Outage Constrained Transmission Optimization for MISO Two-Tier Femtocell Networks

Kun-Yu Wang⋆, Neil Jacklin†, Zhi Ding†, and Chong-Yung Chi⋆

⋆Institute of Communications Engineering
National Tsing Hua University,
Hsinchu, Taiwan 30013
E-mail: kunyuwang7@gmail.com, cychi@ee.nthu.edu.tw

†Department of Electrical and Computer Engineering
University of California,
Davis, CA 95616
E-mail: {najacklin, zding}@ucdavis.edu

***Abstract*—In this paper, we consider a two-tier heterogeneous network that locally consists of a multi-antenna macrocell base station and a multi-antenna femtocell base station (FBS) each serving separate single-antenna users. We investigate an optimal transmission strategy with maximum degree of freedom for transmit power minimization of the FBS under outage-based quality-of-service (QoS) constraints for the femtocell user equipment (FUE) and macrocell user equipment (MUE). Specifically, we examine the scenario that the FBS receives no instantaneous channel estimate from the FUE, and relies on only statistical information of downlink multiple-input single-output (MISO) channels. Although the outage constrained problem has no closed-form probabilistic constraints and may not be convex in general, we propose a transmission strategy and prove its optimality under a given condition. Our simulation results also demonstrate that the proposed transmission strategy can significantly save power compared to beamforming strategies.**

I. Introduction

Femtocells, which can function as operator-certified home base stations, are low power cellular access points and have been recognized as a cost-effective way to strengthen cellular network coverage and quality of service (QoS) to mostly indoor data users [1], [2]. In this paper, we consider a two-tier heterogeneous network where a multi-antenna macrocell base station (MBS) and a multi-antenna femtocell base station (FBS) simultaneously communicate with their respective mobile user equipments while sharing a common spectrum. Spectral sharing between the femtocell user equipment (FUE) and the macrocell user equipment (MUE) will inevitably lead to mutual interference. Such interference may significantly degrade link QoS within the femtocell and the macrocell [3]. Because FBS plays a supporting role in cellular coverage, femtocell interference management is a critical issue in femtocell-augmented heterogeneous networks [1]–[5].

In this paper, we consider an FBS transmit covariance matrix design aiming to optimize FBS transmit power performance under a signal-to-interference-plus-noise ratio (SINR) constraint on the FUE and an interference power constraint on the MUE. When the FBS has perfect channel state information (CSI), it has been shown that beamforming, i.e., rank-one transmit covariance matrix, is an optimal transmission strategy in terms of transmit power minimization [6]. The optimal beamformer can be obtained by solving a convex second-order cone program (SOCP) reformulated from the problem in [4]. However, using beamforming strategy is not necessarily optimal with only imperfect CSI knowledge at the FBS.

Our work here assumes that the FBS has no instantaneous channel estimate feedback from the FUE. In particular, the FBS knows only the statistical information of the downlink channels. Specifically, each downlink channel is assumed to undergo independent and identically distributed (i.i.d.) Rayleigh fading. Due to channel fading, the SINR and interference performance may suffer from outage. To mitigate this outage, we investigate an optimal transmission strategy for transmit power minimization such that the the outage occurs below a (small) preset probability threshold. The probabilistic constraints in general have no closed-form expression, and can be hard to handle. Perhaps because of this reason, optimal transmission strategy for the outage constrained problem remains unknown thus far. In this work, we propose a new transmission strategy and show its optimality provided that a given condition holds true. The simulation results show that the proposed transmission strategy can significantly save transmit power compared to beamforming strategies.

II. System Model and Problem Statement

We consider a two-tier heterogeneous network whose coverage comprises both macrocell and localized femtocells. The MBS, equipped with N_M antennas, communicates with a single-antenna MUE. A closed access FBS, equipped with N_F antennas, serves a single-antenna FUE which shares the MUE spectrum during the downlink transmission by MBS and FBS. The MUE belongs to tier-1 with a higher priority for QoS guarantee while the FUE provides tier-2 service characterized as "best-effort".

