A CONVEX APPROXIMATION APPROACH TO WEIGHTED SUM RATE MAXIMIZATION OF MULTIUSER MISO INTERFERENCE CHANNEL UNDER OUTAGE CONSTRAINTS

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ABSTRACT

This paper considers weighted sum rate maximization of multiuser multiple-input single-output interference channel (MISO-IFC) under outage constraints. The outage-constrained weighted sum rate maximization problem is a nonconvex optimization problem and is difficult to solve. While it is possible to optimally deal with this problem in an exhaustive search manner by finding all the Pareto-optimal rate tuples in the (discretized) outage-constrained achievable rate region, this approach, however, suffers from a prohibitive computational complexity and is feasible only when the number of transmitter-receive pairs is small. In this paper, we propose a convex optimization based approximation method for efficiently handling the outage-constrained weighted sum rate maximization problem. The proposed approximation method consists of solving a sequence of convex optimization problems, and thus can be efficiently implemented by interior-point methods. Simulation results show that the proposed method can yield near-optimal solutions.

Index Terms— Multiuser interference channel, weighted sum rate maximization, outage probability, convex optimization

1. INTRODUCTION

Recently, interference management for improving spectral efficiency of wireless multiuser systems has been a research topic drawing significant attention [1]. This paper considers the K-user multiple-input single-output interference channel (MISO-IFC) where K multi-antenna transmitters simultaneously communicate with K respective single-antenna receivers over a common frequency band. This MISO-IFC arises, for example, in multicell wireless systems where each of the base stations is equipped with multiple antennas and each mobile station has only one antenna. Under the assumption that the transmitters have the perfect channel state information, and that the receivers employ single-user detection, it has been shown that transmit beamforming is an optimal transmission scheme to attain the Pareto boundary of the achievable rate region of MISO-IFC [2]. The structure of the Pareto-optimal beamforming schemes has also been studied in [3, 4]. A game-theoretic approach for MISO-IFC has been presented in [5].

This paper assumes that the channel coefficients are block-faded, and that the transmitters know only the statistical distribution of the channels. Specifically, each channel is assumed to be circularly symmetric complex Gaussian distributed, with a covariance matrix known to the transmitters. Under limited delay constraints and due to channel fading, the receivers' performance may suffer from outage. Assuming that the transmitters employ transmit beamforming, the achievable rate region of MISO-IFC under outage constraints on receivers' performance has been investigated in [6]. While this outage-constrained achievable rate region is not known analytically so far, it has been shown that this region can be found numerically using an exhaustive search method [6]. This method, unfortunately, has a complexity that increases exponentially with $K(K-1)$, and therefore is not feasible in practice.

In this paper, we investigate efficient approaches to achieving Pareto-optimal beamforming solutions that maximize the achievable weighted sum rate. To this end, we study the design formulation that maximizes the weighted sum rate subject to outage constraints and individual power constraints. Due to the nonconvextity of the outage constraints, solving the weighted sum rate maximization problem is a challenging task. To efficiently deal with this problem, we propose a sequential convex approximation method. The proposed approximation method is conservative in the sense that the obtained approximate beamforming solutions are guaranteed to be feasible and satisfy the outage constraints of the original problem. Since the proposed method only involves solving convex optimization problems, it can be efficiently implemented by interior-point methods in a polynomial-time complexity [7]. The presented simulation results show that the proposed approximation method can provide near-optimal performance and outperform the existing maximum-ratio and zero-forcing transmission strategies.

