ON PERFECT CHANNEL IDENTIFIABILITY OF SEMIBLIND ML DETECTION OF ORTHOGONAL SPACE-TIME BLOCK CODED OFDM

Tsung-Hui Chang[†], *Wing-Kin Ma*[⋆], *Chuan-Yuan Huang*[†], *and Chong-Yung Chi*[†]

[†]Institute of Commun. Eng. & Dept. of Elec. Eng.
National Tsing Hua University,
Hsinchu, Taiwan 30013
E-mail: changth@mx.nthu.edu.tw, g9664515@oz.nthu.edu.tw
cychi@ee.nthu.edu.tw

[⋆]Dept. of Electronic Eng.
The Chinese University of Hong Kong,
Shatin, N.T., Hong Kong
E-mail: wkma@ieee.org

ABSTRACT

This paper considers maximum-likelihood (ML) detection of orthogonal space-time block coded OFDM (OSTBC-OFDM) systems without channel state information. Our previous work has shown an interesting identifiability result, that the whole time-domain channel can be uniquely identified by only having one subchannel to transmit pilots. However, this identifiability is in a probability-one sense, under some mild assumptions on the channel statistics. In this paper we establish a "perfect" channel identifiability (PCI) condition under which the channel is always uniquely identifiable. It is shown that PCI can be achieved by judiciously applying the so-called non-intersecting subspace OSTBCs. The resultant PCI achieving scheme has its number of pilots larger than that used in the previous probability-one identifiability achieving scheme, but smaller than that required in conventional pilot-aided channel estimation. Simulation results are presented to show that the proposed scheme can provide a better performance than the other schemes.

***Index Terms*—** OSTBC-OFDM, Maximum-likelihood detection, Channel identifiability

1. INTRODUCTION

In the paper, we consider the semiblind detection problem of orthogonal space-time block coded OFDM (OSTBC-OFDM) systems. This problem has been studied in the literature; e.g., [1, 2], often with an assumption that the multiple-input multiple-output (MIMO) channel remains static over many OSTBC-OFDM blocks. Recently it has been found that semiblind detection can be done within only one OSTBC-OFDM block, by using a deterministic semiblind maximum-likelihood (ML) criterion [3]. This finding is attractive because it enables accommodation of shorter channel coherence time.

A unique channel identifiability condition for the block-wise semiblind ML detector has also been analyzed in [3]. It is shown that the MIMO channel can be uniquely identified *in a probability one sense*, by simply assigning one of the subchannels to transmit a pilot space-time code. While this one-pilot-code scheme is appealing in its low pilot consumption, its probability-one identifiability condition is under the premise that the channel coefficients follow certain Gaussian distributions; e.g., independent and identically distributed (i.i.d.) Gaussian. In this paper we seek to achieve a stronger identifiability condition, namely *perfect channel identifiability (PCI)*, under which the channel is always uniquely identifiable. As we will elaborate upon in Section 3, the idea lies in judicious use of the nonintersecting subspace (NIS) OSTBCs [4] and pilots over the subchannels. It will be shown that the resultant PCI achieving scheme requires an amount of pilots that is more than that of the one-pilot-code scheme, but less than that in pilot-aid least-squares (LS) channel estimation. Its effectiveness over these existing schemes will be demonstrated by simulations in Section 4.

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2. PROBLEM STATEMENT

2.1. OSTBC-OFDM Signal Model and Semiblind ML Detection

In this subsection, we describe the formulation of semiblind (or blind) ML detection of OSTBC-OFDM within one OSTBC-OFDM block [3]. Let N_t and N_r be the numbers of transmitter and receiver antennas, respectively. Denote by N_c the discrete Fourier transform (DFT) size, and by T the employed space-time code length. Under the basic assumption that the channel is static for T OFDM symbols (equivalent to one OSTBC-OFDM block), the received signals can be modeled as

$$\mathbf{Y}_n = \mathbf{C}_n(\mathbf{s}_n)\mathbf{H}_n + \mathbf{W}_n, \quad (1)$$

