OUTAGE CONSTRAINED WEIGHTED SUM RATE MAXIMIZATION FOR MISO INTERFERENCE CHANNEL BY PRICING-BASED OPTIMIZATION

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ABSTRACT

This paper considers beamforming designs for weighted sum rate maximization (WSRM) in a multiple-input single-output interference channel subject to probability constraints on the rate outage. We claim that the outage probability constrained WSRM problem is an NP-hard problem, and therefore focus on devising efficient approximation methods. In particular, inspired by an insightful problem reformulation, a pricing-based sequential optimization (PSO) algorithm is proposed for efficiently handling the considered outage constrained WSRM problem. We show that the proposed PSO algorithm has semi-analytical beamforming solutions in each iteration, and hence can be efficiently implemented. Moreover, the PSO algorithm upon convergence attains a point satisfying Karush-Kuhn-Tucker (KKT) conditions of the original outage constrained problem. Simulation results demonstrate that the proposed PSO algorithm not only yields competing weighted sum rate performance, but also is computationally more efficient than the existing method [1].

Index Terms— Interference channel, weighted sum rate maximization, outage probability, transmit beamforming

1. INTRODUCTION

Inter-cell cooperation has been recognized essential to improving the spectral efficiency of wireless cellular networks [2]. Consider a multiple-input single-output interference channel (MISO IFC) [3] where K multi-antenna transmitters simultaneously communicate with K single-antenna receivers over a common frequency band. When instantaneous channel state information (CSI) is available at the transmitters and the receivers employ single-user detection, it has been shown that transmit beamforming is a Pareto optimal transmission strategy [3, 4]. However, finding such optimal beamformers in practice is a difficult task. In fact, it has been shown that beamforming design problems for maximizing a class of commonly used rate utilities, (e.g., the weighted sum rate) are NP-hard in general [5]. Consequently, many research efforts have been devoted to investigating computationally efficient approximation algorithms [5, 6].

Considering that it is not always feasible to obtain instantaneous CSI, especially in fast fading scenarios, there are parallel works that study the MISO IFC with only channel distribution information (CDI) available at the transmitters [7, 8]. For example, assuming that each MISO channel is (circularly symmetric) complex Gaussian distributed, the authors in [7] characterized the structure of Pareto optimal beamformers for an ergodic achievable rate region. The authors of [8, 9] instead considered an outage constrained scenario where the probability of the rate outage is constrained to be no larger than a predefined, usually small value. More specifically, the works in [8, 9] studied the outage constrained achievable rate region for a two-user MISO IFC, and presented a numerical method for attaining the Pareto boundary. This method, however, has a complexity that increases exponentially with the number of users.

In this paper, we assume that only CDI is available at the transmitters, and study the beamforming design problem for weighted sum rate maximization (WSRM) under outage probability constraints. The goal is to develop efficient algorithms for obtaining the outage constrained optimal beamformers. However, our complexity analysis shows that such outage constrained WSRM problem is intricate – it is NP-hard in general. We thereby focus on devising efficient approximation methods. By carefully inspecting the constraint structure, we reformulate the original outage constrained problem in a form that is analogous to the WSRM problem with instantaneous CSI in [5]. This insightful connection inspires us to propose a pricing-based sequential optimization (PSO) algorithm [10] for efficiently handling the considered outage constrained WSRM problem. We show that the proposed PSO algorithm improves the system sum rate from iteration to iteration, and, upon convergence, reaches a point satisfying the Karush-Kuhn-Tucker (KKT) conditions of the original problem. Moreover, the subproblem involved in each iteration has semi-analytical solutions which can be implemented efficiently. The presented simulation results demonstrate that the PSO algorithm is computationally more efficient than the previously proposed distributed SCA (DSCA) algorithm in [1], though both methods yield almost the same sum rate performance.

