DOA ESTIMATION OF QUASI-STATIONARY SIGNALS VIA KHATRI-RAO SUBSPACE

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ABSTRACT

This paper addresses the problem of direction-of-arrival (DOA) estimation of quasi-stationary signals, which finds applications in array processing of speech and audio. By studying the subspace structures of the local second-order statistics (SOSs) of quasi-stationary signals, we develop a Khatri-Rao (KR) subspace approach that has two notable advantages. First, the approach can operate in underdetermined cases. It is proven that if N is the number of sensors in the array, then the proposed approach can identify up to $2N-2$ source DOAs in an unambiguous fashion. Second, the approach can handle the problem of unknown noise covariance. Essentially, the KR subspace formulation is found to provide a simple and effective way of annihilating the (unknown) noise covariance from the observed signal SOSs. Simulation results, with an emphasis on underdetermined and colored-noise cases, illustrate that the KR subspace approach provides promising mean square estimation error performance.

Index Terms— quasi-stationary signals, Khatri-Rao product, Kruskal rank, underdetermined DOA estimation.

1. INTRODUCTION

This paper concentrates on a direction-of-arrival (DOA) estimation problem where the source signals are assumed quasi-stationary. Quasi-stationary signals represent a class of non-stationary signals in which the statistics are locally static over a short period of time, but exhibit differences from one local period to another. Speech and audio signals, for instance, are often recognized as quasi-stationary signals. In fact, DOA estimation of audio signals has a practically very relevant application where the objective is to monitor birds in an airport for avoiding collisions of birds and aircrafts [1]. It also finds applications in microphone array processing of speech signals [2]. These real-world applications provide strong motivations for studying direction finding of quasi-stationary signals (DF-QSS).

The DF-QSS approach proposed in this paper is based on exploitation of the subspace structures of the time-variant second-order statistics (SOSs) of quasi-stationary signals. While the time-variant SOS natures of quasi-stationary signals have also been utilized in the topic of blind source separation (BSS); e.g., [3–5], the proposed DOA estimation criterion is different from those available in BSS, such as parallel factor analysis (PARAFAC) and joint diagonalization (JD). Basically, the difference lies in that our approach uses subspace concepts in lieu of data fitting. As we will see later, the key part of our subspace approach is in using the subspace characteristics of the self Khatri-Rao (KR) product of the array response. Hence, we call the proposed approach a *KR subspace approach*. In the applications of existing BSS criteria (such as PARAFAC and JD) to DF-QSS, we are faced with a multi-dimensional nonlinear minimization problem. Under the common assumption of uniform linear array structures, the KR subspace approach is a one-dimensional search problem and can be effectively realized using methods such as the popular MUSIC algorithm[1].

We show that the KR subspace approach has two advantages. First, it is proven through an identifiability analysis that for an N-element array, a KR subspace method can handle up to $2N-2$ sources. This is a significant improvement, compared to the conventional subspace DF approach where the identifiability limit is $N-1$ sources. Second, the KR subspace formulation naturally provides an effective way of eliminating the spatial noise covariance from the observed SOSs. It does so without any knowledge of the noise covariance, meaning that we can deal with unknown, possibly colored spatial noise covariance. The above two attractive features will be validated by simulations.

2. PROBLEM STATEMENT

We consider a standard DOA estimation scenario where K narrowband far-field sources are observed by an N-element uniform linear sensor array. We denote by $x_n(t)$ the observed signal of the nth sensor, and $s_k(t)$ the signal emitted by the kth source. By letting $\mathbf{x}(t) = [\, x_1(t), \ldots, x_N(t)\,]^T$ and $\mathbf{s}(t) = [\, s_1(t), \ldots, s_K(t)\,]^T$, the received signal is modeled as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{v}(t), \quad t = 0, 1, 2, \ldots \tag{1}$$

Here, $\mathbf{v}(t) \in \mathbb{C}^N$ represents the spatial noise,

$$\mathbf{A} = [\, \mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)\,] \in \mathbb{C}^{N \times K} \tag{2}$$

is the array response matrix where $\theta_k \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is the direction of arrival (DOA) of source k, and

$$\mathbf{a}(\theta) = [\, 1, e^{-\frac{j2\pi d}{\lambda}\sin(\theta)}, \ldots, e^{-\frac{j2\pi d}{\lambda}(N-1)\sin(\theta)}\,]^T, \tag{3}$$

is the steering vector function with parameters d and λ being the inter-sensor spacing and the signal wavelength respectively. Some basic assumptions are made as follows:

(A1) The source signals $s_k(t), k = 1, \ldots, K$, are mutually uncorrelated and have zero-mean.

