ENERGY-EFFICIENT PRECODING MATRIX DESIGN FOR RELAY-AIDED MULTIUSER DOWNLINK NETWORKS

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ABSTRACT

This paper considers the energy-efficient precoding matrix design for relay-aided multiuser downlink multiple-input single-output wireless systems. The precoders of the base station (BS) and the relay station (RS) are designed to maximize the transmit energy efficiency, defined as the ratio between the system sum rate and the total power consumption, under the quality-of-service constraints of the users and the transmit power constraints on the BS and the RS. In view of the fact that this precoder design problem is a nonconvex fractional programming, a successive Dinkelbach and convex approximation (SDCA) algorithm is proposed to handle this problem. Simulation results are provided to demonstrate the effectiveness of the proposed SDCA algorithm, and significant EE improvement as the number of antennas at the BS and the RS increases.

Index Terms— Convex optimization, energy efficiency, beamforming designs, relay-aided communications, fractional programming.

1. INTRODUCTION

The expeditious expansion of wireless networks has resulted in a tremendous increase in energy consumption. Thus, the issue of energy efficiency (EE) in wireless communications has drawn increasing attention in both academia and industry recently [1]. Among various definitions of EE, the most widely used is the ratio between the achievable transmission rate and the total power consumption, which is usually measured in bits/joule [2, 3]. The resource allocation for EE optimization has been extensively studied under various scenarios, e.g.,frequency-selective interference channel [3], point-to-point parallel AWGN channel [4], point-to-point multiple-input multiple-output (MIMO) channel [5, 6], multiple access channel (MAC) [7].

The aforementioned works focus on one-hop networks. However, wireless relaying is indispensable for reliable transmission with high throughput in the areas with severe shadowing effect, or remote from the base stations (BSs) [8]. The energy-efficient transmission design for relay-aided networks is difficult since the signal-to-interference-plus-noise ratio (SINR) of the users is inevitably a complicated nonconvex function of the product of the transmission precoders of the BS and the relay station (RS), making the transmission design an involved nonconvex problem. Thus, there are few works addressing EE of relay-aided networks. In the literature so far, EE-optimal relay placement is investigated for one-dimensional cellular network [9], energy-efficient noncooperative power control strategy is developed for relay-aided single-input single-output (SISO) interference channel [10], energy-efficient precoder design is devised for relay-aided single-user MIMO downlink transmission [11, 12, 13], and a low complexity EE maximization method is proposed for multiuser uplink networks [14].

In this paper, we consider a relay-aided multiuser downlink system consisting of one BS, one RS, and K users. Assuming amplify-and-forward (AF) relay scheme, we design the precoding matrices of the BS and the RS to maximize the EE under the quality-of-service (QoS) constraints of each individual user and the transmit power constraints of the BS and the RS. The resulting energy efficiency maximization (EEM) problem is a nonconvex fractional programming [15], and is difficult to solve. A special case, i.e., the single-user case, of this EEM problem has been studied in [11] using the Dinkelbach's algorithm [15] and alternating optimization method [16]. This method, however, is not directly applicable to the multiuser case due to the inter-user interference. We hence propose a successive Dinkelbach and convex approximation (SDCA) algorithm to obtain an approximate solution for the multiuser case. We successively approximate the EEM problem by the Dinkelbach's approximation and a conservative approximation based on the inequality of arithmetic and geometric means, leading to an approximation problem that is convex in the the precoding matrices of the BS and the RS, respectively. Then, we can apply the alternating optimization method to handle the resulting problem. Finally, simulation results are provided to demonstrate the efficacy of the SDCA algorithm.

2. SIGNAL MODEL AND PROBLEM STATEMENT

Consider a relay-aided downlink transmission system consisting of one BS, one half-duplex AF RS, and K single-antenna users, where the BS and the RS are equipped with M_B and M_R antennas, respectively. Assuming no direct path between the BS and the users, the downlink transmission is divided into two phases. In the first phase, the BS transmits K data streams (one for each user) to the RS, and the transmitted signal can be expressed as