2. SIGNAL MODEL AND PROBLEM STATEMENT

We consider a MISO IFC consisting of K pairs of multiple-antenna transmitters and single-antenna receivers. Each transmitter is equipped with N_t antennae, and communicates with its intended receiver using transmit beamforming. The transmit signal from transmitter i is given by $\boldsymbol{w}_i s_i$, where $s_i \sim \mathcal{CN}(0,1)$ is the information signal for receiver i, and $\boldsymbol{w}_i \in \mathbb{C}^{N_t}$ is the associated beamforming vector, for $i = 1, \ldots, K$. Let $\boldsymbol{h}_{ik} \in \mathbb{C}^{N_t}$ denote the channel vector between transmitter i and receiver k, for all $i, k = 1, \ldots, K$. Here, we assume that each $\boldsymbol{h}_{ik} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ik})$ with $\mathbf{Q}_{ik} \succeq \mathbf{0}$ (positive semidefinite) denoting the channel covariance matrix. The received signal at receiver i is then given by

$$x_i = \boldsymbol{h}_{ii}^H \boldsymbol{w}_i s_i + \sum_{k=1, k\neq i}^{K} \boldsymbol{h}_{ki}^H \boldsymbol{w}_k s_k + n_i, \tag{1}$$

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where $n_i \sim \mathcal{CN}(0, \sigma_i^2)$ is the additive noise at receiver i with variance $\sigma_i^2 > 0$. Assume that each receiver i decodes the information signal s_i by single user detection, i.e., treating the cross-link interference as noise. Then, the instantaneous achievable rate (in nats/sec/Hz) of the ith transmitter-receiver pair is given by

$$r_i\left(\{\boldsymbol{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K\right) = \log\left(1 + \frac{|\boldsymbol{h}_{ii}^H \boldsymbol{w}_i|^2}{\sum_{k\neq i} |\boldsymbol{h}_{ki}^H \boldsymbol{w}_k|^2 + \sigma_i^2}\right).$$

In this paper, we consider a scenario where the transmitters have knowledge of CDI only, i.e., the channel covariance matrices $\mathbf{Q}_{ik}$, $i, k = 1, \ldots, K$. Under such circumstance, given a transmission rate R_i for the ith transmitter-receiver pair, receiver i may suffer from rate outage, i.e., $r_i(\{\boldsymbol{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K) < R_i$. Our goal is to provide quality of service guaranteed within a tolerable outage probability for the receivers, while maximizing the system throughput (i.e., the sum rate) at the same time. Specifically, given $\epsilon_i \in (0, 1)$ as the maximum tolerable outage probability for each receiver i, we consider the following beamforming design problem [1, 11]

$$\max_{\substack{\boldsymbol{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0, \\ i=1,\ldots,K}} \sum_{i=1}^K \alpha_i R_i \tag{2a}$$

$$\text{s.t. } \text{Prob}\left\{r_i(\{\boldsymbol{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K) < R_i\right\} \leq \epsilon_i, \tag{2b}$$

$$\|\boldsymbol{w}_i\|_2^2 \leq P_i,\ i = 1, \ldots, K, \tag{2c}$$

where $\alpha_1, \ldots, \alpha_K > 0$ are priority weights, and $P_1, \ldots, P_K > 0$ are the power constraints. Notice that, in (2b), the rate outage probabilities are constrained to be no higher than ϵ_i, for $i = 1, \ldots, K$.

3. THE PROBLEM NATURE

It has been shown in [1, 12] that each of the outage constraints in (2b) has an equivalent closed-form expression, given by

$$\rho_i e^{\frac{(2^{R_i}-1)\sigma_i^2}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}} \prod_{k\neq i}\left(1 + \frac{(2^{R_i}-1)\boldsymbol{w}_k^H \mathbf{Q}_{ki} \boldsymbol{w}_k}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}\right) \leq 1, \tag{3}$$

where $\rho_i \triangleq 1 - \epsilon_i$, $i = 1, \ldots, K$. As one can see, the outage constraint in (3) has a non-convex, complicated structure, and thus solving (2) seems to be a challenging task. Therefore, a fundamental question is whether the outage constrained problem (2) is indeed a difficult problem in terms of computational complexity. The following theorem provides the answer.

Theorem 1 *Problem* (2) *is NP-hard in general.*

In fact, one can show that problem (2) is NP-hard even when $N_t = 1$, i.e., when only the transmit powers (without beamforming direction) are optimized. The proof is to construct a polynomial time transformation from the Max-Cut problem, which is known to be NP-hard [13], to problem (2), thereby implying the NP-hardness of the latter problem. Due to space limit, we leave the detailed proof to our future publication. It is worthwhile to note here that Theorem 1 can be regarded as an outage constrained counterpart of the complexity analysis result in [5, 14], where it was shown that the weighted sum rate maximization (WSRM) problem with instantaneous channel state information (CSI) is NP-hard.