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[1]MUltiple SIgnal Classification.

(A2) The source DOAs $\theta_k, k = 1, \ldots, K$, are distinct to one another, i.e., $\theta_k \neq \theta_\ell$ for all $k \neq \ell$.

(A3) The noise $\mathbf{v}(t)$ is zero-mean wide-sense stationary (WSS) with covariance matrix $\mathbf{C} \triangleq \mathrm{E}\{\mathbf{v}(t)\mathbf{v}^H(t)\}$, and it is statistically independent of the source signals.

In addition, we adopt the following QSS assumption [3, 4]:

(A4) Each source signal $s_k(t)$ is wide-sense quasi-stationary with frame length L; that is, for $m = 1, 2, \ldots,$

$$\mathrm{E}\{|s_k(t)|^2\} = d_{mk}, \ \ \forall t \in [(m-1)L, mL-1]. \quad (4)$$

An illustration is given in Fig. 1 to pictorially show how a quasi-stationary signal may behave. Assumption (A4) means that the second order statistics of the source signals are time-varying, but that they remain static over a short period of time.

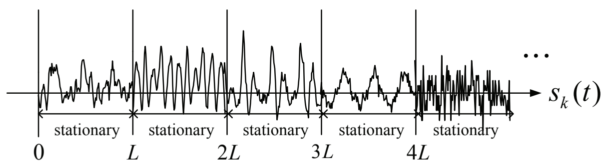

Fig. 1: Illustration of quasi-stationary signals.

Under (A4), we can define a local covariance matrix

$$\mathbf{R}_m = \mathrm{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\}, \quad \forall t \in [(m-1)L, mL-1], \quad (5)$$

where $m = 1, 2, \ldots$ denotes the frame index. In practice, knowledge of these local covariances are acquired by local time averaging; i.e., $\hat{\mathbf{R}}_m = \frac{1}{L}\sum_{t=(m-1)L}^{mL-1} \mathbf{x}(t)\mathbf{x}^H(t)$. With (A1), (A3) and (A4), we can express $\mathbf{R}_m$ as

$$\mathbf{R}_m = \mathbf{A}\mathbf{D}_m\mathbf{A}^H + \mathbf{C} \quad (6)$$

where $\mathbf{D}_m = \mathrm{Diag}(d_{m1}, d_{m2}, \ldots, d_{mK}) \in \mathbb{R}^{K\times K}$ is the source covariance matrix at frame m. Now, suppose that we have acquired local covariance matrices $\mathbf{R}_1, \ldots, \mathbf{R}_M$, where M is the total number of frames. Our goal is to estimate the DOAs $\theta_1, \ldots, \theta_K$ from $\mathbf{R}_1, \ldots, \mathbf{R}_M$, without information of the local source covariances $\mathbf{D}_1, \ldots, \mathbf{D}_M$ and the noise covariance $\mathbf{C}$.

3. KHATRI-RAO SUBSPACE APPROACH

3.1. Khatri-Rao Product

We first review the Khatri-Rao product and its properties, before proceeding to describing the proposed KR subspace approach.