$$\boldsymbol{x}_B = \sum_{k=1}^{K} \boldsymbol{b}_k s_k,$$

where $s_k \in \mathbb{C}$ is the signal intended for user k, and $\boldsymbol{b}_k \in \mathbb{C}^{M_B}$ is the corresponding beamformer. In the second phase, the RS amplifies and forwards the received signal to the K users by the AF precoding matrix $\mathbf{R} \in \mathbb{C}^{M_R \times M_R}$. Let $\mathbf{H} \in \mathbb{C}^{M_R \times M_B}$ denote the MIMO channel between the BS and the RS, and $\mathbf{g}_k \in \mathbb{C}^{M_R}$ denote the multiple-input single-output (MISO) channel between the RS and the kth user for $k = 1, \ldots, K$. Then, the received signal at the kth user is given by

$$\begin{aligned} y_k &= \mathbf{g}_k^H \mathbf{R}(\mathbf{H}\boldsymbol{x}_B + \mathbf{z}_R) + z_k \\ &= \mathbf{g}_k^H \mathbf{R}\mathbf{H}\left(\textstyle\sum_{k=1}^{K} \boldsymbol{b}_k s_k\right) + \mathbf{g}_k^H \mathbf{R}\mathbf{z}_R + z_k \end{aligned}$$

where $\mathbf{z}_R \sim \mathcal{CN}(\mathbf{0}, \sigma_R^2 \mathbf{I}_{M_R})$ and $z_k \sim \mathcal{CN}(0, \sigma_k^2)$ are the additive Gaussian noises at the RS and the kth user, respectively.

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Assume that the information signals are standard complex Gaussian distributed, i.e., $s_k \sim \mathcal{CN}(0,1)$, and that all the users decode their received signals using a single-user detection scheme. Then, the instantaneously achievable data rate to the kth user is given by

$$R_k(\mathbf{B},\mathbf{R}) = \frac{W}{2}\log_2(1+\mathsf{SINR}_k) \text{ (bits/sec)}, \quad (1)$$

where W is the transmission bandwidth (assumed to be 1 for simplcity), $\mathbf{B} \triangleq [\boldsymbol{b}_1,\ldots,\boldsymbol{b}_K]$, and SINR_k is given by

$$\mathsf{SINR}_k = \frac{|\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_k|^2}{\sum_{j\neq k}|\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_j|^2 + \sigma_R^2\|\mathbf{g}_k^H\mathbf{R}\|^2+\sigma_k^2}, \quad (2)$$

where $\|\cdot\|$ denotes the Euclidean norm. On the other hand, the transmit powers of the BS and the RS can be respectively expressed as

$$P_B(\mathbf{B}) = \frac{1}{2\zeta_B}\mathrm{Tr}(\mathbf{B}\mathbf{B}^H), \quad (3)$$

$$P_R(\mathbf{B},\mathbf{R}) = \frac{1}{2\zeta_R}\big(\mathrm{Tr}(\mathbf{R}\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H\mathbf{R}^H) + \sigma_R^2\mathrm{Tr}(\mathbf{R}\mathbf{R}^H)\big), \quad (4)$$

where $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix, ζ_B and ζ_R are the power amplifier efficiencies at BS and RS, respectively, and the factor $1/2$ is due to the two-phase transmission. For simplicity, We assume $\zeta_B = \zeta_R = 1$. Other power consumptions, including circuit power, signal processing power, cooling loss and so on, at BS and RS is also taken into account, and is modeled as [17]

$$P_C = \alpha M + P_{sta}, \quad (5)$$

where αM (a linear function of the number of active antennas M) stands for the dynamic power consumption, and P_{sta} stands for the static power consumption of the baseband signal processing.

The EE of this relay-aided downlink system is defined as the ratio of the achievable sum rate to the total transmitted power $P_T(\mathbf{B},\mathbf{R}) = P_B(\mathbf{B}) + P_R(\mathbf{B},\mathbf{R}) + P_C$, i.e.,

$$\mathsf{EE}(\mathbf{B},\mathbf{R}) = \frac{\sum_{k=1}^K R_k(\mathbf{B},\mathbf{R})}{P_T(\mathbf{B},\mathbf{R})} \text{ (bits/joule)}. \quad (6)$$

Our goal is to maximize the transmission EE under QoS constraints on each user and power constraints on the BS and the RS, i.e.,

$$\max_{\mathbf{B},\mathbf{R}} \mathsf{EE}(\mathbf{B},\mathbf{R}) \quad (7a)$$

$$\text{s.t. } \mathsf{SINR}_k \geq \gamma_k,\ k=1,\ldots,K, \quad (7b)$$

$$P_B(\mathbf{B}) \leq \bar{P}_B,\ P_R(\mathbf{B},\mathbf{R}) \leq \bar{P}_R, \quad (7c)$$

where γ_k is the QoS requirement for user k; $\bar{P}_B$ and $\bar{P}_R$ are the power budgets of BS and RS, respectively. Problem (7) is difficult to solve since it is a noncovex fractional optimization problem.