where $n = 1, \ldots, N_c$, and

$\mathbf{Y}_n \in \mathbb{C}^{T\times N_r}$	received code matrix at subchannel n;
$\mathbf{s}_n \in \{\pm 1\}^{K_n}$	transmitted bit vector for subchannel n where K_n is the number of bits per code;
$\mathbf{C}_n(\cdot) \in \mathbb{C}^{T\times N_t}$	OSTBC assigned to subchannel n;
$\mathbf{H}_n \in \mathbb{C}^{N_t\times N_r}$	MIMO channel frequency response matrix for subchannel n;
$\mathbf{W}_n \in \mathbb{C}^{T\times N_r}$	AWGN matrix for subchannel n where the average power per entry is σ_w^2.

We should emphasize that in a coherent OSTBC-OFDM system it is generally logical to employ the same OSTBC for all subchannels (i.e., $\mathbf{C}_1(\cdot) = \cdots = \mathbf{C}_{N_c}(\cdot)$); while in a blind or semiblind scenario, one can achieve desirable identifiability properties by allowing the transmitted OSTBCs to be different from one subchannel to another [3].

The idea that led to semiblind ML OSTBC-OFDM detection in one OSTBC-OFDM block is to utilize the *time-domain parametrization* of the MIMO frequency responses $\mathbf{H}_n$. In essence, each $\mathbf{H}_n$ is physically dependent on a (MIMO) Fourier transform expression

$$\mathbf{H}_n \triangleq \mathbf{A}_n\boldsymbol{\mathcal{H}} = (\mathbf{I}_{N_t}\otimes \mathbf{f}_n^T)\boldsymbol{\mathcal{H}},\ n = 1, \ldots, N_c, \quad (2)$$

where $\mathbf{A}_n = (\mathbf{I}_{N_t} \otimes \mathbf{f}_n^T)$ in which $\otimes$ is the Kronecker product,

$$\mathbf{f}_n = \frac{1}{\sqrt{N_c}}[1, e^{-j\frac{2\pi}{N_c}(n-1)}, \ ..., e^{-j\frac{2\pi}{N_c}(n-1)(L-1)}]^T, \qquad (3)$$

with $j = \sqrt{-1}$, and

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{h}_{1,1} & \cdots & \mathbf{h}_{1,N_r} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{N_t,1} & \cdots & \mathbf{h}_{N_t,N_r} \end{bmatrix} \in \mathbb{C}^{LN_t \times N_r}, \qquad (4)$$

is the collection of all time-domain MIMO channel coefficients, with each $\mathbf{h}_{m,i} \in \mathbb{C}^L$ standing for the L-order channel impulse response vector between the mth transmit antenna and the ith receive antenna. Using this time-domain channel parametrization, the deterministic blind ML detector for the model in (1) is shown [3] to be

$$\min_{\substack{\mathbf{s}_n \in \{\pm 1\}^{K_n}, \\ n=1,\ldots,N_c}} \left\{ \min_{\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t \times N_r}} \sum_{n=1}^{N_c} \|\mathbf{Y}_n - \mathbf{C}_n(\mathbf{s}_n)\mathbf{A}_n \boldsymbol{\mathcal{H}}\|_F^2 \right\} \qquad (5)$$

and its semiblind counterpart is given by fixing the known, pilot parts of $\{\mathbf{s}_n\}_{n=1}^{N_c}$ in the minimization of (5).

One important issue is the techniques for implementing (5). It was shown [3] that if $\mathbf{C}_n(\cdot)$ are BPSK/QPSK OSTBCs, then (5) can be recast as a Boolean quadratic program which can be handled very effectively by methods such as sphere decoding and semidefinite relaxation (SDR). Divide-and-conquer methods for coping with large-scale OSTBC-OFDM were also illustrated in [3].