Given two matrices $\mathbf{A} \in \mathbb{C}^{n\times k}$ and $\mathbf{B} \in \mathbb{C}^{m\times k}$ of identical number of columns, their Khatri-Rao (KR) product is denoted by

$$\mathbf{A} \odot \mathbf{B} = [\, \mathbf{a}_1 \otimes \mathbf{b}_1, \ldots, \mathbf{a}_k \otimes \mathbf{b}_k \,] \in \mathbb{C}^{nm\times k}, \quad (7)$$

where $\otimes$ denotes the Kronecker product. For two vectors $\mathbf{a} \in \mathbb{C}^n$ and $\mathbf{b} \in \mathbb{C}^m$, the Kronecker product is given by

$$\mathbf{a} \otimes \mathbf{b} = \begin{bmatrix} a_1\mathbf{b} \\ a_2\mathbf{b} \\ \vdots \\ a_n\mathbf{b} \end{bmatrix} = \mathrm{vec}(\mathbf{b}\mathbf{a}^T) \quad (8)$$

where $\mathrm{vec}(\cdot)$ is the vectorization.

The rank properties of KR product has interesting relationships with *Kruskal rank*, or k-rank for short. The k-rank of a matrix $\mathbf{A}$, denoted by $\mathrm{krank}(\mathbf{A})$, is said to be equal to r when every collection of r columns of $\mathbf{A}$ is linear independent but there exists a collection of $r+1$ linearly dependent columns. k-rank presents stronger condition than the standard rank, and thus we have $\mathrm{rank}(\mathbf{A}) \geq \mathrm{krank}(\mathbf{A})$. A k-rank property for KR product is as follows [6]:

Property 1 *For two matrices* $\mathbf{A} \in \mathbb{C}^{n\times k}$ *and* $\mathbf{B} \in \mathbb{C}^{m\times k}$*, with* $\mathrm{krank}(\mathbf{A}) \geq 1$ *and* $\mathrm{krank}(\mathbf{B}) \geq 1$*, it holds true that*

$$\mathrm{krank}(\mathbf{A} \odot \mathbf{B}) \geq \min\{k, \ \mathrm{krank}(\mathbf{A}) + \mathrm{krank}(\mathbf{B}) - 1\}. \quad (9)$$

3.2. Khatri-Rao Subspace Criterion and Algorithm

We now consider the DOA estimation problem formulated in Sec. 2. The local covariance model in (6) can be expressed as

$$\begin{aligned} \mathbf{y}_m &\triangleq \mathrm{vec}(\mathbf{R}_m) = \mathrm{vec}(\mathbf{A}\mathbf{D}_m\mathbf{A}^H) + \mathrm{vec}(\mathbf{C}) \\ &= \sum_{k=1}^{K} d_{mk}\mathrm{vec}(\mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k)) + \mathrm{vec}(\mathbf{C}) \\ &= (\mathbf{A}^* \odot \mathbf{A})\boldsymbol{d}_m + \mathrm{vec}(\mathbf{C}) \end{aligned} \quad (10)$$

where (10) is due to (8). Here we have $\boldsymbol{d}_m = [\, d_{m1}, \ldots, d_{mK} \,]^T$. By stacking $[\, \mathbf{y}_1, \ldots, \mathbf{y}_M \,] \triangleq \mathbf{Y}$, we can write

$$\mathbf{Y} = (\mathbf{A}^* \odot \mathbf{A})\boldsymbol{\Psi}^T + \mathrm{vec}(\mathbf{C})\mathbf{1}_M^T \quad (11)$$

where $\mathbf{1}_M = [\, 1, \ldots, 1 \,]^T \in \mathbb{R}^M$, and

$$\boldsymbol{\Psi} = [\, \boldsymbol{d}_1, \ldots, \boldsymbol{d}_M \,]^T = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1K} \\ d_{21} & d_{22} & \cdots & d_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ d_{M1} & d_{M2} & \cdots & d_{MK} \end{bmatrix}. \quad (12)$$

It is interesting to note from (10) or (11) that $\mathbf{y}_m$ is reminiscent of an array signal model where $(\mathbf{A}^* \odot \mathbf{A}) \in \mathbb{C}^{N^2\times K}$ is virtually the array response matrix and $\boldsymbol{d}_m$ becomes the source signal vector. The virtual array dimension, given by N^2, is greater than the physical array dimension N for $N > 1$, and simply speaking that is the reason why underdetermined DOA estimation is possible.

