BLIND MAXIMUM-LIKELIHOOD DETECTION FOR DECODE-AND-FORWARD RANDOMIZED DISTRIBUTED OSTBC

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ABSTRACT

This paper considers the randomized distributed orthogonal space-time block coding (DOSTBC) system proposed by Sirkeci-Mergen and Scaglione, and considers a situation where some of the cooperative relays can disconnect from the network during the data frame transmission. Such an event translates into abrupt changes of the virtual channel matrix at the receiver; and a coherent detector, which assumes that the virtual channel matrix is perfectly known and static, can be severely degraded in performance. We propose a blind maximum-likelihood (ML) detector and a non-intersecting subspace (NIS) code scheme for the randomized DOSTBC system. With a mild assumption on the transmission protocol, we show that the proposed blind ML detector is robust against the unexpected relay disconnection problem. Moreover, we show that the randomized NIS code scheme can achieve the maximum transmit diversity with the blind ML receiver. Some simulation results are presented to demonstrate the efficacy of the proposed method.

Index Terms— Blind ML detection, distributed space-time coding, relay networks, maximum transmit diversity.

1. INTRODUCTION

Distributed space-time coding schemes have been proposed recently for relay-assisted networks [1–3] in order to obtain capacity and diversity gains as in non-distributed multiple-input multiple-output (MIMO) systems. Among the existing schemes, the randomized distributed orthogonal space-time coding (DOSTBC) scheme proposed by Sirkeci-Mergen and Scaglione [3] has been shown to be effective in achieving the maximum transmit diversity with coherent maximum-likelihood (ML) reception without requiring extra control information overhead for a centralized code allocation procedure.

Since the randomized DOSTBC scheme is based on the so called decode-and-forward strategy, most of the space-time codes and detection methods originally designed for non-distributed MIMO systems can be directly applied. However, due to the *ad hoc* nature of wireless networks, the virtual MIMO channel matrix in the received signal model may change unexpectedly even when the underlying physical channel remains static during the data frame transmission. This happens, for example, when some of the cooperative relays abruptly disconnect from the network due to running out of battery or other human factors. In that case, the coherent ML receiver, which assumes that the virtual channel matrix is perfectly known and static, can be severely degraded in performance, unless the receiver keeps tracking the virtual channel matrix by using additional pilot signals.

Our interest in the paper is to apply the recently proposed non-intersecting subspace (NIS) OSTBC and the associated blind ML detection methods [4, 5] to the randomized DOSTBC system. Since this class of methods requires only one pilot bit for unambiguous channel estimation and data detection, it not only has a higher spectrum efficiency but also has better robustness against unexpected variation of the virtual channel matrix in relay networks. In particular, under a mild assumption on the transmission protocol, we show by computer simulations that the proposed blind ML DOSTBC detector does not suffer much from abrupt disconnection of cooperative relays, and meanwhile outperforms some existing methods. Moreover, we show that the randomized NIS-DOSTBC scheme can attain the maximum transmit diversity with the blind ML receiver, as its coherent counterpart in [3].

2. SIGNAL MODEL AND BACKGROUND

We consider a relay network where a set of single-antenna relays collaborate together to transmit a common message to a destination receiver. We assume that the cooperative relays employ the randomized DOSTBC scheme in [3]. Specifically, let $\mathbf{s}_p \in \{\pm 1\}^{K_p}$ be the data vector intended for the receiver at block p. The ith cooperative relay transmits the following block sequence to the receiver:

$$\mathbf{C}_p(\mathbf{s}_p)\mathbf{r}_i \in \mathbb{C}^T, \tag{1}$$

where $\mathbf{C}_p(\cdot) : \{\pm 1\}^{K_p} \rightarrow \mathbb{C}^{T\times N_t}$ is an OSTBC mapping function, and $\mathbf{r}_i \in \mathbb{C}^{N_t}$ is a random vector independently generated at relay i. Denote by N_s the number of cooperative relays in the network, and assume that there are totally N block codes transmitted in a data frame, during which the physical MIMO channel remains static. The received signal in the destination receiver then is given by

$$\mathbf{Y}_p = \frac{1}{\sqrt{K_p}}\mathbf{C}_p(\mathbf{s}_p)\mathbf{R}\mathbf{H} + \mathbf{W}_p, \tag{2}$$