2.2. Unique Channel Identification Conditions

Our interest in this paper lies in unique channel identifiability conditions, a fundamental aspect that provides important guidelines on the code designs of OSTBC-OFDM. To put this into context, let

$$\boldsymbol{s} = [\mathbf{s}_1^T, \ ..., \ \mathbf{s}_{N_c}^T]^T \in \{\pm 1\}^{\bar{K}}$$

where $\bar{K} = \sum_{n=1}^{N_c} K_n$ is the total number of transmitted bits per block, and consider a general expression for pilot placement

$$\boldsymbol{s} \triangleq \mathbf{\Pi} \begin{bmatrix} \mathbf{s}_{\rm p} \\ \mathbf{s}_{\rm d} \end{bmatrix}, \qquad (6)$$

where $\mathbf{s}_{\rm d} \in \{\pm 1\}^{K_{\rm d}}$ collects the $K_{\rm d}$ (unknown) information bits, $\mathbf{s}_{\rm p} \in \{\pm 1\}^{K - K_{\rm d}}$ contains the (known) pilot bits, and $\mathbf{\Pi} \in \mathbb{R}^{\bar{K} \times \bar{K}}$ is a permutation matrix that describes how the pilots and data are assigned. In a semiblind identifiability analysis, our objective is to determine the *unique identifiability conditions*; that is, conditions under which the ambiguity situation

$$\mathbf{C}_n(\mathbf{s}_n)\mathbf{A}_n \boldsymbol{\mathcal{H}} = \mathbf{C}_n(\mathbf{s}'_n)\mathbf{A}_n \boldsymbol{\mathcal{H}}', \ \ n = 1, \ldots, N_c, \qquad (7)$$

does not hold for any $\boldsymbol{s} \neq \boldsymbol{s}'$ and $\boldsymbol{\mathcal{H}} \neq \boldsymbol{\mathcal{H}}'$ where $\boldsymbol{s}, \boldsymbol{s}' \in \{\pm 1\}^{\bar{K}}$ and $\mathbf{s}_{\rm p} = \mathbf{s}'_{\rm p}$.

There is a simple way of preventing (7) from being satisfied, if the amount of pilots were not a concern. Let us consider an M-pilot-code scheme in which, without loss of generality, the first M subchannels are loaded only with pilots (i.e., $\mathbf{s}_{\rm p} = [\mathbf{s}_1^T, \ ..., \ \mathbf{s}_M^T]^T$). In that case, the first M equations of (7) can be expressed as

$$\underbrace{\begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1)\mathbf{A}_1 \\ \vdots \\ \mathbf{C}_M(\mathbf{s}_M)\mathbf{A}_n \end{bmatrix}}_{\triangleq \boldsymbol{\mathcal{G}}_{\rm p}(\mathbf{s}_{\rm p})} \boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1)\mathbf{A}_1 \\ \vdots \\ \mathbf{C}_M(\mathbf{s}_M)\mathbf{A}_n \end{bmatrix} \boldsymbol{\mathcal{H}}'. \qquad (8)$$

It is not hard to show that $\boldsymbol{\mathcal{G}}_{\rm p}(\mathbf{s}_{\rm p})$ is always of full column rank if $M \geq L$, and thus (8) can never be satisfied. In fact this pilot placement follows the same spirit as in pilot-aided LS channel estimation [5], in which the channel $\boldsymbol{\mathcal{H}}$ is estimated by

$$\hat{\boldsymbol{\mathcal{H}}} = \{\boldsymbol{\mathcal{G}}_{\rm p}^H(\mathbf{s}_{\rm p})\boldsymbol{\mathcal{G}}_{\rm p}(\mathbf{s}_{\rm p})\}^{-1}\boldsymbol{\mathcal{G}}_{\rm p}^H(\mathbf{s}_{\rm p})\boldsymbol{\mathcal{Y}}_{\rm p} \qquad (9)$$

where $\boldsymbol{\mathcal{Y}}_{\rm p} = [\mathbf{Y}_1^T, \ldots, \mathbf{Y}_L^T]^T$.