Theorem 1 implies that it is unlikely to globally solve problem (2) in polynomial time. Therefore, it is necessary to consider approximation methods, in order to deal with the cases wherein there are a large number of transmitter-receiver pairs. In the next section, we propose a pricing-based sequential (block coordinate) optimization method for efficiently handling the considered problem (2).

4. PROPOSED PRICING-BASED ALGORITHM

4.1. Equivalent Formulation

Due to the complex constraint structure in (3), it is difficult to apply general approximation methods to problem (2) (with (2b) replaced by (3)) in its current form. In view of this, we present an alternative formulation to problem (2) which, as one will see, reveals useful insights for efficient approximation.

To this end, let us define

$$\Phi_i(x|\{\boldsymbol{w}_k\}_{k\neq i}) \triangleq \rho_i e^{\sigma_i^2 x} \prod_{k\neq i}(1 + (\boldsymbol{w}_k^H \mathbf{Q}_{ki} \boldsymbol{w}_k)\cdot x). \tag{4}$$

Then, (3) can be written as $\Phi_i((2^{R_i}-1)/\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i | \{\boldsymbol{w}_k\}_{k\neq i}) \leq 1$, $i = 1, \ldots, K$. Furthermore, because both $\sum_{i=1}^K \alpha_i R_i$ and Φ_i are strictly increasing in $(R_1, \ldots, R_K)$, it must be true that (3) holds with equality for problem (2) at the optimal point, i.e.,

$$\Phi_i\left(\frac{2^{R_i}-1}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}\,\middle|\,\{\boldsymbol{w}_k\}_{k\neq i}\right) = 1,\ i = 1, \ldots, K, \tag{5}$$

On the other hand, each $\Phi_i(x|\{\boldsymbol{w}_k\}_{k\neq i})$ is strictly increasing in x; therefore, there exists a unique positive value $\xi_i(\{\boldsymbol{w}_k\}_{k\neq i})$ such that $\Phi_i(\xi_i(\{\boldsymbol{w}_k\}_{k\neq i})|\{\boldsymbol{w}_k\}_{k\neq i}) = 1$. As a result, constraint (5) holds if and only if

$$\frac{2^{R_i}-1}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i} = \xi_i(\{\boldsymbol{w}_k\}_{k\neq i}),\ i = 1, \ldots, K. \tag{6}$$

By (6), problem (2) can be concisely expressed as

$$\max_{\substack{\boldsymbol{w}_i \in \mathbb{C}^{N_t} \\ i=1,\ldots,K}} \sum_{i=1}^K \alpha_i \log(1 + \xi_i(\{\boldsymbol{w}_k\}_{k\neq i}) \boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i) \tag{7a}$$

$$\text{s.t. } \|\boldsymbol{w}_i\|^2 \leq P_i,\ i = 1, \ldots, K, \tag{7b}$$

There are two interesting observations from (7). Firstly, one can observe that each $\xi_i(\{\boldsymbol{w}_k\}_{k\neq i})$, though being implicit, characterizes the impact of cross-link interference plus noise on receiver i, for $i = 1, \ldots, K$. Secondly, by comparing (7) with its instantaneous-CSI counterpart in [5, Eqn. (3)], there is an intriguing analogy between the two problems in mathematical formulation. This motivates us to use a *pricing-based sequential optimization (PSO)* method, which was used in [10, 15] for the instantaneous-CSI case [5, Eqn. (3)], for handling the outage constrained problem (2) in the subsequent two subsections.

4.2. Pricing-based Sequential Optimization Algorithm

The proposed PSO algorithm for problem (2) is an iterative algorithm which optimizes the beamforming vectors $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K$ in a round-robin fashion. Specifically, in an iteration for optimizing $\boldsymbol{w}_i$, given a set of $\bar{\boldsymbol{w}}_1, \ldots, \bar{\boldsymbol{w}}_K$ (that are feasible to (7b)), we seek to update $\bar{\boldsymbol{w}}_i$ by solving the following problem

$$\begin{aligned} \max_{\boldsymbol{w}_i}\ & \log(1 + \xi_i(\{\bar{\boldsymbol{w}}_k\}_{k\neq i}) \boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i) - \sum_{k\neq i} \pi_{ik} \boldsymbol{w}_i^H \mathbf{Q}_{ik} \boldsymbol{w}_i \\ \text{s.t. }\ & \|\boldsymbol{w}_i\|^2 \leq P_i, \end{aligned} \tag{8}$$

where $\xi_i(\{\bar{\boldsymbol{w}}_k\}_{k\neq i})$ is the unique solution to $\Phi_i(x|\{\bar{\boldsymbol{w}}_k\}_{k\neq i}) = 1$[1]. The terms $-\pi_{ik} \boldsymbol{w}_i^H \mathbf{Q}_{ik} \boldsymbol{w}_i$, $k \neq i$, respectively weighted by the unit