for $p = 1, \dots, N$, where $\mathbf{R} = [\mathbf{r}_1, \dots, \mathbf{r}_{N_s}] \in \mathbb{C}^{N_t\times N_s}$ is the randomization matrix, and $\mathbf{W}_p \in \mathbb{C}^{T\times N_r}$ is the AWGN matrix at block p with the average power per entry given by σ_w^2, and N_r denotes the number of antennas at the receiver. The physical flat-fading MIMO channel $\mathbf{H}$ can be expressed as

$$\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_{N_s}]^T \in \mathbb{C}^{N_s\times N_r} \tag{3}$$

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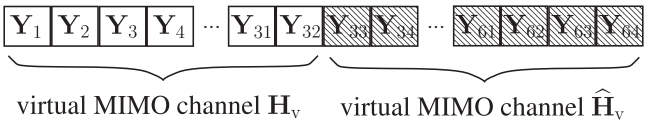

Figure 1. Received block signals at the receiver during a data frame transmission of $N = 64$ blocks. The underlying virtual MIMO channel matrix $\mathbf{H}_{\mathrm{v}}$ has changed for $p \geq 33$ due to disconnection of cooperative relays from the network [see (5)].

where $\mathbf{h}_i \in \mathbb{C}^{N_r}$ denotes the physical channel vector from the ith relay to the receiver. The entries of $\mathbf{H}$ are assumed to be independently and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance.

For the case that the receiver has the perfect knowledge of the virtual MIMO channel matrix

$$\mathbf{H}_{\mathrm{v}} = \mathbf{R}\mathbf{H}, \tag{4}$$

the coherent ML detector can be used to detect the unknown data $\{\mathbf{s}_p\}_{p=1}^{N}$ in a symbol-by-symbol fashion [6]. The performance of the coherent ML detector for the randomized DOSTBC has been analyzed in [3]. In particular, by assuming that

A1) The randomization matrix $\mathbf{R}$ is full rank with probability one, and the value of $\mathrm{E}\{\det^{-1}(\mathbf{R}\mathbf{R}^H)\}$ is finite,

Sirkeci-Mergen and Scaglione showed that the randomized DOSTBC scheme attains the maximum transmit diversity order N_t with the coherent ML detector for $N_s \geq N_t$. However, the performance of ML detection can be impaired if the information of $\mathbf{H}_{\mathrm{v}}$ at the receiver is mismatched [7].

In the paper, we consider a situation that the virtual channel matrix mismatch is caused by the problem that some of the cooperative relays unexpectedly disconnect from the network. For instance, suppose that a data frame contains 64 code blocks ($N = 64$). We assume that the virtual channel matrix $\mathbf{H}_{\mathrm{v}}$ has been trained well in the coherent receiver before the data transmission, and the coherent receiver assumes that the virtual channel matrix $\mathbf{H}_{\mathrm{v}}$ would remain static as the physical channel matrix $\mathbf{H}$ during the data transmission. Under this setting, if the first two relays disconnect from the network without notification after $p = 32$ (as shown in Figure 1), then for $p \geq 33$, the virtual channel matrix changes to

$$\widehat{\mathbf{H}}_{\mathrm{v}} = \left[\mathbf{r}_3, \ldots, \mathbf{r}_{N_s}\right] \begin{bmatrix} \mathbf{h}_3^T \\ \vdots \\ \mathbf{h}_{N_s}^T \end{bmatrix} \neq \mathbf{H}_{\mathrm{v}}. \tag{5}$$

Hence, the coherent ML receiver has a mismatched virtual channel matrix information for $p \geq 33$. In Figure 2, we show some simulation results (average bit-error-rate (BER)) of the *mismatched* coherent ML detector when there were respectively 2 and 4 relays disconnecting from the network after $p = 32$ (i.e., $\Delta N_s = 2$ and $\Delta N_s = 4$) (The simulation setting is described in Section 4). In comparison with the results with no disconnecting relays (i.e., $\Delta N_s = 0$), we can see from the figure that the performance of the mismatched coherent ML detector is much worse and exhibits an error floor for high signal-to-noise ratios (SNRs). It is worthwhile to notice that this performance degradation is much more severe than that due to imperfect channel estimation [7,8]. In the latter case, the mismatched coherent ML detector can still maintain the maximum transmit diversity order N_t [8].