The above described M-pilot-code scheme as well as the LS channel estimator require at least L pilot codes, spanning across L different subchannels. In our previous work we have shown that unique channel identifiability can be achieved *almost surely* by using only one pilot code:

Theorem 1 (One-pilot-code scheme [3]) *Assume $N_c > L$, and that*

A1) *$\boldsymbol{\mathcal{H}}$ is Gaussian distributed and at least one column of $\boldsymbol{\mathcal{H}}$ has a positive definite covariance matrix (e.g., i.i.d. Gaussian).*

The channel $\boldsymbol{\mathcal{H}}$ is uniquely identifiable with probability one if $\mathbf{s}_{\rm p} = \mathbf{s}_1$, that is, only one subchannel is dedicated to transmitting pilots.

It should be pointed out that Theorem 1 does not impose requirements on the choices of OSTBCs $\mathbf{C}_n(\cdot)$ over data subchannels. Theorem 1 provides a very relaxed condition in terms of the amount of pilots used to achieve unique channel identifiability, especially when compared to the M-pilot-code scheme and the pilot-aided LS channel estimator. Its shortcoming, however, lies in the premise **A1)**. While **A1)** is a popular assumption in the space-time-frequency coding literature [6], it may not be satisfied in certain frequency-selective fading models; e.g., the sparse multipath channels. We will provide such an example in the simulation section. Our endeavor in this paper focuses on the analysis problem whether the channel identifiability can be achieved in a stronger sense, namely:

Definition 1 *An OSTBC-OFDM scheme is said to achieve perfect channel identifiability (PCI) if $\boldsymbol{\mathcal{H}}$ is uniquely identifiable for any $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t \times N_r}$, $\boldsymbol{\mathcal{H}} \neq \mathbf{0}$.*

It is noticed from (8) that the M-pilot-code scheme and the LS channel estimator are PCI achieving, but they demand an investment of L pilot codes at least. In the next section, we propose a PCI achieving scheme that uses less pilots.

3. PROPOSED PCI ACHIEVING SCHEME

An important ingredient of constructing a PCI achieving OSTBC-OFDM scheme is to consider the non-intersecting subspace (NIS) OSTBCs:

Definition 2 [4] *Assume BPSK or QPSK constellation. An OSTBC $\mathbf{C}(\cdot)$ is said to be an NIS-OSTBC if*

$$\text{Range}\{\mathbf{C}(\mathbf{s})\} \cap \text{Range}\{\mathbf{C}(\mathbf{s}')\} = \{\mathbf{0}\} \qquad (10)$$

for any $\mathbf{s}, \mathbf{s}' \in \{\pm 1\}^K$, $\mathbf{s}' \neq \pm\mathbf{s}$.

The properties and construction of NIS-OSTBCs have been investigated. Here we give a summary of several key results, and readers are referred to [4] for the complete descriptions.

Property 1 *$T \geq 2N_t$ for NIS-OSTBCs.*

Property 2 *For an OSTBC following the generalized orthogonal designs (GOD)* [1]*, it does not achieve the full rate if it is an NIS-OSTBC.*

Property 3 *Let* $\mathbf{C}(\cdot)$ *be an OSTBC. For any* $\mathbf{H}, \mathbf{H}' \in \mathbb{C}^{N_t \times N_r}$, $\mathbf{H} \neq \mathbf{0}$*, and* $\mathbf{s}, \mathbf{s}' \in \{\pm 1\}^K$*, the ambiguity equation*

$$\mathbf{C}(\mathbf{s})\mathbf{H} = \mathbf{C}(\mathbf{s}')\mathbf{H}'$$

holds only when $(\mathbf{s}', \mathbf{H}') = \pm(\mathbf{s}, \mathbf{H})$*, if and only if* $\mathbf{C}(\cdot)$ *is an NIS-OSTBC.*

Property 3 is particularly important in the study of blind ML OSTBC detection in flat-fading channels, in achieving PCI [4].

