JOINT TRANSMIT BEAMFORMING AND ARTIFICIAL NOISE DESIGN FOR QOS DISCRIMINATION IN WIRELESS DOWNLINK

Wei-Cheng Liao[†], Tsung-Hui Chang[†], Wing-Kin Ma[‡], and Chong-Yung Chi[†]

[†]Institute of Commun. Eng. & Dept. of Elec. Eng.
National Tsing Hua University,
Hsinchu, Taiwan 30013
E-mail: weicliao@gmail.com, changth@mx.nthu.edu.tw
cychi@ee.nthu.edu.tw

[‡]Dept. of Electronic Eng.
The Chinese University of Hong Kong,
Shatin, N.T., Hong Kong
E-mail: wkma@ieee.org

ABSTRACT

This paper considers a downlink wireless system where a multiple-antenna transmitter (Alice) aims to discriminate the reception performances between a legitimate receiver (Bob) and a set of unauthorized receivers (Eves). To this end, there has been great interest in the use of artificial noise (AN) together with transmit beamforming in order to effectively interfere Eves' reception. However, most of the existing works do not optimize the AN but simply allocate it in the left null space of the Alice-to-Bob channel. In the paper, we propose to jointly optimize the beamforming vector and the AN covariance matrix by minimizing the total transmit power subject to a target signal-to-interference-plus-noise ratio (SINR) constraint on Bob and limited SINR constraints on all Eves. While the considered beamforming problem is not convex and may be difficult to solve in general, it can be effectively handled by a convex approximation method called semidefinite program (SDP) relaxation. In addition to showing how SDP relaxation can be applied to this problem, we prove using the KKT optimality that SDP relaxation provides a global optimum solution of the proposed beamforming problem when Alice has perfect information of the channel from Alice to Bob. Simulation results are presented to demonstrate the effectiveness of the proposed beamforming method.

Index Terms— Transmit beamforming, QoS discrimination, artificial noise, SDP relaxation, secure communications

1. INTRODUCTION

This paper considers a transmit beamforming design for achieving quality of service (QoS) discrimination between a legitimate receiver (Bob) and a set of unauthorized receivers (Eves) in downlink wireless systems. This problem appears in many wireless applications, such as reception performance discrimination between paid and unpaid users in TV broadcast systems, and prevention of eavesdropping in secure communications. Conventionally, this QoS discrimination problem is addressed with the use of application level cryptography and user authentication mechanisms, but recent developments in physical-layer secrecy [1] has shown that this problem can also be handled in the physical layer by exploiting the different fading characteristics of Bob and Eves' channels [2].

In secure communications, it has been shown [2] that the transmitter (Alice) can broadcast signals with a nonzero coding rate to Bob without any information being eavesdropped by Eves if the mutual information between Alice and Bob is higher than that between Alice and Eves. This implies that a nonzero secrecy rate is achievable if there is a QoS discrimination between Bob and Eves. These advances in information theory have inspired several recent research efforts [3, 4, 5, 6] that endeavor to enhance the physical-layer secrecy via signal processing techniques. Specifically, with the use of multiple antennas at Alice, transmit beamforming has been shown to be effective in discriminating Bob and Eves' reception performances [3, 4]. Recently, Goel and Negi [5] have proposed an intuitively insightful beamforming scheme where an artificial noise (AN) is purposely added in the transmitted signal for raising the interference level at Eves. Assuming that Alice knows the channel state information (CSI) of Bob only, this AN-aided approach assign the AN in left null space of the Alice-to-Bob channel in a spatially uniform fashion [5, 6]. But suppose that Alice also has knowledge of the CSIs of Eves (either perfectly or imperfectly), which may be available depending on the applications. For example, in a wireless network, other users being served could as well be potential eavesdroppers. Moreover, in scenarios where Eves have to transmit sometimes, Alice can learn Eves' CSI by monitoring Eves' transmission activities. By exploiting the CSIs of Eves, one may in principle concentrate the AN energies on the Eves' directions to make the QoS discrimination even more effective.