Let $s_M(t) \in \mathbb{C}$ denote the information signal for MUE. The transmit signal at the MBS is given by $\boldsymbol{w}_M \cdot s_M(t)$ where $\boldsymbol{w}_M \in \mathbb{C}^{N_M}$ is the beamforming vector for $s_M(t)$. To improve femtocell system performance, we consider the transmit signal at FBS, denoted by $\boldsymbol{x}(t) \in \mathbb{C}^{N_F}$, which has zero mean and covariance matrix $\boldsymbol{Q}_F \succeq \mathbf{0}$ (positive semidefinite). Therefore, the received signal at the FUE can be expressed as

$$y(t) = \boldsymbol{h}_{FF}^H \boldsymbol{x}(t) + \boldsymbol{h}_{FM}^H \boldsymbol{w}_M s_M(t) + n_F(t), \qquad (1)$$

where $\boldsymbol{h}_{FF} \in \mathbb{C}^{N_F}$ and $\boldsymbol{h}_{FM} \in \mathbb{C}^{N_M}$, respectively, denote the channel vectors from FBS to FUE and from MBS to

978-1-4673-3122-7/13/$31.00 ©2013 IEEE

FUE, whereas $n_F(t) \in \mathbb{C}$ is the additive noise at FUE with power $\sigma_F^2 > 0$. The first term in (1) is the intended signal for FUE, and the second term is the interference from macrocell. Assume that $s_M(t)$ has zero mean and $\mathbb{E}[|s_M(t)|^2] = 1$. Then the SINR at FUE can be obtained from (1) as

$$\mathsf{SINR}_F = \frac{\boldsymbol{h}_{FF}^H \boldsymbol{Q}_F \boldsymbol{h}_{FF}}{|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2}, \tag{2}$$

and the interference power at the MUE from the femtocell can be seen to be $\boldsymbol{h}_{MF}^H \boldsymbol{Q}_F \boldsymbol{h}_{MF}$ where $\boldsymbol{h}_{MF} \in \mathbb{C}^{N_F}$ denotes the channel vector between FBS and MUE.

Typically, femtocells are connected to macrocell network via a wired broadband backhaul link such as digital subscriber line (DSL) [7], and thus we assume that the beamforming vector at the MBS, $\boldsymbol{w}_M$, is perfectly known to the FBS. In this work, we consider the scenario that the FBS receives no instantaneous channel estimate from the FUE and only knows statistical channel information. In particular, the channel vectors $\boldsymbol{h}_{FF}$, $\boldsymbol{h}_{FM}$, and $\boldsymbol{h}_{MF}$ are assumed to be circularly symmetric complex Gaussian distributed with zero mean and covariance matrix $\sigma_{FF}^2 \boldsymbol{I}_{N_F}$ (an $N_F \times N_F$ identity matrix), $\sigma_{FM}^2 \boldsymbol{I}_{N_M}$, and $\sigma_{MF}^2 \boldsymbol{I}_{N_F}$, respectively, i.e., $\boldsymbol{h}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_{FF}^2 \boldsymbol{I}_{N_F})$, $\boldsymbol{h}_{FM} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_{FM}^2 \boldsymbol{I}_{N_M})$, and $\boldsymbol{h}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_{MF}^2 \boldsymbol{I}_{N_F})$. Let $\gamma_F \geq 0$ and $\epsilon_M \geq 0$ be the preset target SINR value at the FUE and the preset interference power threshold at the MUE. Due to channel fading, the QoS may suffer from outage, i.e., it would have nonzero probability for $\mathsf{SINR}_F < \gamma_F$ and $\boldsymbol{h}_{MF}^H \boldsymbol{Q}_F \boldsymbol{h}_{MF} > \epsilon_M$. To mitigate this QoS outage, our goal is to design the transmit covariance matrix, $\boldsymbol{Q}_F$, such that the outage probability is kept below a small threshold. Mathematically, the problem can be reformulated as

$$\begin{aligned}
&\min_{\boldsymbol{Q}_F \in \mathbb{H}^{N_F}} \ \mathrm{Tr}(\boldsymbol{Q}_F) && \text{(3a)}\\
&\text{s.t. Prob}\left\{ \frac{\boldsymbol{h}_{FF}^H \boldsymbol{Q}_F \boldsymbol{h}_{FF}}{|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2} \geq \gamma_F \right\} \geq 1 - \rho_F, && \text{(3b)}\\
&\quad \text{Prob}\left\{ \boldsymbol{h}_{MF}^H \boldsymbol{Q}_F \boldsymbol{h}_{MF} \leq \epsilon_M \right\} \geq 1 - \rho_M, && \text{(3c)}\\
&\quad \boldsymbol{Q}_F \succeq \boldsymbol{0}, && \text{(3d)}
\end{aligned}$$

where ρ_F and ρ_M denote the preset maximum tolerable outage probabilities for SINR and interference power constraints, respectively. However, problem (3) is intractable since the probability functions in (3b) and (3c) have no closed-form and may not be convex in general.