Almost all the existing OSTBCs are not NIS. Fortunately, for the BPSK or QPSK constellation, a construction method of NIS-OSTBCs has been proposed [4]. The method works by modifying an existing OSTBC. For example, consider the QPSK Alamouti code

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 + js_4 & s_2 - js_3 \\ s_2 + js_3 & -s_1 + js_4 \end{bmatrix}^T. \tag{11}$$

Then, by [4], we can construct an NIS-OSTBC as

$$\mathbf{C}_{\text{NIS}}(\mathbf{s}) = \begin{bmatrix} s_1 & s_2 - js_3 & s_4 + js_5 & s_6 - js_7 \\ s_2 + js_3 & -s_1 & s_6 + js_7 & -s_4 + js_5 \end{bmatrix}^T. \tag{12}$$

Note that (12) satisfies Properties 1 and 2, the necessary conditions for NIS-OSTBCs.

We now are ready to present the proposed scheme. Define a subcarrier subset $\mathcal{S} \subseteq \{1, \ldots, N_c\}$ and its complementary set $\mathcal{S}^c \subseteq \{1, \ldots, N_c\}$ where $\mathcal{S} \cup \mathcal{S}^c = \{1, \ldots, N_c\}$ and $\mathcal{S} \cap \mathcal{S}^c = \emptyset$. Consider an OSTBC-OFDM scheme as follows:

Proposed Semiblind OSTBC-OFDM Scheme: Part of the subcarriers are using NIS-OSTBC:

$$\mathbf{C}_n(\cdot) = \mathbf{C}_{\text{NIS}}(\cdot) \; \forall \, n \in \mathcal{S} \tag{13}$$

where $\mathbf{C}_{\text{NIS}}(\cdot)$ stands for an NIS-OSTBC. For each $n \in \mathcal{S}$, *one pilot bit is assigned.* For the other part of the subcarriers,

$$\mathbf{C}_n(\cdot) = \mathbf{C}_{\text{O}}(\cdot) \; \forall \, n \in \mathcal{S}^c \tag{14}$$

where $\mathbf{C}_{\text{O}}(\cdot)$ is an arbitrary OSTBC having the same code matrix dimension as $\mathbf{C}_{\text{NIS}}(\cdot)$.

Let us take the 2-transmitter QPSK case as an example to describe the proposed scheme. One can use (12) as the NIS code $\mathbf{C}_{\text{NIS}}(\cdot)$. For the arbitrary code $\mathbf{C}_{\text{O}}(\cdot)$, it is logical to choose a maximal code rate OSTBC with the same dimension as $\mathbf{C}_{\text{NIS}}(\cdot)$. This can be obtained by concatenating two Alamouti codes:

$$\mathbf{C}_{\text{O}}(\mathbf{s}) = \begin{bmatrix} s_1 + js_4 & s_2 - js_3 & s_5 + js_8 & s_6 - js_7 \\ s_2 + js_3 & -s_1 + js_4 & s_6 + js_7 & -s_5 + js_8 \end{bmatrix}^T. \tag{15}$$

The idea of the proposed scheme is based on the following intuition: On one hand, using more NIS-OSTBCs is expected to improve identifiability. But, on the other hand, we should minimize the use of NIS-OSTBCs since they incur rate reduction (by one bit) as a necessity for achieving powerful identifiability (Properties 2 and 3). In the Appendix, we prove the following important result:

[1]Most of the existing OSTBCs are based on the GOD. The designs stipulate that each entry of the code matrix takes on either a symbol, its conjugate, or zero.

Table 1. Data rate (bits/pcu) comparison of the proposed PCI achieving scheme with some existing schemes

	Identifiability	Data rate
One-pilot-code scheme	probability-one	$\left(\frac{N_cK-K}{N_cT}\right)$
L-pilot-code LS channel estimation	perfect	$\left(\frac{N_cK-LK}{N_cT}\right)$
Proposed PCI achieving scheme	perfect	$\left(\frac{N_cK-2L}{N_cT}\right)$

Theorem 2 *The semiblind OSTBC-OFDM scheme in Eqns.* (13) *and* (14) *is PCI achieving if and only if* $|\mathcal{S}| \geq L$.