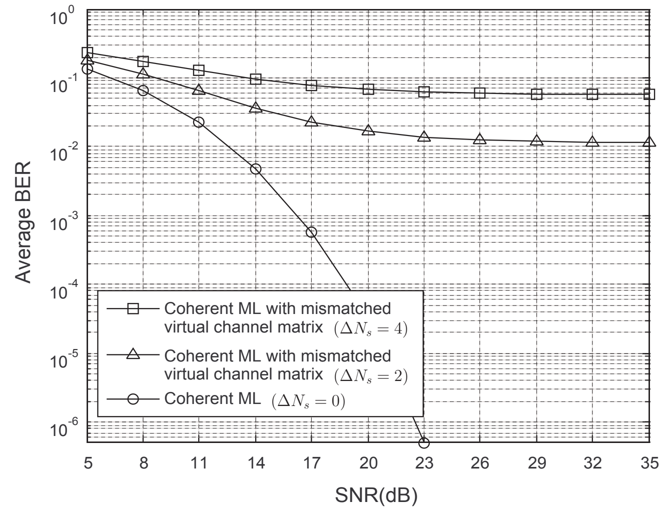

Figure 2. Average BER of the (mismatched) coherent ML detector for $N = 64$, $N_s = 9$ and $N_r = 2$ when the first two and four cooperative relays disconnect from the network for $p \geq 33$.

To alleviate the relay disconnection effects, one of the possible strategies is to allocate additional pilot codes among the 64 block codes for channel tracking. This strategy, however, leads to a spectrum efficiency loss. In the next section, we propose a different strategy based on recent developments on blind ML OSTBC detection [4,5].

3. BLIND ML DOSTBC DETECTION AND UNIQUE DATA IDENTIFIABILITY

In the section, we follow the recent results in [4] and [5] to present a blind ML DOSTBC detector and a randomized NIS-DOSTBC code scheme. We show that, with a mild assumption on the transmission protocol, the proposed blind ML detector can be robust against the relay disconnection problem as mentioned in Section 2.

3.1. Blind ML DOSTBC Detection

Before presenting the proposed blind ML DOSTBC detector, we consider the following assumption on the transmission protocol:

A2) The cooperative relay will not transmit signal if it cannot transmit a complete burst of P code blocks, where P is a small positive integer (e.g., $P = 4 \sim 8$).

Assumption **A2**) implies that the disconnection time of cooperative relays can only be a multiple of P blocks. In view of this, let us define

$$\boldsymbol{s}_m = [\mathbf{s}_{mP+1}^T, \ldots, \mathbf{s}_{(m+1)P}^T]^T \in \{\pm 1\}^{\bar{K}_m}, \tag{6}$$

where $\bar{K}_m = K_{mP+1} + \cdots + K_{(m+1)P}$, and rewrite (2) as the following model consisting of P code blocks:

$$\boldsymbol{\mathcal{Y}}_m \triangleq \begin{bmatrix} \mathbf{Y}_{mP+1} \\ \vdots \\ \mathbf{Y}_{(m+1)P,} \end{bmatrix} = \boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)\mathbf{H}_{\mathrm{v}} + \begin{bmatrix} \mathbf{W}_{mP+1} \\ \vdots \\ \mathbf{W}_{(m+1)P,} \end{bmatrix}, \tag{7}$$

for $m = 0, \ldots, M-1$, where $M = N/P$, and

$$\boldsymbol{\mathcal{G}}(\boldsymbol{s}_m) = \begin{bmatrix} \mathbf{C}_{mP+1}(\mathbf{s}_{mP+1})/\sqrt{K_{mP+1}} \\ \vdots \\ \mathbf{C}_{(m+1)P}(\mathbf{s}_{(m+1)P})/\sqrt{K_{(m+1)P}} \end{bmatrix}. \tag{8}$$

We note from **A2**) that the virtual channel matrix $\mathbf{H}_{\mathrm{v}}$ would remain fixed within $\boldsymbol{\mathcal{Y}}_m$ for each m, but can change from P blocks to the next P blocks if it happens that some of the relays disconnect from the network. We then aim to detect the unknown data $\boldsymbol{s}_m$ solely by $\boldsymbol{\mathcal{Y}}_m$ for each m, so that we can be free from the relay disconnection effects.