We notice that in (12), each column of $\boldsymbol{\Psi}$ describes the power variations of the respective source signal over frames. Let us assume the following:

(A5) The matrix $[\, \boldsymbol{\Psi} \ \mathbf{1}_M \,] \in \mathbb{R}^{M\times(K+1)}$ is of full column rank.

Assumption (A5) physically implies the followings: First, the source power distributions over the time frames (or the columns of $\boldsymbol{\Psi}$) are different so that $\boldsymbol{\Psi}$ can maintain a full column rank condition. Second, any linear combination of the sources cannot result in a WSS source (i.e., for any $c_1, \ldots, c_K \in \mathbb{C}$, $\sum_{k=1}^{K} c_k s_k(t)$ cannot be WSS), otherwise $\mathbf{1}_M$ can be a linear combination of the columns of $\boldsymbol{\Psi}$ which violates (A5). As a necessity for fulfilling (A5), we need $M \geq K+1$. In practice, this should not be an issue since the number of available frames M is generally large.

Under (A5), we can eliminate the unknown noise covariance effectively and easily. Let $\mathbf{P}_{\mathbf{1}_M}^{\perp} = \mathbf{I}_M - \frac{1}{M}\mathbf{1}_M\mathbf{1}_M^T$ be the orthogonal complement projector of $\mathbf{1}_M$. By performing a projection

$$\begin{aligned} \mathbf{Y}\mathbf{P}_{\mathbf{1}_M}^{\perp} &= [(\mathbf{A}^* \odot \mathbf{A})\boldsymbol{\Psi}^T + \mathrm{vec}(\mathbf{C})\mathbf{1}_M^T]\mathbf{P}_{\mathbf{1}_M}^{\perp} \\ &= (\mathbf{A}^* \odot \mathbf{A})(\mathbf{P}_{\mathbf{1}_M}^{\perp}\boldsymbol{\Psi})^T, \end{aligned} \quad (13)$$

we obtain a data model that is free from the noise covariance. Under (A5), we have $\text{rank}(\mathbf{P}^{\perp}_{\mathbf{1}_M}\boldsymbol{\Psi}) = \text{rank}(\boldsymbol{\Psi}) = K$. In other words, (13) does not damage the rank condition of the data model.

Let us assume that $(\mathbf{A}^* \odot \mathbf{A})$ has full column rank. (In the next subsection, further justifications on this assumption will be provided along with the identifiability analysis.) When both $\mathbf{A}^* \odot \mathbf{A}$ and $\mathbf{P}^{\perp}_{\mathbf{1}_M}\boldsymbol{\Psi}$ in (13) have full column rank, we have

$$\mathcal{R}(\mathbf{A}^* \odot \mathbf{A}) = \mathcal{R}(\mathbf{Y}\mathbf{P}^{\perp}_{\mathbf{1}_M}) \tag{14}$$

where $\mathcal{R}(\cdot)$ denotes the range space. Now, consider performing a singular value decomposition (SVD) on $\mathbf{Y}\mathbf{P}^{\perp}_{\mathbf{1}_M}$:

$$\mathbf{Y}\mathbf{P}^{\perp}_{\mathbf{1}_M} = [\mathbf{U}_s \, \mathbf{U}_n] \begin{bmatrix} \boldsymbol{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \tag{15}$$

where $\mathbf{U}_s \in \mathbb{C}^{N^2 \times K}$ and $\mathbf{V}_s \in \mathbb{C}^{M \times K}$ are respectively the left and right singular matrices associated with the nonzero singular values, $\mathbf{U}_n \in \mathbb{C}^{N^2 \times (N^2-K)}$ and $\mathbf{V}_n \in \mathbb{C}^{M \times (N^2-K)}$ are the counterparts for the zero singular values, and $\boldsymbol{\Sigma}_s \in \mathbb{R}^{K \times K}$ is the nonzero singular values matrix. Using the standard SVD results that $\mathcal{R}(\mathbf{A}^* \odot \mathbf{A}) = \mathcal{R}(\mathbf{U}_s)$ and that $\mathbf{U}_n$ is orthogonal to $\mathbf{U}_s$, we conclude that