3. SUCCESSIVE DINKELBACH AND CONVEX APPROXIMATION (SDCA) ALGORITHM

3.1. Dinkelbach's Algorithm

In view of the fractional objective function, we apply the Dinkelbach's algorithm, which has been extensively used to handle fractional programming, to the EEM problem (7). Specifically, given feasible precoding matrices $\mathbf{B}^{(n-1)}$ and $\mathbf{R}^{(n-1)}$ satisfying (7b), (7c), we consider to solve the following optimization problem:

$$\max_{\mathbf{B},\mathbf{R}} \widehat{\mathsf{EE}}(\mathbf{B},\mathbf{R} \mid \mathbf{B}^{(n-1)},\mathbf{R}^{(n-1)}) \quad (8a)$$

$$\text{s.t. } \mathsf{SINR}_k \geq \gamma_k,\ \forall k, \quad (8b)$$

$$P_B(\mathbf{B}) \leq \bar{P}_B,\ P_R(\mathbf{B},\mathbf{R}) \leq \bar{P}_R, \quad (8c)$$

where

$$\widehat{\mathsf{EE}}(\mathbf{B},\mathbf{R} \mid \mathbf{B}^{(n-1)},\mathbf{R}^{(n-1)}) \triangleq \sum_{k=1}^K R_k(\mathbf{B},\mathbf{R}) - \mathsf{EE}^{(n-1)}\cdot P_T(\mathbf{B},\mathbf{R}), \quad (9)$$

in which

$$\mathsf{EE}^{(n-1)} \triangleq \mathsf{EE}(\mathbf{B}^{(n-1)},\mathbf{R}^{(n-1)}). \quad (10)$$

Solving problem (8) is difficult due to the coupling structure of $\mathbf{B}$ and $\mathbf{R}$ and the nonconcave SINR functions, $\mathsf{SINR}_1,\ldots,\mathsf{SINR}_K$.

3.2. SCA-based Algorithm to Problem (8)

Since the variables $\mathbf{B}$ and $\mathbf{R}$ are coupled in problem (8), we consider the alternating optimization method, i.e., alternatingly optimize one variable with the other fixed. However, the subproblems for optimizing $\mathbf{B}$ and $\mathbf{R}$ are still nonconvex due to the nonconcave SINR functions, which appear in the objective function in (8a) and in constraint (8b). To cope with this, let us define $\rho_k(\mathbf{B},\mathbf{R})$ as the interference plus noise power (the denominator of SINR_k given by (2)) at the kth user, i.e.,

$$\rho_k(\mathbf{B},\mathbf{R}) \triangleq \sum_{j\neq k}|\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_j|^2 + \sigma_R^2\|\mathbf{g}_k^H\mathbf{R}\|^2 + \sigma_k^2, \quad (11)$$

and equivalently reformulate constraint (8b) as [18, Appendix II]:

$$\mathrm{Re}\{\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_k\} \geq \gamma_k^{1/2}\cdot\rho_k(\mathbf{B},\mathbf{R})^{1/2},\ k=1,\ldots,K, \quad (12)$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part of a complex number. Note that the constraints in (12) are second-order cone constraints on $\mathbf{B}$ and $\mathbf{R}$, respectively. Next, we tackle the nonconcave objective function based on the successive convex approximation (SCA) method.