III. Optimal Transmission Strategy

To solve problem (3), let us write

$$\boldsymbol{Q}_F = \mathrm{Tr}(\boldsymbol{Q}_F)\tilde{\boldsymbol{Q}}_F, \ \boldsymbol{h}_{FF} = \sigma_{FF} \cdot \boldsymbol{v}_{FF}, \ \boldsymbol{h}_{MF} = \sigma_{MF} \cdot \boldsymbol{v}_{MF}, \tag{4}$$

where $\tilde{\boldsymbol{Q}}_F \succeq \boldsymbol{0}$, $\mathrm{Tr}(\tilde{\boldsymbol{Q}}_F) = 1$; $\boldsymbol{v}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_F})$; and $\boldsymbol{v}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_F})$ are power normalized counterparts of $\boldsymbol{Q}_F$, $\boldsymbol{h}_{FF}$, and $\boldsymbol{h}_{MF}$, respectively. With the expressions in (4), problem (3) can be equivalently written as

$$\begin{aligned}
&\min_{\tilde{\boldsymbol{Q}}_F \in \mathbb{H}^{N_F}, P_F \in \mathbb{R}} \ P_F \triangleq \mathrm{Tr}(\boldsymbol{Q}_F) && \text{(5a)}\\
&\text{s.t. Prob}\left\{ \boldsymbol{v}_{FF}^H \tilde{\boldsymbol{Q}}_F \boldsymbol{v}_{FF} \leq \frac{\gamma_F}{\sigma_{FF}^2 P_F} \times (|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2) \right\} \leq \rho_F, && \text{(5b)}\\
&\quad \text{Prob}\left\{ \boldsymbol{v}_{MF}^H \tilde{\boldsymbol{Q}}_F \boldsymbol{v}_{MF} \leq \frac{\epsilon_M}{\sigma_{MF}^2 P_F} \right\} \geq 1 - \rho_M, && \text{(5c)}\\
&\quad \tilde{\boldsymbol{Q}}_F \succeq \boldsymbol{0}, \ \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F) = 1, \ P_F \geq 0. && \text{(5d)}
\end{aligned}$$

We should point out that constraint (5c) actually provides an upper bound on P_F for which problem (5) is feasible. Therefore, to minimize the transmit power P_F, we need only consider constraints (5b) and (5d).

Let us first consider problem (5) without constraint (5c),

$$\begin{aligned}
&\min_{\tilde{\boldsymbol{Q}}_F \in \mathbb{H}^{N_F}, P_F \in \mathbb{R}} \ P_F && \text{(6a)}\\
&\text{s.t. Prob}\left\{ \boldsymbol{v}_{FF}^H \tilde{\boldsymbol{Q}}_F \boldsymbol{v}_{FF} \leq \frac{\gamma_F}{\sigma_{FF}^2 P_F} \times (|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2) \right\} \leq \rho_F, && \text{(6b)}\\
&\quad \tilde{\boldsymbol{Q}}_F \succeq \boldsymbol{0}, \ \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F) = 1, \ P_F \geq 0. && \text{(6c)}
\end{aligned}$$

One can observe that constraint (6b) must be active when the optimal $\tilde{\boldsymbol{Q}}_F$ and P_F are achieved. Then the resulting equality constraint implies that minimizing the probability function in (6b) (which is a monotonic non-increasing function of P_F) will minimize P_F. For this reason, the optimal $\tilde{\boldsymbol{Q}}_F$ is obtained by solving

$$\min_{\tilde{\boldsymbol{Q}}_F \succeq \boldsymbol{0}, \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F) = 1} \mathbb{E}\left[\mathrm{Prob}\left\{ \boldsymbol{v}_{FF}^H \tilde{\boldsymbol{Q}}_F \boldsymbol{v}_{FF} \leq \frac{\gamma_F}{\sigma_{FF}^2 P_F} \times (z + \sigma_F^2) \ \middle| \ |\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 = z \right\} \right], \tag{7}$$

where the expectation is taken with respect to the exponential random variable $|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2$. With the optimal $\tilde{\boldsymbol{Q}}_F$, denoted by $\tilde{\boldsymbol{Q}}_F^\star$, the optimal $P_F^\star$ to problem (6) can be obtained by solving

$$\mathbb{E}\left[\mathrm{Prob}\left\{ \boldsymbol{v}_{FF}^H \tilde{\boldsymbol{Q}}_F^\star \boldsymbol{v}_{FF} \leq \frac{\gamma_F}{\sigma_{FF}^2 P_F} \times (z + \sigma_F^2) \ \middle| \ |\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 = z \right\} \right] = \rho_F. \tag{8}$$

However, the optimal transmission strategy $\boldsymbol{Q}_F^\star = P_F^\star \tilde{\boldsymbol{Q}}_F^\star$ for problem (6) may not be feasible to problem (5) because it may not satisfy the probability constraint (5c). Due to this concern, in the next subsection, we will propose a feasibility condition under which $\boldsymbol{Q}_F^\star$ is feasible and optimal to problem (5).