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2. SIGNAL MODEL AND PROBLEM STATEMENT

We consider the K-user MISO interference channel where each of the transmitters has N_t antennas and all the receivers are equipped with a single antenna. All the transmitters employ transmit beamforming to transmit information signals to their respective receivers. Let $s_i(t)$ denote the information signal sent from transmitter i, and let $\boldsymbol{w}_i \in \mathbb{C}^{N_t}$ be the associated beamforming vector. The received signal at receiver i is given by

$$x_i(t) = \mathbf{h}_{ii}^H \boldsymbol{w}_i s_i(t) + \sum_{k=1, k\neq i}^{K} \mathbf{h}_{ki}^H \boldsymbol{w}_k s_k(t) + n_i(t), \qquad (1)$$

where $\mathbf{h}_{ki} \in \mathbb{C}^{N_t}$ denotes the channel vector from transmitter k to receiver i, and $n_i(t)$ is the additive noise of receiver i. The noise $n_i(t)$ is assumed to be complex Gaussian distributed with zero mean and variance $\sigma_i^2 > 0$, i.e., $n_i(t) \sim \mathcal{CN}(0, \sigma_i^2)$. Assuming that $s_i(t) \sim \mathcal{CN}(0, 1)$ and that the receivers decode the information message using single-user detection (which treats the cross-link interference as noise), the achievable rate of the ith transmitter-receiver pair is given by

$$r_i\Big(\{\mathbf{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K\Big) = \log_2\left(1 + \frac{\left|\mathbf{h}_{ii}^H \boldsymbol{w}_i\right|^2}{\sum_{k\neq i}\left|\mathbf{h}_{ki}^H \boldsymbol{w}_k\right|^2 + \sigma_i^2}\right).$$

In this paper, we assume that the channel coefficients $\mathbf{h}_{ki}$ are block-faded, and that the transmitters can only acquire the statistical distribution of the channels. In particular, the elements of $\mathbf{h}_{ki}$ are assumed to be circularly symmetric complex Gaussian distributed with covariance matrix equal to $\mathbf{Q}_{ki} \succeq \mathbf{0}$ (positive semidefinite), i.e., $\mathbf{h}_{ki} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ki})$, for all $k, i = 1, \ldots, K$. Let $R_i > 0$ be the target transmission rate of receiver i. Due to channel fading, the receivers' performance may suffer from outage; that is, it would have a nonzero probability such that $r_i\big(\{\mathbf{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K\big) < R_i$. The ϵ_i-outage achievable rate region is defined as follows:

Definition 1 [6] *Let $P_i > 0$ denote the power constraint of transmitter i, and let $\epsilon_i \in (0, 1]$ denote the maximum tolerable outage probability of receiver i, for $i = 1, \ldots, K$. The rate tuple $(R_1, \ldots, R_K)$ is said to be achievable if*

$$\Pr\Big\{r_i\Big(\{\mathbf{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K\Big) < R_i\Big\} \leq \epsilon_i,\ i = 1, \ldots, K$$

for some $(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K) \in \mathcal{W}_1 \times \cdots \times \mathcal{W}_K$ where $\mathcal{W}_i \triangleq \{\boldsymbol{w} \in \mathbb{C}^{N_t} |\ \|\boldsymbol{w}\|^2 \leq P_i\}$. The ϵ_i-outage achievable rate region is given by

$$\mathcal{R} = \bigcup_{\substack{\boldsymbol{w}_i \in \mathcal{W}_i,\\ i=1,\ldots,K}} \left\{ \begin{array}{l} (R_1, \ldots, R_K) |\ \Pr\big\{r_i\big(\{\mathbf{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K\big) < R_i\big\} \\ \qquad \leq \epsilon_i,\ i = 1, \ldots, K \end{array} \right\}.$$

Given the outage specifications $\epsilon_1, \ldots, \epsilon_K$, it is desirable to optimize the beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^K$ such that the system can operate on the so-called Pareto boundary of the achievable rate region $\mathcal{R}$ [6], with system utilities such as the (weighted) sum of $R_1, \ldots, R_K$ being maximized. To this end, we consider the following weighted sum rate maximization problem

$$\max_{\substack{\boldsymbol{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0,\\ i=1,\ldots,K}} \sum_{i=1}^K \alpha_i R_i \tag{2a}$$