$$\mathbf{U}_n^H[\mathbf{A}^* \odot \mathbf{A}]_k = \mathbf{U}_n^H(\mathbf{a}^*(\theta_k) \otimes \mathbf{a}(\theta_k)) = \mathbf{0} \tag{16}$$

for $k = 1, \ldots, K$. From (16) we propose a KR subspace criterion for DOA estimation of quasi-stationary sources, given as follows:

$$\boxed{\begin{array}{rl} \text{find} & \theta \\ \text{such that} & \mathbf{U}_n^H(\mathbf{a}^*(\theta) \otimes \mathbf{a}(\theta)) = \mathbf{0}, \;\; \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] \end{array}} \tag{17}$$

The above KR subspace criterion can be solved by a line search (as one of the possible ways), using the same spirit as MUSIC. The resultant algorithm, called *KR-MUSIC* here, is given in Table 1.

3.3. Identifiability

Certainly, the KR subspace criterion (17) is achieved if θ is one of the true DOAs $\theta_1, \ldots, \theta_K$. But, the more important question is whether (17) is satisfied *only if* θ is a true DOA. To answer this identifiability question, we consider the following lemma:

Lemma 1 *Consider a Vandemonde matrix*

$$\mathbf{V} = \begin{bmatrix} 1 & \cdots & 1 \\ z_1^{-1} & \cdots & z_k^{-1} \\ \vdots & \ddots & \vdots \\ z_1^{-(n-1)} & \cdots & z_k^{-(n-1)} \end{bmatrix} \in \mathbb{C}^{n \times k} \tag{18}$$

where $z_1, \ldots, z_k \in \mathbb{C}$, $z_k \neq z_\ell$ for all $k \neq \ell$. The KR product $\mathbf{V}^* \odot \mathbf{V}$ is of full column rank if $k \leq 2n-1$.

Proof: It has been proven in [7] that for a Vandemonde matrix $\mathbf{V}$, $\text{krank}(\mathbf{V}) = \text{rank}(\mathbf{V})$. And for distinct $z_1, \ldots, z_k$, it must hold true that $\text{rank}(\mathbf{V}) = \min\{n, k\}$. As a consequence, we have

$$\text{rank}(\mathbf{V}^* \odot \mathbf{V}) \geq \text{krank}(\mathbf{V}^* \odot \mathbf{V}) \geq \min\{\, k, \; 2\min\{n, k\} - 1 \,\} \tag{19}$$

where the last inequality is due to Property 1 and $\text{krank}(\mathbf{V}^*) = \text{krank}(\mathbf{V}) = \min\{n, k\}$. For $1 \leq k \leq 2n-1$, (19) reduces to $\text{rank}(\mathbf{V}^* \odot \mathbf{V}) \geq k$. Hence, $\mathbf{V}^* \odot \mathbf{V}$ has full column rank. ∎

An immediate consequence of Lemma 1 is that the virtual array response matrix $\mathbf{A}^* \odot \mathbf{A}$ has full column rank if $K \leq 2N-1$. More importantly, Lemma 1 leads to the following identifiability result:

Table 1: Summary of the KR-MUSIC algorithm.