We now concentrate on solving problem (7). Consider the eigenvalue decomposition of $\tilde{\boldsymbol{Q}}_F = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H$, where $\boldsymbol{U} \in \mathbb{C}^{N_F \times N_F}$ is a unitary matrix and $\boldsymbol{\Lambda} \in \mathbb{R}^{N_F \times N_F}$ is a diagonal matrix with eigenvalues $\lambda_1, \ldots, \lambda_{N_F} \geq 0$ being the diagonal elements. Since unitary transformation of Gaussian random vector will not change their joint probability density distribution, the expectation function in (7) can be equivalently written as

$$\mathbb{E}\left[\operatorname{Prob}\left\{\boldsymbol{v}_{FF}^H\boldsymbol{\Lambda}\boldsymbol{v}_{FF}\leq\frac{\gamma_F}{\sigma_{FF}^2P_F}(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right]$$
$$=\mathbb{E}\left[\operatorname{Prob}\left\{\sum_{i=1}^{N_F}\lambda_i|v_i|^2\leq\frac{\gamma_F}{\sigma_{FF}^2P_F}(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right], \tag{9}$$

where v_i denotes the ith entry of $\boldsymbol{v}_{FF}$. Moreover, it has been shown in [8], [9] that for a given transmission degrees of freedom (DoF) equal to d, i.e., the number of positive eigenvalues in $\tilde{\boldsymbol{Q}}_F$, the probability function in (9) is minimized by uniformly allocating the total power over d DoF, namely,

$$\operatorname{Prob}\left\{\frac{1}{d}\sum_{i=1}^{d}|v_i|^2\leq\frac{\gamma_F}{\sigma_{FF}^2P_F}(z+\sigma_F^2)\right\}$$
$$\leq\operatorname{Prob}\left\{\sum_{i=1}^{d}\lambda_i|v_i|^2\leq\frac{\gamma_F}{\sigma_{FF}^2P_F}(z+\sigma_F^2)\right\}, \tag{10}$$

for all $z\geq 0$, where $\lambda_i>0$, $i=1,\ldots,d$, and $\sum_{i=1}^d\lambda_i=1$.

According to (10), one can show that the optimal $\tilde{\boldsymbol{Q}}_F$ for problem (7) [or, equivalently, problem (6)] can be written as

$$\tilde{\boldsymbol{Q}}_F^\star=\boldsymbol{U}^\star\boldsymbol{\Lambda}(d^\star)(\boldsymbol{U}^\star)^H, \tag{11}$$

where $d^\star$ denotes the optimal DoF (to be presented below); $\boldsymbol{U}^\star\in\mathbb{C}^{N_F\times N_F}$ can be an any arbitrary unitary matrix and $\boldsymbol{\Lambda}(d^\star)\in\mathbb{R}^{N_F\times N_F}$ is a diagonal matrix with the first $d^\star$ diagonal elements being nonzero and equal to $1/d^\star$ (due to $\operatorname{Tr}(\tilde{\boldsymbol{Q}}_F^\star)=1$).

The optimal $d^\star$ is obtained as

$$d^\star=\arg\min\left\{P_F(d),\ d=1,\ldots,N_F\right\}, \tag{12}$$

where $P_F(d)$ is the power by numerically solving (8), which can be equivalently expressed as

$$\mathbb{E}\left[\operatorname{Prob}\left\{\chi_{2d}^2\leq\frac{2d\gamma_F}{\sigma_{FF}^2P_F}(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right]=\rho_F$$
$$\Rightarrow\frac{1}{(d-1)!\sigma_{FM}^2\|\boldsymbol{w}_M\|^2}\int_0^\infty\Gamma\left(d,\frac{d\gamma_F}{\sigma_{FF}^2P_F}(z+\sigma_F^2)\right)\times$$
$$\exp\left(-\frac{z}{\sigma_{FM}^2\|\boldsymbol{w}_M\|^2}\right)dz=\rho_F, \tag{13}$$

where $\|\cdot\|$ denotes Euclidean norm; χ_d^2 denotes the central chi-square random variable with d DoF; and $\Gamma(\alpha,\beta)$ is lower incomplete gamma function, defined as