This paper proposes a new beamforming strategy where the transmit beamformer and the AN are optimized simultaneously, given that the CSIs of Bob and Eves are perfectly known or that the correlation matrices of those CSIs are known. Specifically, the AN is assumed spatially Gaussian distributed, and we manipulate the AN through its covariance matrix. The design formulation is based on total power minimization subject to the constraints that i) Bob achieves a signal-to-interference-plus-noise ratio (SINR) no less than a target QoS requirement; and ii) each of Eves has an SINR no larger than a preset, often low, value. The proposed design formulation is nonconvex. In order to obtain an effective approximate solution in polynomial time, we propose to apply the semidefinite program (SDP) relaxation method [7] to the proposed beamforming problem. In this method, the nonconvex beamforming problem is approximated by a convex SDP which can be efficiently solved by interior point methods [8]. While the SDP relaxation method is in general suboptimal, our analysis shows that this method yields a global optimum solution of the proposed beamforming problem when Alice has perfect CSI of Bob. The proposed beamforming scheme is compared with its no-AN counterpart. Simulation results to be presented will show that the proposed beamforming scheme is

more power efficient than that without AN, in discriminating Bob and Eves' reception performances.

2. SIGNAL MODEL AND PROBLEM STATEMENT

Consider a wireless downlink system that consists of a transmitter (Alice), a legitimate receiver (Bob) and M unauthorized receivers (Eves). We assume that Alice is equipped with N_t transmit antennas, and both Bob and Eves are equipped with only one receive antenna. Let $\mathbf{x}(t) \in \mathbb{C}^{N_t}$ denote the signal vector transmitted from Alice, and let $\mathbf{h} \in \mathbb{C}^{N_t}$ and $\boldsymbol{g}_m \in \mathbb{C}^{N_t}$ denote the channel vectors from Alice to Bob and from Alice to the mth Eve, respectively. The received signals at Bob and Eves can be expressed as

$$y_b(t) = \mathbf{h}^H \mathbf{x}(t) + n(t), \tag{1}$$

$$y_{e,m}(t) = \boldsymbol{g}_m^H \mathbf{x}(t) + v_m(t), \; m = 1, \ldots, M, \tag{2}$$

where $n(t)$ and $v_m(t)$ are the additive noise at Bob and Eve m, respectively. They are assumed to be independent and identically distributed (i.i.d.) zero-mean, complex Gaussian random variables with variances equal to σ_n^2 and $\sigma_{v,m}^2$, respectively.

The aim of Alice is to design the transmit signal vector $\mathbf{x}(t) \in \mathbb{C}^{N_t}$ such that Bob can retrieve the information signal with a desired quality of service (QoS) while all Eves can only have limited reception performance. Taking the received signal-to-noise ratios (SNRs) (or the received signal-to-interference-plus-noise ratios (SINRs) if interference are also present) as the QoS measure, QoS discrimination can be achieved via transmit beamforming [9], that is, by letting $\mathbf{x}(t) = \boldsymbol{w}s(t)$ where $\boldsymbol{w} \in \mathbb{C}^{N_t}$ is the beamforming vector and $s(t)$ is the information signal. Assuming that $s(t)$ has unit variance, one can show by (1) and (2) that the SNRs at Bob and Eves are respectively given by

$$\mathrm{SNR}_b = \boldsymbol{w}^H \mathbf{R}_h \boldsymbol{w}/\sigma_n^2, \;\; \mathrm{SNR}_{e,m} = \boldsymbol{w}^H \mathbf{R}_{g,m} \boldsymbol{w}/\sigma_{v,m}^2, \tag{3}$$