Given	a received signal sequence $\{\mathbf{x}(t)\}_{t=0}^{T-1}$, a source number K, and a frame length L where L divides T.
Step 1.	Compute the local covariance estimates $$\hat{\mathbf{R}}_m = \frac{1}{L} \sum_{t=(m-1)L}^{mL-1} \mathbf{x}(t)\mathbf{x}^H(t)$$ for $m = 1, \ldots, M$, where $M = T/L$. Then, form a data matrix $\hat{\mathbf{Y}} = [\, \text{vec}(\hat{\mathbf{R}}_1), \ldots, \text{vec}(\hat{\mathbf{R}}_M) \,]$.
Step 2.	(noise covariance elimination) $\bar{\mathbf{Y}} = \hat{\mathbf{Y}}\mathbf{P}^{\perp}_{\mathbf{1}_M}$, where $\mathbf{P}^{\perp}_{\mathbf{1}_M} = \mathbf{I}_M - \frac{1}{M}\mathbf{1}_M\mathbf{1}_M^T$.
Step 3.	(subspace extraction) Perform SVD $\bar{\mathbf{Y}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$, and extract the noise subspace matrix $$\mathbf{U}_n = [\, \mathbf{u}_{K+1}, \ldots, \mathbf{u}_{N^2} \,] \in \mathbb{C}^{N^2 \times (N^2-K)}.$$
Step 4.	(MUSIC operation) Compute the spatial spectrum $$\text{P}_{\text{KR-MUSIC}}(\theta) = \frac{1}{\| \mathbf{U}_n^H(\mathbf{a}^*(\theta) \otimes \mathbf{a}(\theta)) \|^2}$$ over $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, and pick the K largest peaks of $\text{P}_{\text{KR-MUSIC}}(\theta)$ as the DOA estimates.

Proposition 1 *Assume that (A1) – (A5) hold. The KR subspace criterion* (17) *is achieved only by the true angles $\theta_1, \ldots, \theta_K$, when*

$$K \leq 2N - 2.$$

Proof: Suppose that there exists an angle $\varphi \notin \{\theta_1, \ldots, \theta_K\}$ such that the KR subspace criterion (17) is satisfied. That implies that $\mathbf{a}^*(\varphi) \otimes \mathbf{a}(\varphi) \in \mathcal{R}(\mathbf{A}^* \odot \mathbf{A})$, or equivalently,

$$\begin{aligned} &[\, \mathbf{A}^* \odot \mathbf{A}, \; \mathbf{a}^*(\varphi) \otimes \mathbf{a}(\varphi) \,] \\ &= [\, \mathbf{A}, \; \mathbf{a}(\varphi) \,]^* \odot [\, \mathbf{A}, \; \mathbf{a}(\varphi) \,] \in \mathbb{C}^{N^2 \times (K+1)} \end{aligned} \tag{20}$$

has linearly dependent columns. But, by Lemma 1, (20) is linearly independent if $K + 1 \leq 2N - 1$. This is a contradiction. ∎

Proposition 1 is an appealing result since it implies that the KR subspace approach can identify the source DOAs even when the number of sensors is about half of the number of the sources.

In fact, the condition $K \leq 2N - 2$ is not only sufficient for the KR subspace approach to provide unambiguous DOA identification (as stated in Proposition 1), but it can also be proven to be a necessary identifiability condition. The latter is further analyzed in [8], but is omitted here due to limit of space.

4. SIMULATION RESULTS

We provide two simulation examples to test the performance of the proposed KR-MUSIC algorithm.

In the first example, we consider an underdetermined case where $(N, K) = (4, 6)$. The true DOA values are $\{\theta_1, \ldots, \theta_K\} = \{-65°, -40°, -20°, 10°, 25°, 50°\}$. Recorded speech is used as the source signals. For ease of demonstrating the feasibility of the technique proposed, we performed a synthetic narrowband simulation with inter-sensor spacing $d = \lambda/2$, and with $s_k(t)$ being speech

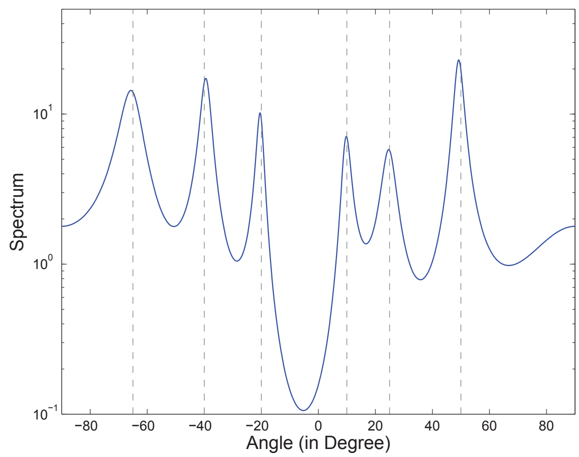

Fig. 2: DOA spectrum of KR-MUSIC for $(N, K) = (4, 6)$. The dashed lines mark the true positions of the DOAs.