Theorem 2 indicates that the minimum number of NIS-OSTBCs for achieving PCI is L. Hence, to maximize the data throughput, it is natural to set $|\mathcal{S}| = L$.

Let us count the data rate of the proposed PCI achieving scheme, defined as the number of information bits transmitted per channel use (bits/pcu). Let K be the number of bits in $\mathbf{C}_{\text{O}}(\cdot)$. According to [4] or the above NIS-OSTBC discussion, the number of bits in $\mathbf{C}_{\text{NIS}}(\cdot)$ is $K-1$. Deducting the L pilot bits, the proposed scheme transmits $(N_cK - 2L)$ data bits per OSTBC-OFDM block. Hence, the data rate is $(N_cK-2L)/(N_cT)$ bits/pcu. In Table I, we compare the data rate of the proposed PCI achieving scheme to that of the one-pilot-code scheme and the LS channel estimator (using L pilot codes). As seen, the proposed PCI achieving scheme has a higher data rate than the LS channel estimator. Moreover, the PCI achieving scheme has a lower data rate than the one-pilot-code scheme (for $L \geq K/2$ which is generally true in practice), but it achieves a stronger identifiability. Next the performance of the proposed PCI achieving scheme is compared to that of the LS channel estimator and the one-pilot-code scheme by simulations.

4. SIMULATION RESULTS AND CONCLUSIONS

In the simulation, we considered a 2-transmitter OSTBC-OFDM system with DFT size equal to 32 ($N_c = 32$), channel length equal to 8 ($L = 8$) and $\mathbf{C}_n(\cdot) = \mathbf{C}_{\text{O}}(\cdot)$ in Eqn. (15) for all $n = 1, \ldots, 32$. We assumed that the channel is sparse by randomly setting 3 out of 8 channel taps in each $\mathbf{h}_{m,i}$ to be zero while letting the other 5 taps to be complex Gaussian distributed with zero mean and unit variance. The proposed PCI achieving scheme was compared with the coherent ML detector (which has the perfect channel state information), the LS channel estimator [5] (see Eqn. (9)) and the one-pilot-code scheme [3]. For the proposed scheme, we set $\mathcal{S} = \{1 + 4q | q = 0, \ldots, 7\}$ ($|\mathcal{S}| = L = 8$), and $\mathbf{C}_n(\cdot) = \mathbf{C}_{\text{NIS}}(\cdot)$ in Eqn. (12) for all $n \in \mathcal{S}$. For the one-pilot-code scheme, we set $\mathbf{s}_{\text{p}} = \mathbf{s}_1$; while for the LS channel estimator all $\mathbf{s}_n$, $n \in \mathcal{S}$ are pilots. The signal-to-noise ratio (SNR) was defined as the ratio of the transmit signal power per bit and the noise power:

$$\text{SNR} = \frac{E\{\sum_{n=1}^{N_c} \|\mathbf{C}_n(\mathbf{s}_n)\mathbf{A}_n\|_F^2\}/\bar{K}}{\sigma_w^2}.$$

For each scheme under test, the associated semiblind ML detector in (5) was implemented by the SDR technique [7]. Each simulation result was obtained from 15,000 trials.

Figures 1(a) and 1(b) present the simulation results (bit error rate (BER) v.s. SNR) for $N_r = 1$ and $N_r = 2$, respectively. It can be observed from these figures that the proposed PCI achieving scheme significantly outperforms the one-pilot-code scheme, especially when $N_r = 1$. Besides, when SNR$\geq$ 20 dB for $N_r = 1$ or when SNR$\geq$ 8 dB for $N_r = 2$, the proposed scheme exhibits a better BER performance than the LS channel estimation method. It is worthwhile to point out that in this simulation example the proposed scheme has a data rate of $15/8 \approx 1.87$ bits/pcu which is lower than the $31/16 \approx 1.93$ bits/pcu of the one-pilot-code scheme, but is higher than the 1.5 bits/pcu of the LS channel estimator.