where $\mathbf{R}_h = \mathbf{hh}^H$ and $\mathbf{R}_{g,m} = \boldsymbol{g}_m \boldsymbol{g}_m^H$ if the instantaneous CSIs of Bob and Eves are available to Alice; while $\mathbf{R}_h = \mathrm{E}\{\mathbf{hh}^H\} \succeq \mathbf{0}$ and $\mathbf{R}_{g,m} = \mathrm{E}\{\boldsymbol{g}_m \boldsymbol{g}_m^H\} \succeq \mathbf{0}$ (here $\boldsymbol{X} \succeq \mathbf{0}$ means that $\boldsymbol{X}$ is Hermitian positive semidefinite (PSD)) if only the channel correlation matrices are known to Alice. Hence, with target SNR values γ_b for Bob and γ_e for Eves ($\gamma_b > \gamma_e$), one can design the beamforming vector $\boldsymbol{w}$ by considering the following optimization problem

$$P^\star = \min_{\boldsymbol{w} \in \mathbb{C}^{N_t}} \|\boldsymbol{w}\|^2 \tag{4a}$$

$$\text{subject to (s.t.) } \boldsymbol{w}^H \mathbf{R}_h \boldsymbol{w}/\sigma_n^2 \geq \gamma_b, \tag{4b}$$

$$\boldsymbol{w}^H \mathbf{R}_{g,m} \boldsymbol{w}/\sigma_{v,m}^2 \leq \gamma_e, \; m = 1, \ldots, M. \tag{4c}$$

We should mention that the same design formulation as in (4) has been used in transmit beamforming for spectrum sharing in cognitive radios [9], a different application. Problem (4) is in general a difficult nonconvex optimization problem, but interestingly when $\mathbf{R}_h = \mathbf{hh}^H$ (i.e., Alice perfectly knows Bob's CSI) problem (4) can be recast as a convex quadratic program as follows (see, e.g., [10]):

$$\min_{\boldsymbol{w} \in \mathbb{C}^{N_t}} \|\boldsymbol{w}\|^2 \tag{5a}$$

$$\text{s.t. } \mathrm{Real}\{\mathbf{h}^H \boldsymbol{w}\} \geq \sqrt{\gamma_b \, \sigma_n^2}, \; \mathrm{Im}\{\mathbf{h}^H \boldsymbol{w}\} = 0, \tag{5b}$$

$$\boldsymbol{w}^H \mathbf{R}_{g,m} \boldsymbol{w} \leq \gamma_e \sigma_{v,m}^2, \; m = 1, \ldots, M, \tag{5c}$$

where $\mathrm{Real}\{\mathbf{h}^H \boldsymbol{w}\}$ and $\mathrm{Im}\{\mathbf{h}^H \boldsymbol{w}\}$ denote the real and imaginary parts of $\mathbf{h}^H \boldsymbol{w}$.

The design formulation in (4) depends on the degree of freedoms (DOFs) of multiple antennas (which is N_t) in discriminating Bob

and Eves' SNRs. Therefore, when M is comparable to N_t or when the target SNR ratio γ_b/γ_e is set too high, the beamformer $\boldsymbol{w}$ has to spend most of its DOFs in fulfilling (4c), but leaves very limited number of DOFs for directing the main beam power toward Bob. In that case, the optimum $\boldsymbol{w}^\star$ of problem (4) inevitably require to scale up the transmit power $\|\boldsymbol{w}^\star\|^2$ in order to meet Bob's SNR demand in (4b).

3. PROPOSED TRANSMIT BEAMFORMING USING ARTIFICIAL NOISE

In the section, we propose a new transmit beamforming strategy by incorporating artificial noise (AN) into signal transmission design. We will show how the associated beamforming problem can be efficiently handled by the SDP relaxation method [7].

3.1. Proposed Beamforming Strategy Using AN

We propose to increase the interference level at Eves by incorporating AN [5] in the transmit beamforming design. In the AN-aided approach, the transmit signal $\mathbf{x}(t)$ is modified as

$$\mathbf{x}(t) = \boldsymbol{w}s(t) + \mathbf{z}(t), \tag{6}$$

where

$$\mathbf{z}(t) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma}), \tag{7}$$

that is, $\mathbf{z}(t)$ follows the zero-mean complex Gaussian distribution with covariance matrix $\boldsymbol{\Sigma} \succeq \mathbf{0}$. Under the AN-aided setting in (6), we can deduce from the models in (1) and (2) that the SINRs at Bob and Eves are