[1]Since $\Phi_i(x|\{\bar{\boldsymbol{w}}_k\}_{k\neq i})$ in (4) is strictly increasing in x, and $\Phi_i(\xi_i(\{\bar{\boldsymbol{w}}_k\}_{k\neq i})|\{\bar{\boldsymbol{w}}_k\}_{k\neq i}) = 1$. The value $\xi_i(\{\bar{\boldsymbol{w}}_k\}_{k\neq i})$ can easily be obtained via a bisection method.

prices $\{\pi_{ik}\}_{k\neq i}$, in the objective function imply that the throughput of user i is maximized at the cost of the interference induced by transmitter i to the other receivers. For notational simplicity, let us denote

$$I_{ik} \triangleq \boldsymbol{w}_i^H \mathbf{Q}_{ik} \boldsymbol{w}_i \;\; (\bar{I}_{ik} \triangleq \bar{\boldsymbol{w}}_i^H \mathbf{Q}_{ik} \bar{\boldsymbol{w}}_i)$$

for all $i, k = 1, \ldots, K$. Furthermore, define

$$U(\{I_{j\ell}\}_{j,\ell=1}^K) \triangleq \sum_{i=1}^K \alpha_i \log(1 + \xi_i(\{I_{ki}\}_{k\neq i}) I_{ii}) \qquad (9)$$

as an alternative expression of the objective function of (7), where $\xi_i(\{I_{ki}\}_{k\neq i}) \triangleq \xi_i(\{\boldsymbol{w}_k\}_{k\neq i})$ (by (4), (5) and (6)) for all $i = 1, \ldots, K$. According to [10, 15], the unit prices are given by

$$\pi_{ik} = -\frac{1}{\alpha_i} \frac{\partial U(\{I_{j\ell}\}_{j,\ell=1}^K)}{\partial I_{ik}} \Bigg|_{I_{j\ell}=\bar{I}_{j\ell}\ \forall j,\ell} \qquad (10)$$

for all $k \neq i$. Specifically, by (9) and by applying the implicit function theorem [16] for computing the gradient of $\xi_k(\{I_{jk}\}_{j\neq k})$ with respect to I_{ik}, one can show that π_{ik} has an explicit form as

$$\pi_{ik} = \frac{\alpha_k}{\alpha_i} \frac{\bar{I}_{kk}\xi_k(\{\bar{I}_{jk}\}_{j\neq k})}{1 + \bar{I}_{kk}\xi_k(\{\bar{I}_{jk}\}_{j\neq k})} \Bigg[\Bigg(\sigma_k^2 + \sum_{\ell\neq i,k} \frac{\bar{I}_{\ell k}}{1 + \bar{I}_{\ell k}\xi_k(\{\bar{I}_{jk}\}_{j\neq k})} \Bigg) \times \big(1 + \bar{I}_{ik}\xi_k(\{\bar{I}_{jk}\}_{j\neq k})\big) + \bar{I}_{ik} \Bigg]^{-1}. \qquad (11)$$

The detailed description of our proposed PSO algorithm for handling problem (7) (i.e., problem (2)) is presented in Algorithm 1.

Algorithm 1 Proposed PSO algorithm for problem (7)

