 24. *Averaged output SINR for user 1 associated with the nonblind linear MMSE equalizer (solid line), the MV receiver (□), and the BMMD-BMRC(1) algorithm with Model III used (○) for (a)* NFR = 0 *dB and (b)* NFR = 10 *dB, respectively.*



▲ 25. *Averaged output SINR of user 1 for (a) NFR =* 0 *dB and (b) NFR =* 10 *dB, respectively, associated with the BMMD-BMRC(J) algorithm with Model III used for single antenna (○), two antennas (×), and four antennas (△), and the nonblind linear MMSE equalizer for single antenna (dotted lines), two antennas (dashed lines), and four antennas (solid lines).*

for finite SNR regardless of the channel having zeros on the unit circle or not. As for the MIMO case, whether similar results are also applicable to the M-IFC-based algorithm and M-SEA needs further studies though they seem to be. On the other hand, due to the "mobilities" of users and the wireless characteristics, the channel between transmitter and receiver is naturally time varying. Fast real-time batch or adaptive signal processing with low complexity for blind channel estimation and equalization is still challenging research. Moreover, for practical utilization of the blind equalization algorithms in wireless communication systems, the available data length (such as the number of samples in a data frame) and the allowed equalizer length (or the number of equalizer coefficients) are the key points to be taken into account. Therefore, the effects of finite data length, finite equalizer length, and number of users on the performance of the blind equalization algorithms reviewed in this article also need to be further studied.

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