$$\text{s.t. } \Pr\Big\{r_i\Big(\{\mathbf{h}_{ki}\}_{k=1}^K, \{\boldsymbol{w}_k\}_{k=1}^K\Big) < R_i\Big\} \leq \epsilon_i,$$
$$i = 1, \ldots, K, \tag{2b}$$
$$\|\boldsymbol{w}_i\|^2 \leq P_i,\ \ i = 1, \ldots, K, \tag{2c}$$

where $\alpha_i \geq 0$ is the priority weight for the ith transmitter-receiver pair. Solving problem (2) is challenging because the outage constraints in (2b) are difficult to handle. One possible approach to solving problem (2) is to first obtain a set of Pareto-optimal rate tuples $(R_1, \ldots, R_K)$ by discretizing $\mathcal{R}$ using an exhaustive search method reported in [6], followed by picking the one that corresponds to the largest value of $\sum_{i=1}^K \alpha_i R_i$. The complexity of this approach, however, increases exponentially with $K(K-1)$[1]. In the next section, based on convex approximation techniques, we present a suboptimal approach for efficiently handling problem (2).

[1]The exhaustive search method in [6] samples the achievable rate region $\mathcal{R}$ by discretizing the cross-link interference into a finite number of levels. Let M be the number of discretization levels. This method then needs to list a total number of $M^{K(K-1)}$ rate tuples, and finds the one with maximum $\sum_{i=1}^K \alpha_i R_i$. For a rough case of $M = 10$ and $K = 3$, this method requires to search over 10^6 rate tuples, which is computationally prohibitive.

3. PROPOSED CONVEX APPROXIMATION METHOD

3.1. Closed-Form Expression of Outage Probability

While the probability constraints in (2b) seem intractable, there actually exist closed-form expressions. To show this, it is noted that each of the probability in (2b) can be expressed as

$$\Pr\left\{\frac{\left|\mathbf{h}_{ii}^H \boldsymbol{w}_i\right|^2}{\sum_{k\neq i}\left|\mathbf{h}_{ki}^H \boldsymbol{w}_k\right|^2 + \sigma_i^2} < 2^{R_i} - 1\right\} \tag{3}$$

which is the left tail probability of the ratio of the exponential random variable $|\mathbf{h}_{ii}^H \boldsymbol{w}_i|^2$ to the sum of independent exponential random variables $|\mathbf{h}_{ki}^H \boldsymbol{w}_k|^2$ for $k \neq i$. According to [8, Appendix I], (3) has a closed-form expression as

$$1 - e^{\frac{-(2^{R_i}-1)\sigma_i^2}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}} \prod_{k\neq i} \frac{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i + (2^{R_i}-1)\boldsymbol{w}_k^H \mathbf{Q}_{ki} \boldsymbol{w}_k}. \tag{4}$$

Hence problem (2) can be equivalently represented by

$$\max_{\substack{\boldsymbol{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0,\\ i=1,\ldots,K}} \sum_{i=1}^K \alpha_i R_i \tag{5}$$

$$\text{s.t. } \rho_i e^{\frac{(2^{R_i}-1)\sigma_i^2}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}} \prod_{k\neq i}\left(1 + \frac{(2^{R_i}-1)\boldsymbol{w}_k^H \mathbf{Q}_{ki} \boldsymbol{w}_k}{\boldsymbol{w}_i^H \mathbf{Q}_{ii} \boldsymbol{w}_i}\right) \leq 1,$$
$$\|\boldsymbol{w}_i\|^2 \leq P_i,\ i = 1, \ldots, K,$$

where $\rho_i \triangleq 1 - \epsilon_i$. It can be seen that (5) is a nonconvex optimization problem. Next, we show how to approximate problem (5) by a convex optimization problem.