Given an initial set of $\bar{\boldsymbol{w}}_1, \ldots, \bar{\boldsymbol{w}}_K$ satisfying (7b);
Set $\bar{I}_{ik} := \bar{\boldsymbol{w}}_i^H \mathbf{Q}_{ik} \bar{\boldsymbol{w}}_i\ \forall i, k = 1, \ldots, K$, and compute $\xi_k(\{\bar{I}_{jk}\}_{j\neq k}), k = 1, \ldots, K$, by bisection;
repeat
  for $i = 1, \ldots, K$ **do**
    Compute the unit prices $\{\pi_{ik}\}_{k\neq i}$ by (11);
    Solve problem (8) by Proposition 2 below to obtain an optimal solution $\boldsymbol{w}_i^\star$, followed by updating $\bar{\boldsymbol{w}}_i$ with $\boldsymbol{w}_i^\star$;
    Update $\bar{I}_{ik} = \bar{\boldsymbol{w}}_i^H \mathbf{Q}_{ik} \bar{\boldsymbol{w}}_i, k = 1, \ldots, K$, and compute $\xi_k(\{\bar{I}_{jk}\}_{j\neq k}), k = 1, \ldots, K$;
  end for
until the predefined stopping criterion is met.
Output $(\bar{\boldsymbol{w}}_1, \ldots, \bar{\boldsymbol{w}}_K)$ as an approximate solution to (7).

While Algorithm 1 seems to be a straightforward application of the pricing-based method in [10, 15] to problem (7), it is actually not obvious to see whether Algorithm 1 can reach any interesting point of problem (7). This is mainly because $\xi_1(\{I_{k1}\}_{k\neq 1}), \ldots, \xi_K(\{I_{kK}\}_{k\neq K})$ are implicit. To answer the above question, let us analyze the relation between problem (8) and the original problem (7). The following lemma is needed in the subsequent analysis.

Lemma 1 *For each $i \in \{1, \ldots, K\}$ and $k \in \{1, \ldots, K\}\backslash\{i\}$, the individual rate $\log(1 + \xi_k(\{I_{jk}\}_{j\neq k}) I_{kk})$ is strictly convex in $I_{ik} \geq 0$ for any given $\{I_{jk} \geq 0\}_{j\neq i}$.*

The proof of Lemma 1 is presented in the Appendix. By Lemma 1, (10), and by the first-order condition of convex functions, we have

$$\begin{aligned}
&\alpha_k \log\big(1 + \xi_k(\{\bar{I}_{jk}\}_{j\neq k,i}, I_{ik})\bar{I}_{kk}\big) \\
&\quad \geq \alpha_k \log(1 + \xi_k(\{\bar{I}_{jk}\}_{j\neq k,i}, \bar{I}_{ik})\bar{I}_{kk}) - \alpha_i \pi_{ik}(I_{ik} - \bar{I}_{ik}),
\end{aligned}$$

for all $k \in \{1, \ldots, K\}\backslash\{i\}$. Hence, it follows from (9) and the above inequality that

$$\begin{aligned}
&U(I_{i1}, \ldots, I_{iK}, \{\bar{I}_{j1}, \ldots, \bar{I}_{jK}\}_{j\neq i}) \\
&\quad \geq \alpha_i \log(1 + \xi_i(\{\bar{I}_{ki}\}_{k\neq i}) I_{ii}) - \alpha_i \sum_{k\neq i} \pi_{ik}(I_{ik} - \bar{I}_{ik}) \\
&\qquad + \sum_{k\neq i} \alpha_k \log(1 + \xi_k(\{\bar{I}_{jk}\}_{j\neq k,i}, \bar{I}_{ik})\bar{I}_{kk}) \qquad (12) \\
&\quad \triangleq U_{\rm LB}^{(i)}(\{I_{i\ell}\}_{\ell=1}^K \big| \{\bar{I}_{j1}, \ldots, \bar{I}_{jK}\}_{j\neq i}).
\end{aligned}$$

Since the sum of the first two terms on the right hand side of (12) is proportional to the objective function in (8), optimizing problem (8) for user i is equivalent to maximizing the lower bound $U_{\rm LB}^{(i)}$.

More importantly, one can check that $U_{\rm LB}^{(i)}$ is locally tight, in the sense that

$$U_{\rm LB}^{(i)}(\{\bar{I}_{i\ell}\}_{\ell=1}^K \big| \{\bar{I}_{j1}, \ldots, \bar{I}_{jK}\}_{j\neq i}) = U(\{\bar{I}_{j\ell}\}_{j,\ell=1}^K), \qquad (13)$$

for $i = 1, \ldots, K$. Therefore, using an argument similar to [15, Lemma 1], one can show that the weighted sum rate $U(\{\bar{I}_{j\ell}\}_{j,\ell=1}^K)$ achieved by $\bar{\boldsymbol{w}}_1, \ldots, \bar{\boldsymbol{w}}_K$ in Algorithm 1 would be non-decreasing from one iteration to another. This, together with the fact that $U(\{\bar{I}_{j\ell}\}_{j,\ell=1}^K)$ is bounded due to the power constraints (7b), implies that $U(\{\bar{I}_{j\ell}\}_{j,\ell=1}^K)$ eventually converges. Further, $U_{\rm LB}^{(i)}$ has locally tight gradients, i.e.,