In summary, we have presented a PCI achieving scheme for block-wise semiblind ML OSTBC-OFDM detection in the paper. The proposed scheme uses a smaller number of pilots than that required by the pilot-aided LS channel estimator. The presented simulation results have demonstrated that the proposed scheme outperforms the pilot-aided LS channel estimator as well as the one-pilot-code scheme.

5. APPENDIX: PROOF OF THEOREM 2

To prove sufficiency, we show that for the proposed scheme with $|\mathcal{S}| \geq L$, (7) holds only when $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}'$. Note in (7) that if $\mathbf{s}_n = \mathbf{s}'_n$ for some $n \in \mathcal{S}$, then $\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}'$; whereas if $\mathbf{s}_n \neq \mathbf{s}'_n$, then we must have $\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}' = \mathbf{0}$ by Definition 2 and due to the presence of one pilot bit. For both cases, we have $\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}' \;\forall\, n \in \mathcal{S}$. Let $\mathcal{S} = \{n_1, n_2, \ldots, n_{|S|}\}$. We note that

$$\left[\mathbf{A}_{n_1}^T \cdots \mathbf{A}_{n_{|S|}}^T\right]^T = \bar{\boldsymbol{\Pi}}\left(\mathbf{I}_{N_t} \otimes \left[\mathbf{f}_{n_1} \cdots \mathbf{f}_{n_{|S|}}\right]^T\right) \in \mathbb{C}^{|\mathcal{S}|N_t \times LN_t},$$

where $\bar{\boldsymbol{\Pi}} \in \mathbb{R}^{|\mathcal{S}|N_t \times |\mathcal{S}|N_t}$ is a permutation matrix, has the full rank for $|\mathcal{S}| \geq L$ due to the Vandermonde structure of $[\,\mathbf{f}_{n_1}, \ldots, \mathbf{f}_{n_{|S|}}\,]^T$. Consequently, we can only have $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}'$ in (7) if $|\mathcal{S}| \geq L$.

To prove necessity, we show that if $|\mathcal{S}| < L$, one can find a pair $\boldsymbol{s}, \boldsymbol{s}' \in \{\pm 1\}^{\bar{K}}$, $\boldsymbol{s} \neq \boldsymbol{s}'$ such that (7) holds for some $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{H}}'$ where $\boldsymbol{\mathcal{H}} \neq \boldsymbol{\mathcal{H}}'$. Without loss of generality, assume that $|\mathcal{S}| = L-1$ and $\mathcal{S} = \{1, \ldots, L-1\}$. In addition, assume $N_r = 1$, and construct a channel pair $\boldsymbol{\mathcal{H}} = \mathbf{g} \otimes \mathbf{h}$ and $\boldsymbol{\mathcal{H}}' = \mathbf{g}' \otimes \mathbf{h}$, where $\mathbf{h} \in \mathbb{C}^{N_t}$ satisfies

$$\mathbf{f}_n^T\mathbf{h} = 0 \;\forall\, n \in \mathcal{S}, \tag{16}$$

and $\mathbf{g}, \mathbf{g}' \in \mathbb{C}^{N_t}$, $\mathbf{g} \neq \pm\mathbf{g}'$, are chosen such that

$$\mathbf{C}_{\mathrm{O}}(\mathbf{u})\mathbf{g} = \mathbf{C}_{\mathrm{O}}(\mathbf{u}')\mathbf{g}' \tag{17}$$