3.2. Proposed Convex Approximation Formulation

The approximation method to be presented is conservative, in the sense that the obtained approximate solution is guaranteed to be feasible to problem (2). To illustrate the proposed method, let us define

$$e^{x_{ki}} \triangleq \text{Tr}(\mathbf{W}_k \mathbf{Q}_{ki}),\ \ e^{y_i} \triangleq 2^{R_i} - 1, \tag{6a}$$

$$z_i \triangleq \frac{2^{R_i} - 1}{\text{Tr}(\mathbf{W}_i \mathbf{Q}_{ii})} = e^{y_i - x_{ii}}, \tag{6b}$$

$$\mathbf{W}_i \triangleq \boldsymbol{w}_i \boldsymbol{w}_i^H, \tag{6c}$$

where $x_{ki}, y_i, z_i \in \mathbb{R}$ are introduced slack variables for $k, i = 1, \ldots, K$, and $\text{Tr}(\cdot)$ denotes the trace of a matrix. Substituting (6) into (5) yields the following problem

$$\max_{\substack{\mathbf{W}_i \in \mathbb{H}^{N_t}, R_i \geq 0,\\ x_{ki}, y_i, z_i \in \mathbb{R},\\ k,i=1,\ldots,K}} \sum_{i=1}^K \alpha_i R_i, \tag{7a}$$

$$\text{s.t. } \rho_i e^{\sigma_i^2 z_i} \prod_{k\neq i}\big(1 + e^{-x_{ii}+x_{ki}+y_i}\big) \leq 1, \tag{7b}$$
$$\text{Tr}(\mathbf{W}_k \mathbf{Q}_{ki}) \leq e^{x_{ki}},\ \ k \in \mathcal{K}_i^c, \tag{7c}$$
$$\text{Tr}(\mathbf{W}_i \mathbf{Q}_{ii}) \geq e^{x_{ii}}, \tag{7d}$$
$$2^{R_i} \leq e^{y_i} + 1, \tag{7e}$$
$$e^{y_i - x_{ii}} \leq z_i, \tag{7f}$$
$$\text{Tr}(\mathbf{W}_i) \leq P_i, \tag{7g}$$
$$\mathbf{W}_i \succeq \mathbf{0},\ \text{rank}(\mathbf{W}_i) = 1,\ i = 1, \ldots, K, \tag{7h}$$

where $\mathcal{K}_i^c \triangleq \{1,\ldots,K\}\backslash\{i\}$, and (7h) is due to (6c). Notice that we have replaced the equalities in (6a) and (6b) with inequalities as in (7c) to (7f). It is not difficult to verify that all the inequalities in (7c) to (7f) would hold with equalities at the optimum; otherwise a larger optimal weighted sum rate can always be obtained. Therefore, problem (7) is equivalent to problem (5).

One can see that the objective function and most of the constraints of problem (7) are convex, except the constraints in (7c) and (7e), and the nonconvex rank-one constraints in (7h). Let $(\{\bar{\boldsymbol{w}}_i\}_{i=1}^K, \{\bar{R}_i\}_{i=1}^K)$ be a feasible point of problem (2). Define

$$\bar{x}_{ki} \triangleq \ln(\bar{\boldsymbol{w}}_k^H \mathbf{Q}_{ki} \bar{\boldsymbol{w}}_k),\ k \in \mathcal{K}_i^c, \tag{8a}$$

$$\bar{y}_i \triangleq \ln(2^{\bar{R}_i} - 1), \tag{8b}$$

for $i = 1,\ldots,K$. Then $\{\{\bar{x}_{ki}\}_{k\neq i}, \bar{y}_i\}_{i=1}^K$ together with $\bar{R}_i$, $\bar{x}_{ii} \triangleq \ln(\bar{\boldsymbol{w}}_i^H \mathbf{Q}_{ii} \bar{\boldsymbol{w}}_i)$, $\bar{\mathbf{W}}_i \triangleq \bar{\boldsymbol{w}}_i \bar{\boldsymbol{w}}_i^H$ and $\bar{z}_i \triangleq e^{\bar{y}_i - \bar{x}_{ii}}$ for $i = 1,\ldots,K$, are feasible to problem (7). We aim to conservatively approximate (7c) and (7e) with respective to the point $\{\{\bar{x}_{ki}\}_{k\neq i}, \bar{y}_i\}_{i=1}^K$. Since $e^{x_{ki}}$ is convex, its first-order approximation at $\bar{x}_{ki}$, i.e., $e^{\bar{x}_{ki}}(x_{ki} - \bar{x}_{ki} + 1)$, is a global underestimate of $e^{x_{ki}}$. Hence it is sufficient to achieve (7c) by considering the following linear constraint