$$\Gamma(\alpha,\beta)=\int_0^\beta t^{\alpha-1}\exp(-t)\,dt. \tag{14}$$

As a result, the optimal transmission strategy for problem (6) can be expressed as [see (4)]

$$\boldsymbol{Q}_F^\star=P_F(d^\star)\tilde{\boldsymbol{Q}}_F^\star. \tag{15}$$

On the other hand, constraint (5c) with $d=d^\star$ can be represented by

$$\operatorname{Prob}\left\{\frac{1}{2d^\star}\chi_{2d^\star}^2\leq\frac{\epsilon_M}{\sigma_{MF}^2P_F}\right\}\geq 1-\rho_M$$
$$\Rightarrow P_F\leq\frac{\epsilon_M}{\sigma_{MF}^2F_{2d^\star}^{-1}(1-\rho_M)}\triangleq\bar{P}_F(d^\star), \tag{16}$$

where $F_d^{-1}(\cdot)$ is the inverse function of $F_d(\cdot)$, and

$$F_d(x)\triangleq\operatorname{Prob}\left\{\frac{1}{d}\chi_d^2\leq x\right\}, \tag{17}$$

which is a continuous, monotonically increasing, and invertible function. Thus, as long as the condition $P_F(d^\star)\leq\bar{P}_F(d^\star)$ is satisfied, the transmission strategy in (15) is also optimal to problem (5).

Remark 1 *We should emphasize that when $P_F(d^\star)>\bar{P}_F(d^\star)$, the transmission strategy in* (15) *is not feasible to problem* (5) *any more. For this case, one can determine the transmission DoF by solving* (13) *for $d=1,\ldots,N_F$, and then choose the transmission DoF with the minimum power satisfying* (16). *However, the obtained $\boldsymbol{Q}_F$ is a suboptimal solution to problem* (5) *since there may exist a non-uniform power allocation strategy such that constraint* (16) *is satisfied with lower power. The optimal solution for this case is still an open problem.*

A. Feasibility Condition for Transmission Strategy (15)

Next, we present a sufficient condition under which the transmission strategy in (15) is feasible, and thus is optimal to the optimization problem (5). To proceed, we need the following definition:

Definition 1 *For positive integers $d\geq 1$ and $\ell\geq 1$, let $x(d,d+\ell)$ represent the point at which the CDFs $F_d(x)$, which is defined in* (17), *and $F_{d+\ell}(x)$ intersect. The quantity*

$$p(d,d+\ell)=F_d(x(d,d+\ell))=F_{d+\ell}(x(d,d+\ell)) \tag{18}$$

represents the corresponding probability value of $x(d,d+\ell)$.

It has been shown in [8] that $p(d,d+\ell)$ is unique, greater than 0.5, and decreases to 0.5 as d increases, and precise values of $p(d,d+\ell)$ for $d\geq 1$ and $\ell\geq 1$ can be computed numerically. Using the quantities in Definition 1, we can prove the following proposition.

Proposition 1 *The transmission strategy in* (15) *is a feasible solution to problem* (5) *if*

$$\exp\left(-\frac{\gamma_F\sigma_F^2\sigma_{MF}^2F_{2\theta}^{-1}(1-\rho_M)}{\sigma_{FF}^2\epsilon_M}\right)\times$$
$$\frac{\sigma_{FF}^2\epsilon_M}{\sigma_{FF}^2\epsilon_M+\gamma_F\sigma_{MF}^2\sigma_{FM}^2\|\boldsymbol{w}_M\|^2F_{2\theta}^{-1}(1-\rho_M)}\geq 1-\rho_F \tag{19}$$

holds true, where

$$\theta = \begin{cases} n, & \forall (1-\rho_M) \in [p(2n, 2n+2), p(2n-2, 2n)), \\ & \qquad n = 1, \ldots, N_F - 1, \\ N_F, & \forall (1-\rho_M) \in [0, p(2N_F - 2, 2N_F)), \end{cases}$$

in which $p(0, 2) \triangleq 1$.