For ease of exposition, let us introduce the slack variables $\{t_k\}_{k=1}^K$ and equivalently reformulate problem (8) as

$$\max_{\mathbf{B},\mathbf{R},\{t_k\}_{k=1}^K} \sum_{k=1}^K \frac{1}{2}\log_2(1+t_k) - \mathsf{EE}^{(n-1)}\cdot P_T(\mathbf{B},\mathbf{R}) \quad (13a)$$

$$\text{s.t. } \mathsf{SINR}_k \geq t_k,\ \forall k, \quad (13b)$$

$$\mathrm{Re}\{\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_k\} \geq \gamma_k^{1/2}\cdot\rho_k(\mathbf{B},\mathbf{R})^{1/2},\ \forall k, \quad (13c)$$

$$P_B(\mathbf{B}) \leq \bar{P}_B,\ P_R(\mathbf{B},\mathbf{R}) \leq \bar{P}_R. \quad (13d)$$

Similar to (8b), the constraints in (13b) can be written as

$$\mathrm{Re}\{\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_k\} \geq t_k^{1/2}\cdot\rho_k(\mathbf{B},\mathbf{R})^{1/2},\ \forall k. \quad (14)$$

However, the constraints in (14) are still hard to handle since, in contrast to γ_k, t_k is a variable for $k=1,\ldots,K$. In view of this, we consider the inequality of arithmetic and geometric means [19]:

$$\frac{\phi^{-1}a+\phi b}{2} \geq \sqrt{ab},\ \forall a,b\geq 0,\ \phi>0, \quad (15)$$

where the equality holds when $\phi = \sqrt{a/b}$. Therefore, by applying (15) to (14), problem (13) can be conservatively approximated by

$$\max_{\mathbf{B},\mathbf{R},\{t_k\}_{k=1}^K} \sum_{k=1}^K \frac{1}{2}\log_2(1+t_k) - \mathsf{EE}^{(n-1)}\cdot P_T(\mathbf{B},\mathbf{R}) \quad (16a)$$

$$\text{s.t. } \mathrm{Re}\{\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_k\} \geq \frac{\phi_k^{-1}\rho_k(\mathbf{B},\mathbf{R})+\phi_k t_k}{2},\ \forall k, \quad (16b)$$

$$\mathrm{Re}\{\mathbf{g}_k^H\mathbf{R}\mathbf{H}\boldsymbol{b}_k\} \geq \gamma_k^{1/2}\cdot\rho_k(\mathbf{B},\mathbf{R})^{1/2}, \forall k, \quad (16c)$$

$$P_B(\mathbf{B}) \leq \bar{P}_B,\ P_R(\mathbf{B},\mathbf{R}) \leq \bar{P}_R, \quad (16d)$$

where $\phi_k > 0$, $k = 1, \ldots, K$, are parameters to be judiciously assigned. Observing that the optimal $\{t_k\}_{k=1}^K$ must satisfy the constraints in (16b) with equality, we further rewrite problem (16) as

$$\max_{\mathbf{B},\mathbf{R}} \sum_{k=1}^{K} \frac{1}{2}\log_2(1+T_k(\mathbf{B},\mathbf{R},\phi_k)) - \mathsf{EE}^{(n-1)} P_T(\mathbf{B},\mathbf{R}) \quad (17a)$$

$$\text{s.t. } \mathrm{Re}\{\mathbf{g}_k^H \mathbf{R}\mathbf{H}\boldsymbol{b}_k\} \geq \gamma_k^{1/2} \cdot \rho_k(\mathbf{B},\mathbf{R})^{1/2}, \forall k, \quad (17b)$$

$$P_B(\mathbf{B}) \leq \bar{P}_B,\ P_R(\mathbf{B},\mathbf{R}) \leq \bar{P}_R, \quad (17c)$$

where we have replaced t_k by

$$T_k(\mathbf{B},\mathbf{R},\phi_k) \triangleq \left[2\mathrm{Re}\{\mathbf{g}_k^H \mathbf{R}\mathbf{H}\boldsymbol{b}_k\} - \phi_k^{-1}\rho_k(\mathbf{B},\mathbf{R})\right] \cdot \phi_k^{-1}. \quad (18)$$

Note that problem (17) is convex if either $\mathbf{B}$ or $\mathbf{R}$ is fixed, and hence can be handled by alternating optimization method. Moreover, according to (13b) and (15), we have

$$T_k(\mathbf{B},\mathbf{R},\phi_k) \leq \mathsf{SINR}_k,\ \forall \phi_k > 0,\ k = 1, \ldots, K. \quad (19)$$