$$\mathrm{Tr}(\mathbf{W}_k \mathbf{Q}_{ki}) \leq e^{\bar{x}_{ki}}(x_{ki} - \bar{x}_{ki} + 1), \tag{9}$$

for $k \in \mathcal{K}_i^c$. To approximate (7e), we consider the following lower bound for $e^{y_i} + 1$:

$$\left(\frac{e^{y_i}}{\theta_{i1}(\bar{y}_i)}\right)^{\theta_{i1}(\bar{y}_i)} \left(\frac{1}{\theta_{i2}(\bar{y}_i)}\right)^{\theta_{i2}(\bar{y}_i)} \leq e^{y_i} + 1, \tag{10}$$

where $\theta_{i1}(\bar{y}_i) = e^{\bar{y}_i}/(e^{\bar{y}_i} + 1)$ and $\theta_{i2}(\bar{y}_i) = 1/(e^{\bar{y}_i} + 1)$. Equation (10) is obtained from the inequality of arithmetic and geometric means. By (10), a sufficient condition for (7e) can be obtained as

$$(\theta_{i1}(\bar{y}_i))^{\theta_{i1}(\bar{y}_i)} (\theta_{i2}(\bar{y}_i))^{\theta_{i2}(\bar{y}_i)} e^{(\ln 2)R_i - \theta_{i1}(\bar{y}_i) y_i} \leq 1, \tag{11}$$

for $i = 1,\ldots,K$, which are convex constraints. By replacing (7c) and (7e) with (9) and (11), respectively, and by ignoring the nonconvex rank-one constraints in (7h), we obtain the following approximation formulation for problem (7):

$$\begin{aligned}
\max_{\substack{\mathbf{W}_i \in \mathbb{H}^{N_t}, R_i \geq 0,\\ x_{ki}, y_i, z_i \in \mathbb{R},\\ k,i=1,\ldots,K}} \quad & \sum_{i=1}^K \alpha_i R_i, && (12)\\
\text{s.t.} \quad & \rho_i e^{\sigma_i^2 z_i} \prod_{k\neq i} \left(1 + e^{-x_{ii} + x_{ki} + y_i}\right) \leq 1,\\
& \mathrm{Tr}(\mathbf{W}_k \mathbf{Q}_{ki}) \leq e^{\bar{x}_{ki}}(x_{ki} - \bar{x}_{ki} + 1),\ \ k \in \mathcal{K}_i^c,\\
& \mathrm{Tr}(\mathbf{W}_i \mathbf{Q}_{ii}) \geq e^{x_{ii}},\\
& \Theta_i(\bar{y}_i) e^{(\ln 2)R_i - \theta_{i1}(\bar{y}_i) y_i} \leq 1,\\
& e^{y_i - x_{ii}} \leq z_i,\\
& \mathrm{Tr}(\mathbf{W}_i) \leq P_i,\ \mathbf{W}_i \succeq \mathbf{0},\ i = 1,\ldots,K,
\end{aligned}$$

where $\Theta_i(\bar{y}_i) \triangleq (\theta_{i1}(\bar{y}_i))^{\theta_{i1}(\bar{y}_i)} (\theta_{i2}(\bar{y}_i))^{\theta_{i2}(\bar{y}_i)}$. Problem (12) is a convex optimization problem; it can be efficiently solved by standard convex solvers such as CVX [7].