Proof: We prove Proposition 1 by considering problem (5) with DoF $d = 1$, which can be equivalently expressed as [10]:

$$\min_{P_F \in \mathbb{R}} \; P_F \tag{20a}$$

$$\text{s.t.} \;\; \exp\left(-\frac{\gamma_F \sigma_F^2}{\sigma_{FF}^2 P_F}\right) \frac{\sigma_{FF}^2 P_F}{\sigma_{FF}^2 P_F + \gamma_F \sigma_{FM}^2 \|\boldsymbol{w}_M\|^2} \geq 1 - \rho_F, \tag{20b}$$

$$P_F \leq \frac{\epsilon_M}{\sigma_{MF}^2 \ln(1/\rho_M)}, \tag{20c}$$

$$P_F \geq 0. \tag{20d}$$

To present the proof, let us first consider the following problem [i.e., problem (20) with (20c) replaced by (21c)]:

$$\min_{P_F \in \mathbb{R}} \; P_F \tag{21a}$$

$$\text{s.t.} \;\; \exp\left(-\frac{\gamma_F \sigma_F^2}{\sigma_{FF}^2 P_F}\right) \frac{\sigma_{FF}^2 P_F}{\sigma_{FF}^2 P_F + \gamma_F \sigma_{FM}^2 \|\boldsymbol{w}_M\|^2} \geq 1 - \rho_F, \tag{21b}$$

$$P_F \leq \frac{\epsilon_M}{\sigma_{MF}^2 F_{2\theta}^{-1}(1-\rho_M)} \triangleq \bar{P}_F(\theta), \tag{21c}$$

$$P_F \geq 0, \tag{21d}$$

where

$$\theta \triangleq \arg\max\{F_{2d}^{-1}(1-\rho_M), \; d = 1, \ldots, N_F\} = \begin{cases} n, & \forall (1-\rho_M) \in [p(2n, 2n+2), p(2n-2, 2n)), \\ & \qquad n = 1, \ldots, N_F - 1, \\ N_F, & \forall (1-\rho_M) \in [0, p(2N_F - 2, 2N_F)), \end{cases} \tag{22}$$

which has been proven in [11]. We will use the following lemma [10]:

Lemma 1 *Problem* (21) *is feasible if and only if*

$$\exp\left(-\frac{\gamma_F \sigma_F^2 \sigma_{MF}^2 F_{2\theta}^{-1}(1-\rho_M)}{\sigma_{FF}^2 \epsilon_M}\right) \times \frac{\sigma_{FF}^2 \epsilon_M}{\sigma_{FF}^2 \epsilon_M + \gamma_F \sigma_{MF}^2 \sigma_{FM}^2 \|\boldsymbol{w}_M\|^2 F_{2\theta}^{-1}(1-\rho_M)} \geq 1 - \rho_F. \tag{23}$$

If problem (21) is feasible, i.e., (23) holds true, then problem (20) is also feasible since the feasible set of constraint (21c) is a subset of constraint (20c) since $F_{2\theta}^{-1}(1-\rho_M) \geq F_2^{-1}(1-\rho_M) = \ln(1/\rho_M)$ [by the definition in (22)], i.e.,

$$\bar{P}_F(\theta) \leq \frac{\epsilon_M}{\sigma_{MF}^2 \ln(1/\rho_M)}. \tag{24}$$

That is to say, problem (5) with DoF $d = 1$ is feasible if problem (21) is feasible.

Next, we show that the optimal transmit power to problem (5) with DoF $d = 1$, denoted as $P_F(1)$ [which is obtained by solving (13)], must be less than or equal to $\bar{P}_F(\theta)$, i.e.,

$$P_F(1) \leq \bar{P}_F(\theta). \tag{25}$$

Since one can always find a feasible power to problem (21), say $\tilde{P}_F$, which is also feasible to problem (5) with DoF $d = 1$ [due to (24)], if $P_F(1) > \bar{P}_F(\theta)$ holds, then we have $\tilde{P}_F \leq \bar{P}_F(\theta) < P_F(1)$, which is a contradiction to $P_F(1)$ being the optimal (minimum) power to problem (5) with DoF $d = 1$.

Since $P_F(1) \geq P_F(d^\star)$ [by (12)] and $\bar{P}_F(\theta) \leq \bar{P}_F(d^\star)$ [by (21c) and (22)], according to (25), we have $P_F(d^\star) \leq \bar{P}_F(d^\star)$, which means that the transmission strategy in (15) is feasible to problem (5). The proof is thus complete. ∎

IV. Simulation Results

We consider FBS and MBS each equipped with four transmit antennas, i.e., $N_F = N_M = 4$. Let maximum tolerable outage probabilities $\rho_F = \rho_M = 0.1$, requiring that the SINR constraint at FUE and the interference power constraint on MUE be satisfied over 90% of the time. Assume $\sigma_F^2 = 0.01$ and $\epsilon_M = -3$ dB. The beamforming vector $\boldsymbol{w}_M$ at MBS is randomly generated uniformly on the unit-norm sphere $\|\boldsymbol{w}_M\|^2 = 1$. Considering the indoor scenario for FBS where the channel strength of $\boldsymbol{h}_{MF}$ and $\boldsymbol{h}_{FM}$ are much lower than that of $\boldsymbol{h}_{FF}$ due to penetration losses [3], we set $\sigma_{FF}^2 = 1$, and $\sigma_{MF}^2 = \sigma_{FM}^2 = 0.01$.