Among the existing blind (noncoherent) OSTBC detection methods [4,5,9–11], the blind ML detection method in [4,5] based on the deterministic blind ML criterion [12] has been shown to be particularly effective for situations with small to moderate number of P. For the DOSTBC signal model in (7), the blind ML DOSTBC detection problem is given by

$$\hat{\boldsymbol{s}}_m = \arg \min_{\boldsymbol{s}_m \in \{\pm 1\}^{\bar{K}_m}} \left\{ \min_{\mathbf{H}_{\mathrm{v}} \in \mathbb{C}^{N_t \times N_r}} \|\boldsymbol{\mathcal{Y}}_m - \boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)\mathbf{H}_{\mathrm{v}}\|_F^2 \right\} \tag{9}$$

for $m = 0, \ldots, M-1$, where the unknown data and channel are jointly detected and estimated, respectively. By exploiting the orthogonal property of OSTBCs (i.e., $\boldsymbol{\mathcal{G}}^H(\boldsymbol{s}_m)\boldsymbol{\mathcal{G}}(\boldsymbol{s}_m) = P\mathbf{I}_{N_t}$ where $\mathbf{I}_{N_t}$ is an N_t by N_t identity matrix), (9) can be further reformulated as

$$\hat{s}_m = \arg \max_{\boldsymbol{s}_m \in \{\pm 1\}^{\bar{K}_m}} \mathrm{tr}\left(\boldsymbol{\mathcal{Y}}_m^H \boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)\boldsymbol{\mathcal{G}}^H(\boldsymbol{s}_m)\boldsymbol{\mathcal{Y}}_m \right), \tag{10}$$

where $\mathrm{tr}(\cdot)$ denotes the matrix trace. While (10) is a difficult optimization problem for general space-time block codes, Ma *et. al* [4] have shown that for OSTBCs, (10) can be implemented by several efficient methods. Interested readers are referred to [4] for the details.

3.2. Unique Data Identifiability by NIS-DOSTBC

One of the important aspects of blind detection methods is to guarantee the transmitted data to be uniquely identifiable by the blind ML receiver. That is, in the noise-free case, the following equation

$$\boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)\mathbf{H}_{\mathrm{v}} = \boldsymbol{\mathcal{G}}(\boldsymbol{s}'_m)\mathbf{H}'_{\mathrm{v}} \tag{11}$$

must not hold for any $(\boldsymbol{s}'_m, \mathbf{H}'_{\mathrm{v}}) \neq \pm(\boldsymbol{s}_m, \mathbf{H}_{\mathrm{v}})$. Otherwise, $\boldsymbol{s}_m$ and $\mathbf{H}_{\mathrm{v}}$ cannot be uniquely identified (up to a sign) by the blind ML detector in (9), even in the noiseless case.

While the randomization matrix $\mathbf{R}$ might lead to a rank deficient virtual channel matrix $\mathbf{H}_{\mathrm{v}} = \mathbf{RH}$ when $N_s < N_t$ (even with $N_r \geq N_t$), unique identification of $\boldsymbol{s}_m$ and $\mathbf{H}_{\mathrm{v}}$ is still possible if $\boldsymbol{\mathcal{G}}(\cdot)$ is an non-intersecting subspace (NIS) code:

Definition 1 *The code $\boldsymbol{\mathcal{G}}(\cdot)$ in* (8) *is said to be an NIS code if*

$$\mathrm{Range}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)) \cap \mathrm{Range}(\mathcal{G}(\boldsymbol{s}'_m)) = \{\mathbf{0}\} \tag{12}$$

for any $\boldsymbol{s}_m, \boldsymbol{s}'_m \in \{\pm 1\}^{\bar{K}_m}$, $\boldsymbol{s}'_m \neq \pm \boldsymbol{s}_m$.