The idea of removing the nonconvex rank-one constraints of $\{\mathbf{W}_i\}_{i=1}^K$ in (12) is known as semidefinite relaxation (SDR) in convex optimization theory [9]. SDR is in general an approximation because the optimal $\{\mathbf{W}_i\}_{i=1}^K$ of problem (12) may not be of rank one. Surprisingly, it is found that, for all the problem instances we tested in simulations, problem (12) always yields rank-one optimal solution, $\{\mathbf{W}_i\}_{i=1}^K$, i.e., $\mathbf{W}_i = \boldsymbol{w}_i(\boldsymbol{w}_i)^H$ for all i, provided that $\mathbf{W}_i \neq \mathbf{0}$. This implies that an approximate beamforming solution to (2) can be directly obtained by decomposing the optimal $\{\mathbf{W}_i\}_{i=1}^K$ of (12).

3.3. Sequential Convex Approximations

The formulation (12) is obtained by approximating problem (2) with respect to the feasible point $(\{\bar{\boldsymbol{w}}_i\}_{i=1}^K, \{\bar{R}_i\}_{i=1}^K)$ [see (8)]. It is possible to further improve the approximation performance by solving problem (12) iteratively with the optimal $(\{\boldsymbol{w}_i\}_{i=1}^K, \{R_i\}_{i=1}^K)$ at the current iteration used as the feasible point $(\{\bar{\boldsymbol{w}}_i\}_{i=1}^K, \{\bar{R}_i\}_{i=1}^K)$ for the next iteration. The proposed sequential approximation algorithm is summarized in the following Algorithm 1:

Algorithm 1 Proposed sequential convex approximation algorithm for solving problem (2)

1: **Input** a feasible point $(\{\bar{\boldsymbol{w}}_i\}_{i=1}^K, \{\bar{R}_i\}_{i=1}^K)$ of problem (2), and a solution accuracy $\delta > 0$.
2: Obtain $\{\{\bar{x}_{ki}\}_{k\neq i}, \bar{y}_i\}_{i=1}^K$ by (8) and obtain $\theta_{i1}(\bar{y}_i) = e^{\bar{y}_i}/(e^{\bar{y}_i} + 1)$ and $\theta_{i2}(\bar{y}_i) = 1/(e^{\bar{y}_i} + 1)$ for $i = 1,\ldots,K$.
3: Solve problem (12) to obtain the optimal beamforming matrices $\{\mathbf{W}_i^\star\}_{i=1}^K$ and rates $\{R_i^\star\}_{i=1}^K$.
4: Obtain $\boldsymbol{w}_i^\star$ by decomposition of $\mathbf{W}_i^\star = \boldsymbol{w}_i^\star(\boldsymbol{w}_i^\star)^H$ for $i = 1,\ldots,K$.
5: **Output** the approximate beamforming solution $(\boldsymbol{w}_1^\star,\ldots,\boldsymbol{w}_K^\star)$ and achievable rate tuple $(R_1^\star,\ldots,R_K^\star)$ if $|\sum_{i=1}^K \alpha_i R_i^\star - \sum_{i=1}^K \alpha_i \bar{R}_i| / \sum_{i=1}^K \alpha_i \bar{R}_i < \delta$; otherwise update $\bar{\boldsymbol{w}}_i := \boldsymbol{w}_i^\star$ and $\bar{R}_i := R_i^\star$ for all i, and go to Step 2.

A feasible point to initialize Algorithm 1 can be easily obtained by some heuristic transmission strategies. For example, one can obtain a feasible point $(\{\bar{\boldsymbol{w}}_i\}_{i=1}^K, \{\bar{R}_i\}_{i=1}^K)$ of problem (2) through the simple maximum-ratio transmission (MRT) strategy. In this strategy, the beamforming vectors $\{\bar{\boldsymbol{w}}_i\}_{i=1}^K$ are simply set to $\bar{\boldsymbol{w}}_i = \sqrt{P_i}\boldsymbol{q}_i$ where $\boldsymbol{q}_i \in \mathbb{C}^{N_t}$, $\|\boldsymbol{q}_i\| = 1$, is the principal eigenvector of $\mathbf{Q}_{ii}$ for $i = 1,\ldots,K$. For the ith transmitter-receiver pair, the associated ϵ_i-outage achievable rate of MRT is given by the maximum $\bar{R}_i$ that satisfies the following inequality [see (5)]