$$\frac{\partial U_{\rm LB}^{(i)}(\{\bar{I}_{i\ell}\}_{\ell=1}^K \big| \{\bar{I}_{j1}, \ldots, \bar{I}_{jK}\}_{j\neq i})}{\partial I_{ik}} = \frac{\partial U(\{\bar{I}_{j\ell}\}_{j,\ell=1}^K)}{\partial I_{ik}}, \qquad (14)$$

for all $k, i = 1, \ldots, K$. This property can be exploited to show that Algorithm 1, upon the convergence of $(\bar{\boldsymbol{w}}_1, \ldots, \bar{\boldsymbol{w}}_K)$, attains a point satisfying the KKT conditions of problem (7). The detailed derivations are omitted here due to space limitation. We summarize the above analyses in the following proposition.

Proposition 1 *The weighted sum rate $U(\{\bar{I}_{j\ell}\}_{j,\ell=1}^K)$ achieved in each iteration of Algorithm 1 converges monotonically. Moreover, any convergent point of $(\bar{\boldsymbol{w}}_1, \ldots, \bar{\boldsymbol{w}}_K)$ is a KKT point of (7).*

4.3. Efficient Implementation of PSO Algorithm

An important aspect regarding the computational complexity of Algorithm 1 depends on solving the nonconvex problem (8) efficiently. A similar problem was studied in [10] for the WSRM problem with instantaneous CSI; however, the approach to handling such problem is via linear approximation, and hence is suboptimal. We herein provide the global optimal solution to problem (8) in a semi-analytical form.

The proposed approach is based on the popular semidefinite relaxation (SDR) technique [17]. In particular, we relax the rank-one matrix $\boldsymbol{w}_i \boldsymbol{w}_i^H$ to a rank-unconstrained positive semidefinite matrix $\mathbf{W}_i \succeq \mathbf{0}$, and consider the following convex problem

$$\begin{aligned}
\max_{\mathbf{W}_i \succeq \mathbf{0}}\ & \log\big(1 + \xi_i(\{\bar{I}_{ki}\}_{k\neq i}) {\rm Tr}(\mathbf{W}_i \mathbf{Q}_{ii})\big) - {\rm Tr}\Big(\mathbf{W}_i \sum_{k\neq i} \pi_{ik} \mathbf{Q}_{ik}\Big) \\
\text{s.t. }\ & {\rm Tr}(\mathbf{W}_i) \leq P_i. \qquad (15)
\end{aligned}$$

A key finding is that the SDR problem (15) has a rank-one optimal solution, as stated in the following proposition.

Proposition 2 *Let $\mu_i^\star \geq 0$ denote the optimal dual variable associated with the power constraint in* (15)*, and let $\mathbf{R}_i = \mu_i^\star \mathbf{I}_{N_t} + \sum_{k\neq i} \pi_{ik}\mathbf{Q}_{ik}$, where $\mathbf{I}_{N_t}$ is an $N_t \times N_t$ identity matrix. Then, there exists a principal eigenvector of $\mathbf{R}_i^{-1/2}\mathbf{Q}_{ii}\mathbf{R}_i^{-1/2}$, denoted by $\boldsymbol{\nu}_i^\star$, such that $\mathbf{W}_i^\star = \boldsymbol{w}_i^\star(\boldsymbol{w}_i^\star)^H$ is optimal to problem* (15)*, where*

$$\boldsymbol{w}_i^\star = \sqrt{p_i^\star}\mathbf{R}_i^{-1/2}\boldsymbol{\nu}_i^\star \quad \textit{(optimal to problem (8))}$$

$$p_i^\star = \max\left(1 - [\lambda_i^\star \xi_i(\{\bar{I}_{ki}\}_{k\neq i})]^{-1}, 0\right),$$

and $\lambda_i^\star$ is the maximum eigenvalue of $\mathbf{R}_i^{-1/2}\mathbf{Q}_{ii}\mathbf{R}_i^{-1/2}$.