Since the optimal solution of (17) must satisfy $\mathrm{Re}\{\mathbf{g}_k^H \mathbf{R}\mathbf{H}\boldsymbol{b}_k\} = |\mathbf{g}_k^H \mathbf{R}\mathbf{H}\boldsymbol{b}_k|$, $k = 1, \ldots, K$, (19) holds with equality when

$$\phi_k = \frac{\rho_k(\mathbf{B},\mathbf{R})}{\mathrm{Re}\{\mathbf{g}_k^H \mathbf{R}\mathbf{H}\boldsymbol{b}_k\}} \triangleq \Phi_k(\mathbf{B},\mathbf{R}),\ k = 1, \ldots, K. \quad (20)$$

Therefore, given the feasible point $(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})$, of problem (8), we choose $\phi_k = \Phi_k(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})$ for $k = 1, \ldots, K$, such that, by optimizing either $\mathbf{B}$ or $\mathbf{R}$ in (17), we can achieve a higher objective value to problem (8) compared with that achieved by $(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})$, which is $\widehat{\mathsf{EE}}(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)} \mid \mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)}) = 0$. To see this, let $J(\mathbf{B},\mathbf{R},\{\phi_k\}_{k=1}^K)$ denote the objective function of problem (17). By (9) and (19), we have

$$\begin{aligned}
&\widehat{\mathsf{EE}}(\mathbf{B}^{(n)}, \mathbf{R}^{(n-1)} \mid \mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)}) \\
&\geq J(\mathbf{B}^{(n)}, \mathbf{R}^{(n-1)}, \{\Phi_k(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})\}_{k=1}^K) \\
&\geq J(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)}, \{\Phi_k(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})\}_{k=1}^K) \\
&= \widehat{\mathsf{EE}}(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)} \mid \mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)}) = 0, \quad (21)
\end{aligned}$$

where $\mathbf{B}^{(n)}$ is obtained by optimizing problem (17) with $\mathbf{R}$ fixed to $\mathbf{R}^{(n-1)}$ and $\phi_k = \Phi_k(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})$, $k = 1, \ldots, K$, i.e.,

$$\mathbf{B}^{(n)} = \arg\max_{\mathbf{B}} J(\mathbf{B}, \mathbf{R}^{(n-1)}, \{\Phi_k(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)})\}_{k=1}^K) \quad (22a)$$

$$\text{s.t. } \mathrm{Re}\{\mathbf{g}_k^H \mathbf{R}^{(n-1)}\mathbf{H}\boldsymbol{b}_k\} \geq [\gamma_k \rho(\mathbf{B}, \mathbf{R}^{(n-1)})]^{\frac{1}{2}}, \forall k, \quad (22b)$$

$$P_B(\mathbf{B}) \leq \bar{P}_B,\ P_R(\mathbf{B}, \mathbf{R}^{(n-1)}) \leq \bar{P}_R. \quad (22c)$$

Analogous to (21) and (22), updating $\mathbf{R}$ by

$$\mathbf{R}^{(n)} = \arg\max_{\mathbf{R}} J(\mathbf{B}^{(n)}, \mathbf{R}, \{\Phi_k(\mathbf{B}^{(n)}, \mathbf{R}^{(n-1)})\}_{k=1}^K) \quad (23a)$$

$$\text{s.t. } \mathrm{Re}\{\mathbf{g}_k^H \mathbf{R}\mathbf{H}\boldsymbol{b}_k^{(n)}\} \geq [\gamma_k \rho(\mathbf{B}^{(n)}, \mathbf{R})]^{\frac{1}{2}}, \forall k, \quad (23b)$$

$$P_R(\mathbf{B}^{(n)}, \mathbf{R}) \leq \bar{P}_R, \quad (23c)$$

we have

$$\begin{aligned}
&\widehat{\mathsf{EE}}(\mathbf{B}^{(n)}, \mathbf{R}^{(n)} \mid \mathbf{B}^{(n)}, \mathbf{R}^{(n-1)}) \\
&\geq \widehat{\mathsf{EE}}(\mathbf{B}^{(n)}, \mathbf{R}^{(n-1)} \mid \mathbf{B}^{(n)}, \mathbf{R}^{(n-1)}) = 0. \quad (24)
\end{aligned}$$

From (21), (24) and (9), one can further prove that

$$\mathsf{EE}(\mathbf{B}^{(n)}, \mathbf{R}^{(n)}) \geq \mathsf{EE}(\mathbf{B}^{(n-1)}, \mathbf{R}^{(n-1)}).$$

Hence, alternatively solving problem (22) and problem (23) achieves a nondecreasing sequence of energy efficiency values, $\{\mathsf{EE}^{(n)}\}_{n=0}^{\infty}$, which eventually converges since the achievable energy efficiency is upper bounded. Therefore, we come up with the SDCA algorithm constituted by the above successive optimization procedures to handle problem (7) as summarized in Algorithm 1.