$$\mathrm{SINR}_b = \frac{\boldsymbol{w}^H \mathbf{R}_h \boldsymbol{w}}{\mathrm{Tr}(\mathbf{R}_h \boldsymbol{\Sigma}) + \sigma_n^2}, \; \mathrm{SINR}_{e,m} = \frac{\boldsymbol{w}^H \mathbf{R}_{g,m} \boldsymbol{w}}{\mathrm{Tr}(\mathbf{R}_{g,m} \boldsymbol{\Sigma}) + \sigma_{v,m}^2}, \tag{8}$$

respectively, and the average transmit power is given by

$$\mathrm{E}\{\|\mathbf{x}(t)\|^2\} = \|\boldsymbol{w}\|^2 + \mathrm{Tr}(\boldsymbol{\Sigma}), \tag{9}$$

where $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix. While the SINR_b in (8) seems degraded compared to the SNRs in (3) due to the added AN, by joint design of the beamforming vector $\boldsymbol{w}$ and AN covariance matrix $\boldsymbol{\Sigma}$, we shall be able to alleviate the degradation and in the meantime effectively enhance the performance discrimination between Bob and Eves. Following the power minimization criterion in (4), the design formulation for the AN-aided case is as follows:

$$P_{\mathrm{AN}}^\star = \min_{\boldsymbol{w}, \boldsymbol{\Sigma}} \|\boldsymbol{w}\|^2 + \mathrm{Tr}(\boldsymbol{\Sigma}) \tag{10a}$$

$$\text{s.t. } \frac{\boldsymbol{w}^H \mathbf{R}_h \boldsymbol{w}}{\mathrm{Tr}(\mathbf{R}_h \boldsymbol{\Sigma}) + \sigma_n^2} \geq \gamma_b, \tag{10b}$$

$$\frac{\boldsymbol{w}^H \mathbf{R}_{g,m} \boldsymbol{w}}{\mathrm{Tr}(\mathbf{R}_{g,m} \boldsymbol{\Sigma}) + \sigma_{v,m}^2} \leq \gamma_e, \; m = 1, \ldots, M, \tag{10c}$$

$$\boldsymbol{\Sigma} \succeq 0. \tag{10d}$$

A benefit that we can immediately see from (10) is that $P_{\mathrm{AN}}^\star \leq P^\star$; i.e., the proposed formulation (10) is more power efficient than its no-AN counterpart (4), given the same specifications of γ_b and γ_e. The reason for this is that the feasible set of problem (4), together with $\boldsymbol{\Sigma} = \mathbf{0}$, is merely a subset of the feasible set of (10). Problem (10) is more difficult to handle than (4), however. In particular, one can check that, unlike problem (4), problem (10) does not admit a convex reformulation as in (5) when $\mathbf{R}_h = \mathbf{hh}^H$. However, we will present next that problem (10) can be efficiently handled by the SDP relaxation technique [7].

3.2. Approximation by SDP Relaxation

To illustrate the idea of SDP relaxation [7], let us define $\mathbf{W} = \boldsymbol{w}\boldsymbol{w}^H$ and rewrite problem (10) in terms of $\mathbf{W}$ as follows

$$\min_{\mathbf{W}, \boldsymbol{\Sigma}} \ \mathrm{Tr}(\mathbf{W}) + \mathrm{Tr}(\boldsymbol{\Sigma}) \tag{11a}$$

$$\text{s.t. } \gamma_b \mathrm{Tr}(\mathbf{R}_h \boldsymbol{\Sigma}) + \gamma_b \sigma_n^2 - \mathrm{Tr}(\mathbf{R}_h \mathbf{W}) \le 0, \tag{11b}$$

$$\mathrm{Tr}(\mathbf{R}_{g,m}\mathbf{W}) - \gamma_e \mathrm{Tr}(\mathbf{R}_{g,m}\boldsymbol{\Sigma}) - \gamma_e \sigma_{v,m}^2 \le 0, \tag{11c}$$
$$m = 1, \ldots, M,$$