$$\rho_i e^{\frac{(2^{\bar{R}_i}-1)\sigma_i^2}{\bar{\boldsymbol{w}}_i^H \mathbf{Q}_{ii} \bar{\boldsymbol{w}}_i}} \prod_{k\neq i} \left(1 + \frac{(2^{\bar{R}_i} - 1)\bar{\boldsymbol{w}}_k^H \mathbf{Q}_{ki} \bar{\boldsymbol{w}}_k}{\bar{\boldsymbol{w}}_i^H \mathbf{Q}_{ii} \bar{\boldsymbol{w}}_i}\right) \leq 1.$$

Analogously, one can also obtain a feasible point of (2) by the zero-forcing (ZF) transmission strategy, provided that the column space of $\mathbf{Q}_{ii}$ is not subsumed by the column space of $\sum_{k\neq i}^K \mathbf{Q}_{ik}$, for all $i = 1,\ldots,K$. In the next section, we present some simulation results to demonstrate the efficacy of the proposed approximation algorithm.

4. SIMULATION RESULTS AND DISCUSSIONS

In the simulations, we consider the multiuser MISO-IFC as described in Section 2. For simplicity, all the receivers are assumed to have the same noise power, i.e., $\sigma_1^2 = \cdots = \sigma_K^2 \triangleq \sigma^2$, and all the power constraints are set to one, i.e., $P_1 = \cdots = P_K = 1$. The channel covariance matrices $\mathbf{Q}_{ki}$ were randomly generated. We normalize the maximum eigenvalue of $\mathbf{Q}_{ii}$, i.e., $\lambda_{\max}(\mathbf{Q}_{ii})$, to one for all i, and normalize $\lambda_{\max}(\mathbf{Q}_{ki})$ to a value $\eta \in (0, 1]$ for all $k \in \mathcal{K}_i^c$, $i = 1,\ldots,K$. The parameter η, thereby, represents the relative cross-link interference level. If not mentioned specifically, the ranks of $\mathbf{Q}_{ki}$ are all set to N_t. We consider the sum rate maximization problem by setting $\alpha_1 = \cdots = \alpha_K = 1$ for problem (2),

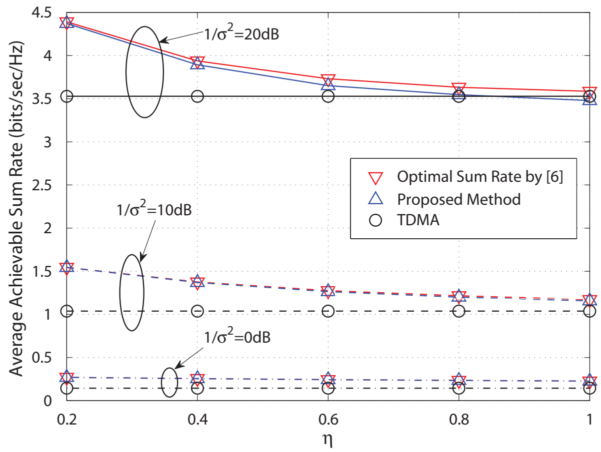

Fig. 1. Average achievable sum rate versus η for $K = 2$, $N_t = 4$, and $\mathrm{rank}(\mathbf{Q}_{ki}) = 4$ for all k, i.

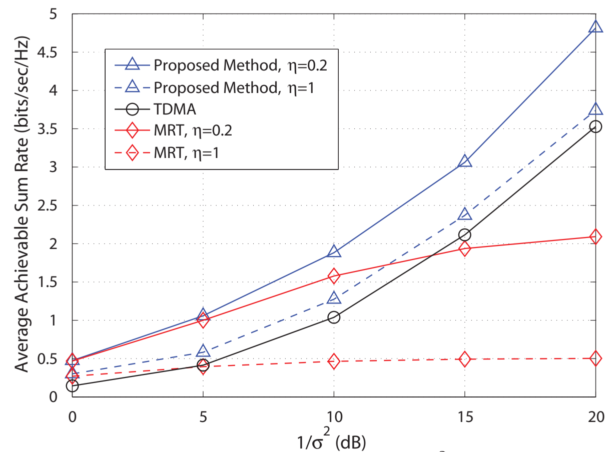

Fig. 2. Average achievable sum rate versus $1/\sigma^2$ for $K = N_t = 4$, and $\mathrm{rank}(\mathbf{Q}_{ki}) = 4$ for all k, i.