Algorithm 1 SDCA algorithm to problem (7)

1: **Input** $(\mathbf{B}^{(0)}, \mathbf{R}^{(0)})$ satisfying (7b) and (7c); set solution accuracy $\epsilon > 0$;
2: Set $n := 0$;
3: **repeat**
4: $n = n + 1$;
5: Obtain $\mathbf{B}^{(n)}$ by (22);
6: Obtain $\mathbf{R}^{(n)}$ by (23);
7: **until**

$$\frac{\mathsf{EE}^{(n)} - \mathsf{EE}^{(n-1)}}{\mathsf{EE}^{(n-1)}} \leq \epsilon;$$

8: **Output** $(\mathbf{B}^{(n)}, \mathbf{R}^{(n)})$ as an approximate solution to (7).

3.3. Initialization of the SDCA Algorithm

The SDCA algorithm needs to be initialized by a feasible point of problem (7). However, finding a feasible point of the nonconvex constraint set, (7b) and (7c), is difficult. Next, we present a heuristic approach based on the idea of zero-forcing beamforming to find a feasible point.

Assume that $M_B \geq K$ and $M_R \geq K$. Let $\hat{\mathbf{g}}_k = \mathbf{g}_k/\|\mathbf{g}_k\|$ and $\hat{\mathbf{h}}_k = \mathbf{h}_k/\|\mathbf{h}_k\|$, where $\mathbf{h}_k$ is the kth column of $\mathbf{H}$, for $k = 1, \ldots, K$, and define

$$\hat{\mathbf{G}}_{-k} \triangleq [\hat{\mathbf{g}}_1, \ldots, \hat{\mathbf{g}}_{k-1}, \hat{\mathbf{g}}_{k+1}, \ldots, \hat{\mathbf{g}}_K],\ k = 1, \ldots, K,$$

$$\hat{\mathbf{H}}_{-k} \triangleq [\hat{\mathbf{h}}_1, \ldots, \hat{\mathbf{h}}_{k-1}, \hat{\mathbf{h}}_{k+1}, \ldots, \hat{\mathbf{h}}_K],\ k = 1, \ldots, K.$$

Under the assumption that $M_B \geq K$ and $M_R \geq K$, we can eliminate the inter-user interference by making $\mathbf{R}^{(0)}$ and $\mathbf{B}^{(0)}$ in the following structure:

$$\mathbf{B}^{(0)} = \begin{bmatrix} \mathbf{Diag}(\sqrt{p_{B1}}, \ldots, \sqrt{p_{BK}}) \\ \mathbf{0}_{(M_B - K)\times K} \end{bmatrix} \quad (25a)$$

$$\mathbf{R}^{(0)} = \sum_{k=1}^{K} \sqrt{p_{Rk}}\, \hat{\mathbf{g}}_k^{\perp} (\hat{\mathbf{h}}_k^{\perp})^H, \quad (25b)$$

where $\hat{\mathbf{g}}_k^{\perp}$ and $\hat{\mathbf{h}}_k^{\perp}$ are the unit-norm zero-forcing vectors obtained through complement orthogonal projection associated with $\hat{\mathbf{G}}_{-k}$ and $\hat{\mathbf{H}}_{-k}$, respectively, i.e.,

$$\hat{\mathbf{g}}_k^{\perp} \triangleq \frac{(\mathbf{I}_{M_R} - \hat{\mathbf{G}}_{-k}(\hat{\mathbf{G}}_{-k}^H \hat{\mathbf{G}}_{-k})^{-1}\hat{\mathbf{G}}_{-k}^H)\hat{\mathbf{g}}_k}{\|(\mathbf{I}_{M_R} - \hat{\mathbf{G}}_{-k}(\hat{\mathbf{G}}_{-k}^H \hat{\mathbf{G}}_{-k})^{-1}\hat{\mathbf{G}}_{-k}^H)\hat{\mathbf{g}}_k\|},$$

$$\hat{\mathbf{h}}_k^{\perp} \triangleq \frac{(\mathbf{I}_{M_R} - \hat{\mathbf{H}}_{-k}(\hat{\mathbf{H}}_{-k}^H \hat{\mathbf{H}}_{-k})^{-1}\hat{\mathbf{H}}_{-k}^H)\hat{\mathbf{h}}_k}{\|(\mathbf{I}_{M_R} - \hat{\mathbf{H}}_{-k}(\hat{\mathbf{H}}_{-k}^H \hat{\mathbf{H}}_{-k})^{-1}\hat{\mathbf{H}}_{-k}^H)\hat{\mathbf{h}}_k\|}.$$