$$\boldsymbol{\Sigma} \succeq 0, \ \mathbf{W} \succeq 0, \ \mathrm{rank}(\mathbf{W}) = 1, \tag{11d}$$

where (11d) is due to $\mathbf{W} = \boldsymbol{w}\boldsymbol{w}^H$. SDP relaxation works by neglecting the hard constraint $\mathrm{rank}(\mathbf{W}) = 1$ and uses the problem below

$$\min_{\mathbf{W}, \boldsymbol{\Sigma}} \ \mathrm{Tr}(\mathbf{W}) + \mathrm{Tr}(\boldsymbol{\Sigma}) \tag{12a}$$

$$\text{s.t. } \gamma_b \mathrm{Tr}(\mathbf{R}_h \boldsymbol{\Sigma}) + \gamma_b \sigma_n^2 - \mathrm{Tr}(\mathbf{R}_h \mathbf{W}) \le 0, \tag{12b}$$

$$\mathrm{Tr}(\mathbf{R}_{g,m}\mathbf{W}) - \gamma_e \mathrm{Tr}(\mathbf{R}_{g,m}\boldsymbol{\Sigma}) - \gamma_e \sigma_{v,m}^2 \le 0, \tag{12c}$$
$$m = 1, \ldots, M,$$

$$\boldsymbol{\Sigma} \succeq 0, \ \mathbf{W} \succeq 0. \tag{12d}$$

to approximate the original problem. Problem (12) is a convex SDP, and can be solved very efficiently by interior point methods [8].

It is known [7] that SDP relaxation does not guarantee a rank-one solution in general, and thus it is a suboptimal solver of the original design problem (10). For those situations, one can apply a Gaussian randomization procedure to turn the SDP relaxation solution to an approximate solution to problem (10). Interested readers are referred to [9] for the details. Remarkably, we will show in the next subsection that when $\mathbf{R}_h = \mathbf{hh}^H$, the optimum $\mathbf{W}^\star$ of problem (12) must have rank one, implying that SDP relaxation is a global optimum solver of the design formulation (10) when Alice has perfect CSI of Bob.

3.3. Rank-One Optimality Condition

We investigate the rank condition of $\mathbf{W}^\star$ by considering the Karush-Kuhn-Tucker (KKT) conditions of problem (12). Specifically, let $\lambda^\star \ge 0$, $\mu_m^\star \ge 0$, $m = 1, \ldots, M$, be the optimum dual variables associated with the constraints in (12b) and (12c), and let $\mathbf{Y}^\star \succeq \mathbf{0}$ be the optimum dual variable associated with $\mathbf{W}^\star$. Here we list the two KKT conditions of (12) which are directly related to $\mathbf{W}^\star$:

$$\text{(K1) } \mathbf{Y}^\star = \mathbf{I}_{N_t} + \sum_{m=1}^{M} \mu_m^\star \mathbf{R}_{g,m} - \lambda^\star \mathbf{R}_h \succeq \mathbf{0},$$

$$\text{(K2) } \mathbf{Y}^\star \mathbf{W}^\star = \mathbf{0},$$

where $\mathbf{I}_{N_t}$ is the N_t-by-N_t identity matrix. We seek to prove that $\mathrm{rank}(\mathbf{W}^\star) = 1$ when $\mathbf{R}_h = \mathbf{hh}^H$ and $\mathbf{R}_{g,m} \succeq \mathbf{0}$ for all $m = 1, \ldots, M$ (the latter condition is always true). First, note from the KKT condition (K2) that the columns of $\mathbf{W}^\star$ must lie in the null space of $\mathbf{Y}^\star$. Therefore, it suffices to show that $\mathrm{Nullity}(\mathbf{Y}^\star) = 1$. Substituting $\mathbf{R}_h = \mathbf{hh}^H$ and $\mathbf{R}_{g,m} \succeq \mathbf{0}$ for all m into (K1) yields