In our first example, we show the distribution of achievable SINR of FUE, i.e., the value of SINR_F in (2), obtained over 10^5 realizations of true i.i.d. channels $\boldsymbol{h}_{FF} \sim \mathcal{CN}(\mathbf{0}, \sigma_{FF}^2 \boldsymbol{I}_{N_F})$, $\boldsymbol{h}_{MF} \sim \mathcal{CN}(\mathbf{0}, \sigma_{MF}^2 \boldsymbol{I}_{N_F})$, and $\boldsymbol{h}_{FM} \sim \mathcal{CN}(\mathbf{0}, \sigma_{FM}^2 \boldsymbol{I}_{N_M})$. The target SINR γ_F is set to 15 dB. One can show that the transmission strategy in (15) is optimal to problem (3), or equivalently problem (5), according to Proposition 1, and the optimal DoF $d^\star$, defined in (12), can be obtained numerically as $d^\star = N_F$. Figure 1 shows the distribution of the achievable SINR of FUE, where "Optimal Transmission Strategy" and "Optimal Beamforming Strategy" denote the results from using the transmission strategy in (15) with $d = N_F$ and $d = 1$, respectively. The optimal transmit beamformer can be determined as $\boldsymbol{w}_F \triangleq \sqrt{P_F(1)}\boldsymbol{u}_F$, where $P_F(1)$ is the transmit power obtained by solving (13) with $d = 1$, and $\boldsymbol{u}_F \in \mathbb{C}^{N_F}$ is an arbitrary column vector with $\|\boldsymbol{u}_F\| = 1$. From this figure, we can see that both methods can achieve 90% SINR satisfaction probability. In addition, the mean SINR value of the FUE using the optimal transmission strategy is 19.85 dB, which is lower than the corresponding 25.45 dB from using the optimal beamforming strategy, and this indicates that FBS can save transmit power while achieving the same desired SINR performance when using the optimal transmission strategy.

Next, we compare the transmit power performance of the optimal transmission strategy and the optimal beamforming strategy under different target SINR values γ_F. The obtained results are shown in Fig. 2, where "Optimal Transmission Strategy" and "Optimal Beamforming Strategy" denote the

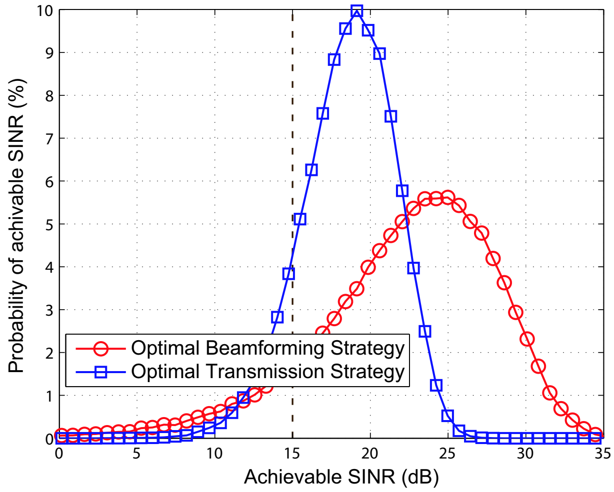

Fig. 1: Probability distributions of achievable SINR values of FUE for $\gamma_F = 15$ dB, both at outage probabilities are 10%.

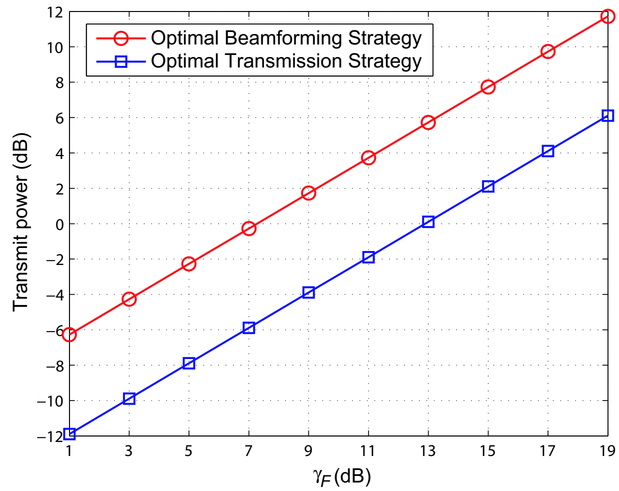

Fig. 2: Transmit power versus target SINR γ_F.