By (25), we have

$$|\mathbf{g}_k^H \mathbf{R}^{(0)}\mathbf{H}\boldsymbol{b}_j^{(0)}|^2 = \begin{cases} p_{Rk}p_{Bj} \cdot \|\mathbf{g}_k\|^2\|\mathbf{h}_j\|^2, & \text{if } j = k, \\ 0, & \text{if } j \neq k. \end{cases}$$

With the precoder structures given in (25), we aim to allocate the transmission powers p_{Bk}, p_{Rk}, $k = 1, \ldots, K$, such that $\mathbf{B}^{(0)}$ and

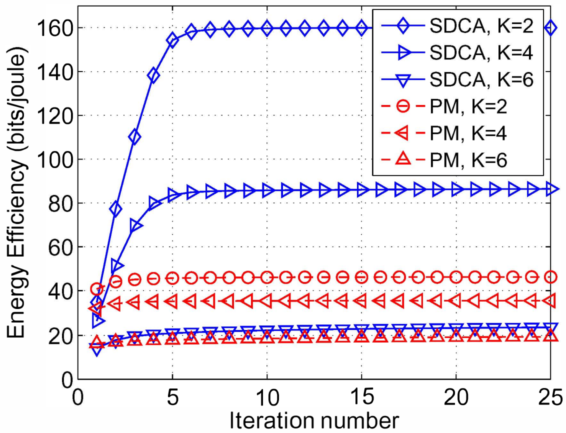

Fig. 1. Performance of energy efficiency versus iteration number of the SDCA algorithm and the PM algorithm for $K = 2, 4, 6$ and $M_B = M_R = 6$.

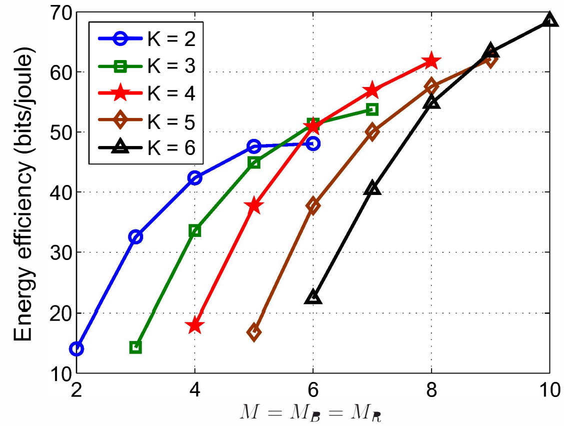

Fig. 2. Performance of energy efficiency versus $M_B = M_R \triangleq M$ of the SDCA algorithm for $K = 2, ..., 6$, and $M \geq K$.

$\mathbf{R}^{(0)}$ satisfy constraints (7b) and (7c). This can be formulated as the following optimization problem

$$\min_{\substack{p_{Bk}, p_{Rk} \geq 0, \\ k=1,\ldots,K}} P_R(\mathbf{B}^{(0)}, \mathbf{R}^{(0)}) = \sum_{k=1}^{K} p_{Rk}(p_{Bk}\|\mathbf{h}_k\|^2 + \sigma_R^2) \quad (26a)$$

$$\text{s.t. } \frac{p_{Rk}p_{Bk} \cdot \|\mathbf{g}_k\|^2\|\mathbf{h}_k\|^2}{\sigma_R^2\|\mathbf{g}_k\|^2 p_{Rk} + \sigma_k^2} \geq \gamma_k, \ \forall k, \quad (26b)$$

$$\sum_{k=1}^{K} p_{Bk} \leq \bar{P}_B. \quad (26c)$$

By the change of variables, $\tilde{p}_{Rk} = \ln(p_{Rk})$ and $\tilde{p}_{Bk} = \ln(p_{Bk})$, $k = 1, \ldots, K$, problem (26) can be converted into the following convex optimization problem:

$$\min_{\substack{\tilde{p}_{Bk}, \tilde{p}_{Rk} \in \mathbb{R} \\ k=1,\ldots,K}} \sum_{k=1}^{K} \left(\|\mathbf{h}_k\|^2 e^{\tilde{p}_{Rk}+\tilde{p}_{Bk}} + \sigma_R^2 e^{\tilde{p}_{Rk}} \right) \quad (27a)$$

$$\text{s.t. } \frac{\gamma_k \sigma_R^2}{\|\mathbf{h}_k\|^2} e^{-\tilde{p}_{Bk}} + \frac{\gamma_k \sigma_k^2}{\|\mathbf{g}_k\|^2\|\mathbf{h}_k\|^2} e^{-\tilde{p}_{Rk}-\tilde{p}_{Bk}} \leq 1, \ \forall k, \quad (27b)$$

$$\sum_{k=1}^{K} e^{\tilde{p}_{Bk}} \leq \bar{P}_B. \quad (27c)$$

Consequently, if the optimal value of problem (27) is less than or equal to $\bar{P}_R$, then the associated precoder given by (25) is a feasible point of problem (7).

Note that, since the associated feasibility problem of problem (7) is itself a nonconvex problem, there is no efficient method to efficiently determine the feasibility or obtain a feasible point of problem (7). Nevertheless, the above zero-forcing beamforming scheme successfully yields feasible points for more than 95% of the randomly generated channel realizations in our simulations.

4. SIMULATION RESULTS AND CONCLUSIONS

This section shows some simulation results to demonstrate the efficacy of the proposed SDCA algorithm. In the simulation, we set the power budgets of the BS and the RS to $\bar{P}_B = \bar{P}_R = 10$ dB, the SINR requirements for all the users are $\gamma_1 = \cdots = \gamma_K = 5$ dB, and the noise variances are $\sigma_R^2 = \sigma_1^2 = \cdots = \sigma_K^2 = 0.01$. The general circuit power model is given by $P_C = 0.005M + 0.005$. All the simulation results are obtained by averaging over 200 sets of channel realizations, in which every component of $\mathbf{H}$ and $\mathbf{g}_1, \ldots, \mathbf{g}_K$ is independently generated according to the standard complex Gaussian distribution. We set solution accuracy $\epsilon = 10^{-3}$. To the best of our knowledge, there is no existing state-of-the-art algorithm for performance comparison. Thus we compare the performance of the proposed algorithm with that of a heuristic algorithm motivated by [18]. In [18], the total power $P_T(\mathbf{B}, \mathbf{R})$ is minimized under the users' QoS constraints (7b). Adding the transmit power constraint (7c) to this problem results in an optimization problem that is convex in $\mathbf{B}$ and in $\mathbf{R}$, respectively. Therefore, the heuristic algorithm, which is referred to as power minimization (PM) algorithm below, is to solve this PM problem by alternating optimization and is initialized by the scheme presented in Subsection 3.3.

Fig. 1 shows the EE performances of the SDCA algorithm and the PM algorithm for $M_B = M_R = 6$ and $K = 2, 4, 6$. It is observed that the SDCA algorithm outperforms the PM algorithm; moreover, the performance difference is significant when K is small because, in this case, there are sufficient spatial degrees of freedom to enhance the transmission rate without significantly increasing the transmit power. On the other hand, both of the two algorithms converge quickly, indicating promising computational efficiency.

Fig. 2 demonstrates the EE performance of the proposed SDCA algorithm versus the number of antennas, $M_B = M_R \triangleq M$, at the BS and the RS. It is observed that the achieved EE increases with the number of antennas M, demonstrating the efficiency of the SDCA algorithm in exploiting the spatial degrees of freedom. However, the increment eventually saturates as M increases. The reason is that the increasing circuit power consumption (cf. (5)) would constrain the effectiveness of activating more antennas.

In conclusion, we have presented an SDCA algorithm for the design of the precoding matrices of the multiple-antenna BS and RS by maximizing the EE under each user's QoS constraint and the transmit power constraints of the BS and the RS. Some simulation results were provided to demonstrate its effectiveness and fast monotone convergence.

5. REFERENCES

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