$$\mathbf{Y}^\star = \mathbf{I}_{N_t} + \sum_{m=1}^{M} \mu_m^\star \mathbf{R}_{g,m} - \lambda^\star \mathbf{hh}^H \triangleq \mathbf{B} - \lambda^\star \mathbf{hh}^H \succeq \mathbf{0}, \tag{14}$$

where $\mathbf{B} \triangleq \mathbf{I}_{N_t} + \sum_{m=1}^{M} \mu_m^\star \mathbf{R}_{g,m} \succ \mathbf{0}$ (positive definite). Note from (K1) and (K2) that we must have $\mathbf{Y}^\star \not\succ \mathbf{0}$; otherwise we would have $\mathbf{W}^\star = \mathbf{0}$ which contradicts (12b). It follows from (14) that

$$\mathrm{rank}(\mathbf{Y}^\star) = \mathrm{rank}\left(\mathbf{B}^{-1/2}(\mathbf{B} - \lambda^\star \mathbf{hh}^H)\mathbf{B}^{-1/2}\right)$$
$$= \mathrm{rank}\left(\mathbf{I}_{N_t} - \lambda^\star (\mathbf{B}^{-1/2}\mathbf{h})(\mathbf{B}^{-1/2}\mathbf{h})^H\right) = N_t - 1.$$

Hence we have $\mathrm{Nullity}(\mathbf{Y}^\star) = 1$, and $\mathrm{rank}(\mathbf{W}^\star) = 1$ by (K2). We here summarize the above results as the following proposition:

Proposition 1 *Consider the AN-aided transmit beamforming problem in (10) and its SDP relaxation problem in (12). Let $\{\mathbf{W}^\star, \boldsymbol{\Sigma}^\star\}$ be an optimum solution of problem (12). Suppose that $\mathbf{R}_h = \mathbf{hh}^H$, or, in other words, the CSI of Bob is perfectly known to Alice. Then $\mathbf{W}^\star$ must be of rank one taking the form*

$$\mathbf{W}^\star = \boldsymbol{w}^\star (\boldsymbol{w}^\star)^H, \tag{15}$$

and subsequently $\{\boldsymbol{w}^\star, \boldsymbol{\Sigma}^\star\}$ is an optimum solution of problem (10).

We should point out that Proposition 1 is a sufficient optimality condition for SDP relaxation. There may be other operating conditions under which SDP relaxation yields optimum solutions to problem (10). This direction is currently under investigation.

4. SIMULATION RESULTS AND DISCUSSIONS

In the simulations, we considered the downlink system as described in Sec. 2 with $N_t = 4$ and $M = 3$. If not mentioned specifically, we set $1/\sigma_n^2 = 10$ dB, $\sigma_v^2 \triangleq \sigma_{v,1}^2 = \sigma_{v,2}^2 = \sigma_{v,3}^2$ and target SINR $\gamma_b = 10$ dB. Problem (12) was solved by SeDuMi [11].

In the first simulation example, we assumed that Alice perfectly knows both the Alice-to-Bob and Alice-to-Eves CSIs, i.e., $\mathbf{R}_h = \mathbf{hh}^H$ and $\mathbf{R}_{g,m} = \boldsymbol{g}_m \boldsymbol{g}_m^H$ for all $m = 1, 2, 3$. The coefficients of $\mathbf{h}$ and $\boldsymbol{g}_m$, $m = 1, 2, 3$, were complex Gaussian distributed $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t}/N_t)$. Figure 1 presents the simulation result regarding the average transmit power versus $1/\sigma_v^2$, with $\gamma_e = 3$ dB. Note that the result was obtained by averaging over 1000 simulation trials. As seen from the figure, for $1/\sigma_v^2 \le 3$ dB, both methods consume the same transmit power because the DOFs provided by multiple antennas are already sufficient to limit Eves' performance; whereas for $1/\sigma_v^2 > 3$ dB the difference between transmit powers of (4) and (10) increases with increased $1/\sigma_v^2$ and can be as large as 15 dB for $1/\sigma_v^2 = 30$ dB.