The proof of Proposition 2 is omitted due to space limitation. It can be shown that $\mu_i^\star$ can be computed by simple bisection, and thus $\boldsymbol{w}_i^\star$ can be obtained efficiently. More specifically, since the major computation load of computing $\boldsymbol{w}_i^\star$ lies in matrix inversion and eigenvalue decomposition, each having a complexity order of $\mathcal{O}(N_t^3)$, the complexity order of solving (8) is roughly given by $\mathcal{O}(N_t^3 \log(1/\varepsilon_1))$, where $\varepsilon_1 > 0$ is the solution accuracy of the bisection search for $\mu_i^\star$. As a result, the overall complexity order of Algorithm 1 is $\kappa_1 K\mathcal{O}(N_t^3 \log(1/\varepsilon_1))$, where κ_1 denotes the total number of round-robin iterations (steps 3 to 8 in Algorithm 1). Note that, for the DSCA algorithm in [1], the subproblem for each transmitter is implemented by interior-point methods. Hence, the DSCA algorithm has an overall complexity order of $\kappa_2 K\mathcal{O}((N_t^{6.5} + K^{3.5})\log((N_t + K)/\varepsilon_2))$ where κ_2 is the total number of round-robin iterations and $\varepsilon_2 > 0$ is the solution accuracy of interior-point methods [18]. One can see that the complexity order of PSO algorithm is lower than that of the DSCA algorithm. Thus, it is expected that the PSO algorithm is computationally more efficient than the DSCA algorithm, which will be verified by our simulation results.

5. SIMULATION RESULTS AND DISCUSSIONS

For simplicity, we set $\sigma_1^2 = \cdots = \sigma_K^2 \triangleq \sigma^2$ and $P_1 = \cdots = P_K = 1$. The tolerable outage probability is set to 10%, i.e., $\epsilon_1 = \cdots = \epsilon_K = 0.1$. The channel covariance matrices $\{\mathbf{Q}_{ik}\}_{i,k=1}^K$ are randomly generated with full rank, and the maximal eigenvalues of $\{\mathbf{Q}_{ik}\}_{i,k=1}^K$ are normalized to $\lambda_{\max}(\mathbf{Q}_{ii}) = 1$ and $\lambda_{\max}(\mathbf{Q}_{ik}) = \eta$ for all $k \neq i$; therefore, $0 \leq \eta \leq 1$ reflects the strength of cross-link channels. Algorithm 1 stops when the difference between the weighted sum rates $U(\{\bar{I}_{j,\ell}\}_{j,\ell=1}^K)$ of two consecutive round-robin iterations is no larger than 0.1% of that in the previous iteration. All simulation results are averaged over 500 realizations of $\{\mathbf{Q}_{ik}\}_{i,k=1}^K$, and, for each problem instance, both the PSO and DSCA algorithms are initialized at a randomly generated feasible point.

We first examine the efficacy of the PSO algorithm by comparing it with the DSCA algorithm in [1] and the naive maximum-ratio transmission (MRT) strategy. Figure 1 shows the average weighted sum rate versus $1/\sigma^2$ for $K = N_t = 4$. It can be seen that the PSO algorithm and the DSCA algorithm yield almost the same weighted sum rate performance, and both outperform the MRT strategy.

In Fig. 2, we compare the average computation time (in seconds) of the PSO algorithm and the DSCA algorithm versus the number of users K, for $1/\sigma^2 = 10$ dB, $\eta = 0.5$, and $N_t = 4$ and 8. The subproblems involved in the DSCA algorithm are handled by CVX [19]. Note that the computation time of the PSO algorithm increases almost linearly with K, whereas that of the DSCA algorithm increases much faster with K. According to Fig. 2, the PSO algorithm is about 10^3 times faster than the DSCA algorithm.

6. APPENDIX: PROOF OF LEMMA 1

We show that $\partial \log(1 + \xi_k(\{I_{jk}\}_{j\neq k})I_{kk})/\partial I_{ik}$ is strictly increasing in $I_{ik} \geq 0$, which implies that $\log(1+\xi_k(\{I_{jk}\}_{j\neq k})I_{kk})$ is convex in $I_{ik} \geq 0$ [18], using the two properties of $\xi_k(\{I_{jk}\}_{j\neq k})$: 1) $\xi_k(\{I_{jk}\}_{j\neq k})$ is strictly decreasing in $I_{ik} \geq 0$; 2) $I_{ik}\cdot\xi_k(\{I_{jk}\}_{j\neq k})$ is strictly increasing in $I_{ik} \geq 0$.