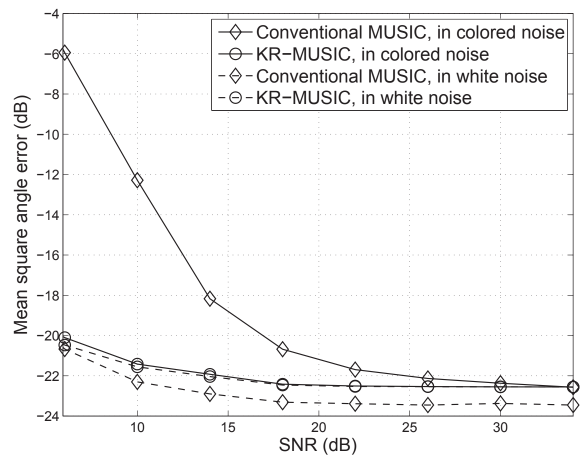

Fig. 3: Mean square error performance of KR-MUSIC and MUSIC.

signals. (In reality, speech signals fall into the wideband case. The wideband array processing requires more complex processing procedures on top of KR-MUSIC, which is too lengthy to describe in this paper. The wideband extension is considered and tested in [8].) Noise is spatially white Gaussian. The SNR, defined as the total signal power versus the noise power, is 20dB. The frame length is $L = 200$ (which is set to be commensurate with the standard stationary period of speech, 25ms), and the number of frames $M = 125$. Fig. 2 shows the KR-MUSIC DOA spectrum ($\mathrm{P}_{\mathrm{KR-MUSIC}}(\theta)$ in Table 1). We see that KR-MUSIC shows spectral peaks at the true DOA positions and those peaks are quite distinguishable.

In the second example, we model the noise as spatially non-white Gaussian where the covariance $\mathbf{C}$ follows the structure

$$C_{i,k} = \sigma_v^2 \rho^{|i-k|}$$

for some $0 \leq \rho \leq 1$ and σ_v. We set up an overdetermined situation $(N, K) = (4, 3)$, thereby enabling comparisons with the conventional MUSIC. The true DOA values are $\{\theta_1, \ldots, \theta_K\} = \{-30^\circ, 0^\circ, 30^\circ\}$. The other simulation parameter settings are the same as those of the last example. Both MUSIC and KR-MUSIC used the same total data length. We ran a 1000-trial Monte Carlo simulation, and the results obtained are shown in Fig. 3. In the legend, 'colored noise' stands for $\rho = 0.8$ while 'white noise' $\rho = 0$. We observe the followings: At very high SNRs, the mean square error (MSE) of the angle estimates of KR-MUSIC or MUSIC can no longer improve. That is due to the finite sample covariance estimation errors which may be reduced by using a larger L or M. In the presence of white noise, the conventional MUSIC yields better performance than KR-MUSIC by about 1dB in MSE. However, in the presence of colored noise, MUSIC is seen to exhibit substantial performance degradation at low SNRs. For KR-MUSIC, the MSEs in the colored and white noise cases are almost the same. This indicates that KR-MUSIC is insensitive to noise covariance. In particular, in the colored noise case, KR-MUSIC yields considerably better MSE performance than MUSIC at low to moderate SNRs.

5. CONCLUSION AND DISCUSSION

We have presented a KR subspace approach to DOA estimation of quasi-stationary signals. The proposed approach is effective in implementations, can deal with underdetermined DOA estimation cases, and can cope with the effects of unknown noise covariance. These benefits are obtained by carefully utilizing the subspace structures of the SOSs of quasi-stationary signals.

The full version of this paper [8] provides more results. For instance, we describe an additional dimension reduction method that improves upon the algorithm proposed here. Moreover, numerical comparisons with other existing underdetermined DOA estimation methods such as [9] (not based on quasi-stationarity) are given.

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