To further understand the efficacy of the proposed method, we considered the uniform linear array (ULA) channel model for the space between successive array elements is half of the carrier wavelength where the channel vectors $\mathbf{h}$ and $\boldsymbol{g}_m$ posses a Vandermonde structure; i.e., $\mathbf{v}(\phi) = [1 \ e^{j\theta} \ e^{j2\theta} \ \ldots \ e^{j(N_t-1)\theta}]^T/\sqrt{N_t}$ where $j = \sqrt{-1}$, $\theta = -\pi \sin(\phi\pi/180)$, and $\phi \in [-90°, 90°)$. In Fig. 2, the beam patterns of the optimized beamforming vectors by problems (4) and (10) are displayed for $\gamma_e = 0$ dB, $\gamma_b = 20$ dB, $1/\sigma_n^2 = 0$ dB, $1/\sigma_v^2 = 20$ dB, $\mathbf{R}_h = \mathbf{v}(0)\mathbf{v}^H(0)$ (Bob is along the direction of zero degree), and $\mathbf{R}_{g,m} = \mathbf{v}(\phi_m)\mathbf{v}^H(\phi_m)$ where $\{\phi_1, \phi_2, \phi_3\} = \{20°, -20°, 30°\}$. Specifically, we present the beam pattern of $\boldsymbol{w}^\star$ of problem (4) (solid red line) in Fig. 2(a), and plotted both of the beam patterns of $\boldsymbol{w}^\star$ (solid red line) and $\boldsymbol{\Sigma}^\star$ (dashed blue line) of problem (10) in Fig. 2(b). From Fig. 2(a), one can observe that the optimum beamformer in the no-AN design forms nulls along the directions of Eves to degrade their reception performance. On the other hand, one can see from Fig. 2(b) that the AN-aided design directs the ANs main (interfering) power toward the directions of Eves; while the beamformer $\boldsymbol{w}^\star$ focuses its main (information) beam power toward Bob. The total powers achieved by the no-AN and AN-aided designs are 29.9 dB and 20.8 dB, respectively. Apparently, the AN-aided design is more power efficient than the no-AN under the same SINR specification.
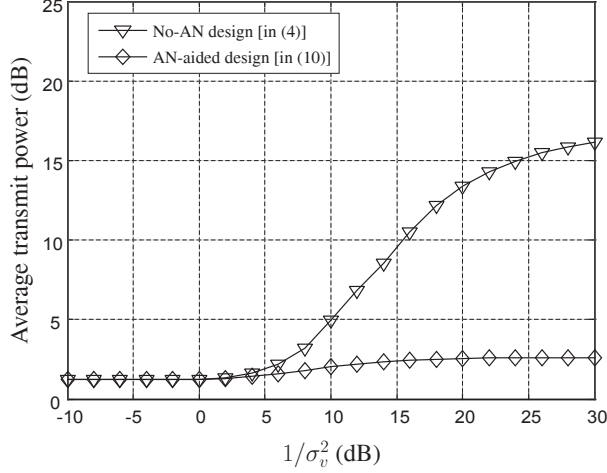
Fig. 1. Simulation results (average transmit power versus $1/\sigma_v^2$) for $\gamma_b = 10$ dB, $\gamma_e = 3$ dB, $1/\sigma_n^2 = 10$ dB, $\mathbf{R}_h = \mathbf{h}\mathbf{h}^H$ and $\mathbf{R}_{g,m} = \boldsymbol{g}_m \boldsymbol{g}_m^H$ for all $m = 1, 2, 3$.