results obtained by solving problem (5) using the transmission strategy in (15) with $d = N_F$ and $d = 1$, respectively. From this figure, one can see that the difference of transmit power between the optimal beamforming strategy and the optimal transmission strategy is approximately 5.6 dB. We also show the power performance under different number of transmit antennas at FBS in Fig. 3, where the target SINR γ_F is set to 15 dB. As can be seen from this figure, the performance gap increases as the number of transmit antennas, N_F, increases. Specifically, the difference of transmit power between the optimal beamforming strategy and the optimal transmission strategy becomes 6.8 dB at $N_F = 10$. From Fig. 2 and Fig. 3, we observe that the optimal transmission strategy can significantly save transmit power than the optimal beamforming strategy. Naturally, beamforming strategy has lower implementation complexity at the cost of higher power, which is a typical performance-complexity tradeoff.

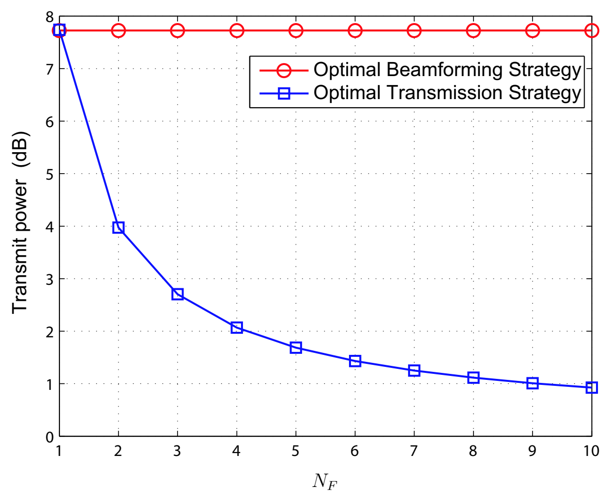

Fig. 3: Transmit power versus N_F for $\gamma_F = 15$ dB.

Acknowledgments

This material is based upon works supported by the National Science Council, R.O.C. under Grant NSC-99-2221-E-007-052-MY3, and by the National Science Foundation under Grants 1147930 and 0917251.

References

[1] T. Zahir, K. Arshad, A. Nakata, and K. Moessner, "Interference management in femtocells," *to appear in IEEE Commun. Surveys & Tutorials*, 2012.

[2] J. G. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. C. Reed, "Femtocells: Past, present, and future," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 497–508, Apr. 2012.

[3] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, "Femtocell networks: A survey," *IEEE Commun. Mag.*, vol. 46, no. 9, pp. 59–67, Sep. 2008.

[4] Y. Jeong, T. Q. S. Quek, and H. Shin, "Beamforming optimization for multiuser two-tier networks," *J. Commun. and Networks*, vol. 13, no. 4, pp. 327–338, Aug. 2011.

[5] V. Chandrasekhar, J. G. Andrews, T. Muharemovic, Z. Shen, and A. Gatherer, "Power control in two-tier femtocell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4316–4328, Aug. 2009.

[6] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.

[7] M. Yavuz, F. Meshkati, S. Nanda, A. Pokhariyal, N. Johnson, B. Raghothaman, and A. Richardson, "Interference management and performance analysis of UMTS/HSPA+ femtocells," *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 102–109, Sep. 2009.

[8] G. J. Székely and N. K. Bakirov, "Extremal probabilities for Gaussian quadratic forms," *Probability Theory and Related Fields*, vol. 126, no. 2, pp. 184–202, Jun. 2003.

[9] E. A. Jorswieck and H. Boche, "Outage probability in multiple antenna systems," *European Trans. Telecommun.*, vol. 18, no. 3, pp. 217–233, Apr. 2007.

[10] K.-Y. Wang, N. Jacklin, Z. Ding, and C.-Y. Chi, "Robust MISO transmit optimization under outage-based QoS constraints in two-tier heterogeneous networks," *to appear in IEEE Trans. Wireless Commun.*

[11] K.-Y. Wang, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "Optimal transmission strategy for outage rate maximization in MISO fading channels with training," in *Proc. IEEE ICASSP 2012*, Kyoto, Japan, Mar. 25-30, 2012, pp. 2945–2948.