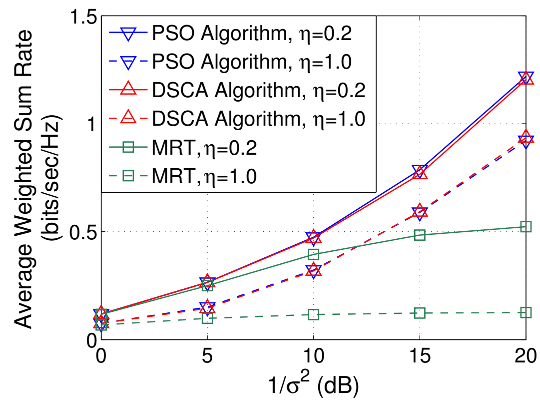

Fig. 1. Average achievable sum rate versus $1/\sigma^2$ for $K = N_t = 4$, and $\mathrm{rank}(\mathbf{Q}_{ki}) = 4$ for all k, i.

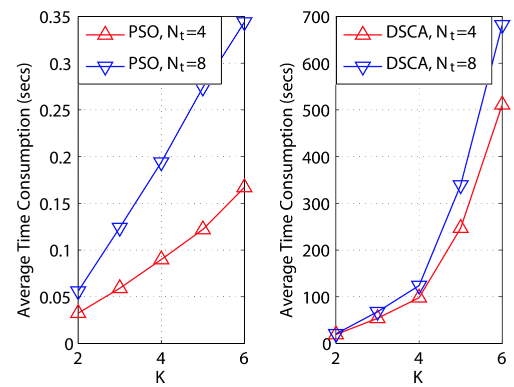

Fig. 2. Average computation time of the PSO algorithm and the DSCA algorithm versus K, for $N_t = 4, 8$, $1/\sigma^2 = 10$ dB, and $\eta = 0.5$.

To prove the first property, observe from (4) that $\Phi_k(x|\{I_{jk}\}_{j\neq k})$ is strictly increasing in $x > 0$ and $I_{ik} \geq 0$. Thus, the function $\xi_k(\{I_{jk}\}_{j\neq k})$, which satisfies $\Phi_k(\xi_k(\{I_{jk}\}_{j\neq k})|\{I_{jk}\}_{j\neq i}) = 1$ uniquely, is strictly decreasing in $I_{ik} \geq 0$. To show the second property, suppose that $I'_{ik} < I''_{ik}$, and define $\xi'_k = \xi_k(\{I_{jk}\}_{j\neq i,k}, I'_{ik})$ and $\xi''_k = \xi_k(\{I_{jk}\}_{j\neq i,k}, I''_{ik})$. By the definition of $\xi_k(\{I_{jk}\}_{j\neq k})$, we have $\Phi_k(\xi'_k|\{I_{jk}\}_{j\neq i,k}, I'_{ik}) = \Phi_k(\xi''_k|\{I_{jk}\}_{j\neq i,k}, I''_{ik}) = 1$. Moreover, $\xi'_k > \xi''_k$ by the first property. Therefore, the following chain holds

$$\begin{aligned} 1 =& \rho_k \exp(\sigma_k^2\xi'_k)(1 + I'_{ik}\xi'_k)\prod_{\ell\neq i,k}(1 + I_{\ell k}\xi'_k) \quad \text{(by (4))} \\ =& \rho_k \exp(\sigma_k^2\xi''_k)(1 + I''_{ik}\xi''_k)\prod_{\ell\neq i,k}(1 + I_{\ell k}\xi''_k) \\ <& \rho_k \exp(\sigma_k^2\xi'_k)(1 + I''_{ik}\xi''_k)\prod_{\ell\neq i,k}(1 + I_{\ell k}\xi'_k) \end{aligned}$$

which implies $I'_{ik}\xi'_k < I''_{ik}\xi''_k$. By these two properties, and by (11) and the fact of $\partial \log(1 + \xi_k(\{I_{jk}\}_{j\neq k})I_{kk})/\partial I_{ik} = -\frac{\alpha_i}{\alpha_k}\pi_{ik}$ (see (9) and (10)), one can verify that $\partial \log(1 + \xi_k(\{I_{jk}\}_{j\neq k})I_{kk})/\partial I_{ik}$ is strictly increasing in $I_{ik} \geq 0$. This completes the proof. ■

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