As the final simulation example, we considered problems (4) and (10) with $\mathbf{R}_h = \mathbf{h}\mathbf{h}^H$ and $\mathbf{R}_{g,m} = \mathrm{E}\{\boldsymbol{g}_m \boldsymbol{g}_m^H\}$ for $m = 1, 2, 3$; i.e., Alice has the perfect Alice-to-Bob CSI but only knows the correlation matrices of Alice-to-Eves channels. The channel vector $\mathbf{h}$ was complex Gaussian distributed $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t}/N_t)$ and the covariance matrices $\mathrm{E}\{\boldsymbol{g}_m \boldsymbol{g}_m^H\}$ were given by

$$\mathrm{E}\{\boldsymbol{g}_m \boldsymbol{g}_m^H\} = \alpha \frac{\bar{\boldsymbol{g}}_m \bar{\boldsymbol{g}}_m^H}{||\bar{\boldsymbol{g}}_m||^2} + (1 - \alpha)\boldsymbol{\Lambda}, \ m = 1, 2, 3, \quad (16)$$

where $\bar{\boldsymbol{g}}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t})$, $[\boldsymbol{\Lambda}]_{i,j} = 0.1^{|i-j|}/N_t$ specifies the degree of correlation between transmit antennas, and $\alpha \in [0, 1]$ was used to control the level of uncertainty of channel coefficients. In this particular example, we found that problem (4) or problem (10) may be infeasible (or have no solution) sometimes, especially if the given problem instances $\mathbf{R}_h, \mathbf{R}_{g,1}, \dots \mathbf{R}_{g,M}$ are harsh. In Fig. 3, we present the percentage of infeasibility (%) of problems (4) and (10) out of 1000 simulation trials for $\gamma_e = 3$ dB. One can see from the figure that for $\alpha = 0.8$ and $1/\sigma_v^2 \geq 10$ dB problem (4) has about 30% infeasibility rate, and it can increase to about 55% for $\alpha = 0.6$. The proposed problem (10) by contrast is always feasible for both $\alpha = 0.6$ and $\alpha = 0.8$. The presented simulation results well demonstrate the effectiveness of the proposed beamforming strategy in power consumption as well as in feasibility.

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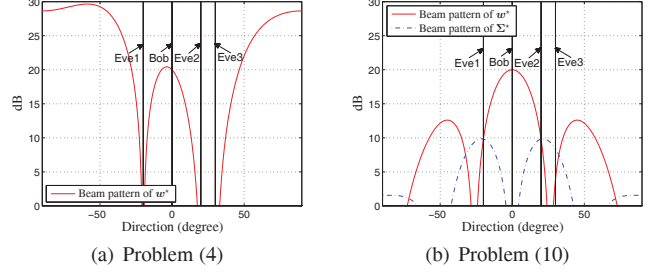
(a) Problem (4)

(b) Problem (10)

Fig. 2. Optimum beam patterns of problems (4) and (10) for $\gamma_b = 20$ dB, $\gamma_e = 0$ dB, $1/\sigma_n^2 = 0$ dB, $1/\sigma_v^2 = 20$ dB, $\mathbf{R}_h = \mathbf{v}(0)\mathbf{v}^H(0)$, and $\mathbf{R}_{g,m} = \mathbf{v}(\phi_m)\mathbf{v}^H(\phi_m)$ where $\{\phi_1, \phi_2, \phi_3\} = \{20°, -20°, 30°\}$.
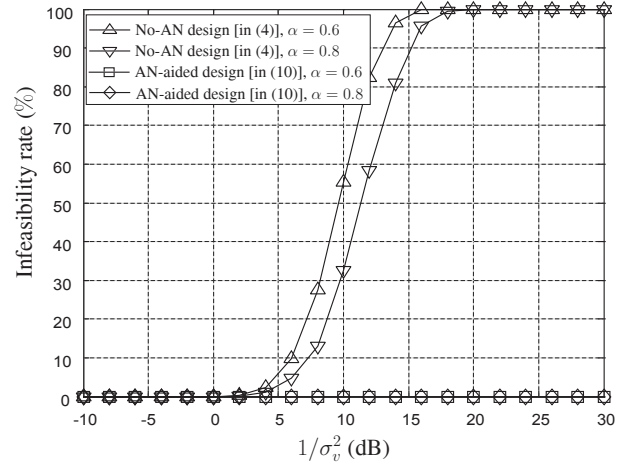

Fig. 3. Infeasibility rate (%) of problems (4) and (10) for $\gamma_b = 10$ dB, $\gamma_e = 3$ dB and $1/\sigma_n^2 = 10$ dB.

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