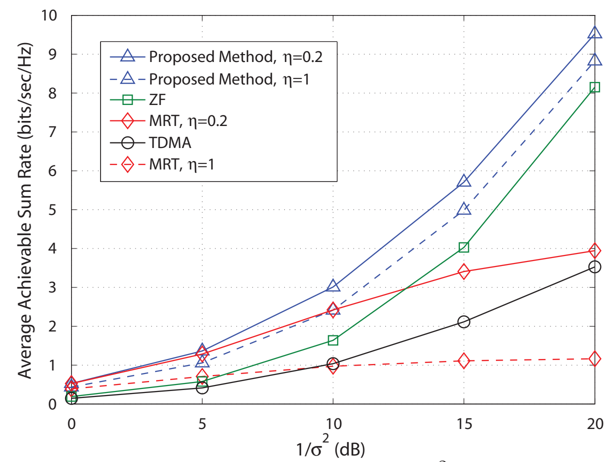

Fig. 3. Average achievable sum rate versus $1/\sigma^2$ for $K = 4$, $N_t = 8$, and $\mathrm{rank}(\mathbf{Q}_{ki}) = 2$ for all k, i.

and set $\epsilon_1 = \cdots = \epsilon_K = 0.1$, i.e., 10% outage probability. For the proposed approximation algorithm (Algorithm 1), we set $\delta = 10^{-2}$ and use CVX [7] to handle the associated problem (12). All the simulation results were obtained by averaging over 500 trials.

In the first example, we examine the approximation accuracy of the proposed method by comparing with the optimal sum rate obtained by the exhaustive search method in [6]. Figure 1 shows the simulation results of average achievable rate versus η for $K = 2$ and $N_t = 4$. The achievable rate of the simple TDMA scheme is also shown in this figure. Firstly, one can see from this figure that the sum rate achieved by the proposed method approaches that of TDMA with increased η; TDMA exhibits a constant sum rate for all η because there is no cross-link interference for this scheme. Secondly, we observe that the proposed method can exactly attain the average optimal sum rate for $1/\sigma^2 = 0$ dB and $1/\sigma^2 = 10$ dB. For $1/\sigma^2 = 20$ dB and for $\eta \geq 0.5$ (interference dominated scenarios), it can be observed that there is a small gap between the rate achieved by the proposed method and the optimal rate. Nevertheless, this gap is within 3% of the optimal sum rate on average.

In the second example, we compare the proposed method with the MRT scheme and TDMA for $N_t = K = 4$. Figure 2 shows the results of average sum rate versus $1/\sigma^2$. Note that, for the case of $K = 4$, the exhaustive search method in [6] is too complex to implement, and thus no result for the optimal sum rate is shown. From Fig. 2, we can observe that the proposed method achieves the highest sum rate among the three methods, no matter when $\eta = 0.2$ or $\eta = 1$. One can also see that, for $\eta = 0.2$ and $1/\sigma^2 < 5$ dB, MRT can yield a sum rate comparable to the proposed method and outperforms TDMA; whereas TDMA performs better for $\eta = 1$.

In order to compare with the ZF scheme, in the third example, we extend the number of antennas to 8 ($N_t = 8$) and constrain the ranks of all channel covariance matrices to 2. The simulation results are shown in Fig. 3. As seen from this figure, the proposed method still performs best compared to the other three schemes. On the other hand, one can see that ZF can achieve a higher average sum rate than TDMA, and also outperforms MRT for $\eta = 1$.

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