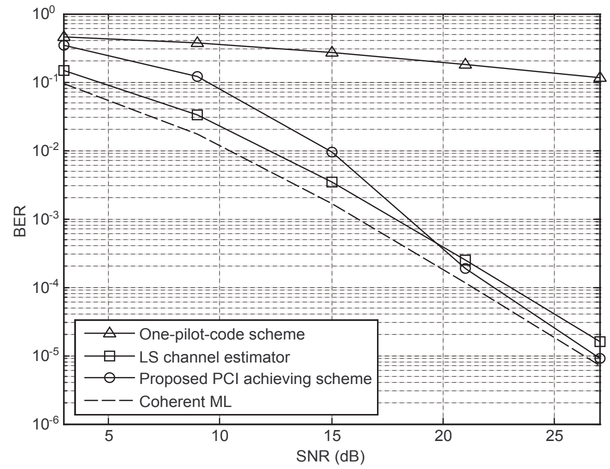
for some $\mathbf{u}, \mathbf{u}' \in \{\pm 1\}^K$, $\mathbf{u} \neq \pm\mathbf{u}'$. We should emphasize that since $[\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_{L-1}]^T \in \mathbb{C}^{(L-1)\times L}$ has nullity equal to 1, the $\mathbf{h}$ in (16) must exist. On the other hand, since $\mathbf{C}_{\mathrm{O}}(\cdot)$ is not NIS, according to Property 3, (17) can hold true. For $n \in \mathcal{S}$, we then have $\mathbf{A}_n\boldsymbol{\mathcal{H}} = (\mathbf{g} \otimes \mathbf{f}_n^T\mathbf{h}) = \mathbf{A}_n\boldsymbol{\mathcal{H}}' = (\mathbf{g}' \otimes \mathbf{f}_n^T\mathbf{h}) = \mathbf{0}$, and therefore, for any $\mathbf{s}_n, \mathbf{s}'_n$, $n \in \mathcal{S}$, we have

$$\mathbf{C}_{\mathrm{NIS}}(\mathbf{s}_n)\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{C}_{\mathrm{NIS}}(\mathbf{s}'_n)\mathbf{A}_n\boldsymbol{\mathcal{H}}' = \mathbf{0}. \tag{18}$$

Let $\mathbf{s}_n = \mathbf{u}$ and $\mathbf{s}'_n = \mathbf{u}'$ for all $n \in \mathcal{S}^c$. Then it can be shown that

$$\begin{aligned}\mathbf{C}_{\mathrm{O}}(\mathbf{s}_n)\mathbf{A}_n\boldsymbol{\mathcal{H}} &= \mathbf{C}_{\mathrm{O}}(\mathbf{s}_n)\mathbf{g}(\mathbf{f}_n^T\mathbf{h}) = \mathbf{C}_{\mathrm{O}}(\mathbf{s}'_n)\mathbf{g}'(\mathbf{f}_n^T\mathbf{h}) \\ &= \mathbf{C}_{\mathrm{O}}(\mathbf{s}'_n)(\mathbf{g}' \otimes \mathbf{f}_n^T\mathbf{h}) = \mathbf{C}_{\mathrm{O}}(\mathbf{s}'_n)\mathbf{A}_n\boldsymbol{\mathcal{H}}',\end{aligned} \tag{19}$$

for all $n \in \mathcal{S}^c$. Therefore, by (18) and (19), we see that the channel $\boldsymbol{\mathcal{H}}$ cannot be uniquely identified if $|\mathcal{S}| < L$. ■



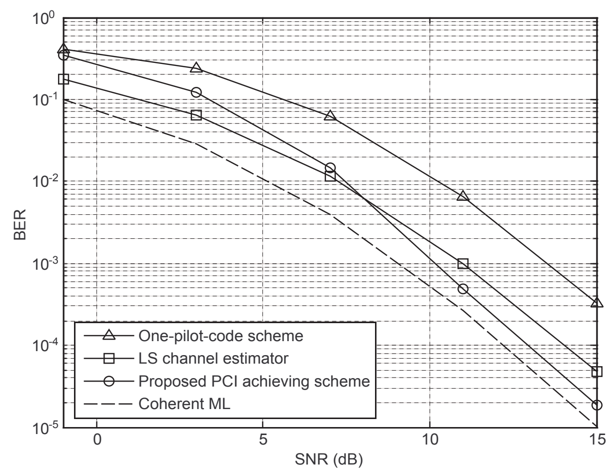


Figure 1: Performance (BER) of the proposed PCI achieving scheme for $N_c = 32$, $L = 8$, and (a) $N_r = 1$ and (b) $N_r = 2$.

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