The properties and construction of NIS codes based on OSTBC have been investigated. In [5], it is shown that the use of NIS codes guarantees the so called perfect data identifiability which allows the data symbols $\boldsymbol{s}_m$ to be uniquely identifiable as long as the virtual channel $\mathbf{H}_{\mathrm{v}}$ is not a zero matrix. One shortcoming of the NIS code, however, is that it cannot achieve full transmission rate if the underlying OSTBC codes $\mathbf{C}_p(\cdot)$ are based on the generalized orthogonal design (GOD)[1]. Fortunately, Ma [5] has proposed a simple NIS code construction method which only has a loss of one bit for BPSK/QPSK NIS OSTBCs. To see this method, let us take the following 4×3 QPSK OSTBC ($T = 4,\ N_t = 3,\ K = 6$) as an example

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 + js_4 & 0 & s_2 + js_5 & -s_3 - js_6 \\ 0 & s_1 + js_4 & s_3 - js_6 & s_2 - js_5 \\ -s_2 + js_5 & -s_3 - js_6 & s_1 - js_4 & 0 \end{bmatrix}^T, \tag{13}$$

and also consider its one-bit-dropped version

$$\widetilde{\mathbf{C}}(\mathbf{s}) = \begin{bmatrix} s_1 + js_4 & 0 & s_2 + js_5 & -s_3 \\ 0 & s_1 + js_4 & s_3 & s_2 - js_5 \\ -s_2 + js_5 & -s_3 & s_1 - js_4 & 0 \end{bmatrix}^T \tag{14}$$

(where the bit s_6 has been removed). For the concatenated codeword $\boldsymbol{\mathcal{G}}(\cdot)$ in (8), we set

$$\mathbf{C}_{mP+1}(\cdot) \triangleq \widetilde{\mathbf{C}}(\cdot), \tag{15a}$$

$$\mathbf{C}_{mP+2}(\cdot) = \cdots = \mathbf{C}_{(m+1)P}(\cdot) \triangleq \mathbf{C}(\cdot). \tag{15b}$$

for $m = 0, \ldots, M-1$. Then, it can be shown [5] that $\boldsymbol{\mathcal{G}}(\cdot)$ is an NIS code. Hence we can have the following result:

Theorem 1 *Consider the blind ML DOSTBC decoder in* (9)*. The transmitted data vectors $\{\mathbf{s}_p\}_{p=1}^N$ are uniquely identifiable up to a sign if the transmitted DOSTBCs $\{\mathbf{C}_p(\cdot)\}_{p=1}^N$ are constructed according to* (15)*.*

One should notice from Theorem 1 that one bit in $\boldsymbol{s}_m$ is required to be known to the receiver (i.e., a pilot bit) to solve the inherent sign ambiguity problem. Without loss of generality, we can set the first bit of $\boldsymbol{s}_m$ as the pilot bit. The incurred data rate loss, however, is much less compared to that using pilot codes for channel tracking.

3.3. Full Blind Transmit Diversity

The presented randomized NIS-DOSTBC scheme in (15) can be shown to attain the maximum transmit diversity order N_t with the blind ML detector, the same as its coherent counterpart in [3]. Although similar results have been presented for the non-distributed OSTBC systems [5] and for the (distributed) unitary space-time modulation systems [13, 14], they are not directly applicable to the randomized DOSTBC scenario considered in the paper (due to the presence of the randomization matrix $\mathbf{R}$). In the Appendix, We provide such a proof for the considered randomized NIS-DOSTBC scheme in (15). The main result is summarized in the following theorem:

Theorem 2 *Assume that* **A1**) *holds and that the randomized NIS-DOSTBC scheme in* (15) *is employed. The blind ML DOSTBC detector* (10) *achieves the maximum transmit diversity order N_t for $N_s \geq N_t$.*

[1] Most of the existing OSTBCs are based on the GOD. The designs stipulate that each entry of the code matrix takes on either a symbol, its conjugate, or zero.

4. SIMULATION RESULTS

In the section, we present some simulation results to demonstrate the performance of the proposed blind ML decoder and the randomized NIS-DOSTBC scheme [in (15)]. We considered a relay network with 9 cooperative relays ($N_s = 9$), and set $N = 64$, $N_r = 2$ and $P = 8$. Both the physical channel matrix $\mathbf{H}$ and randomization matrix $\mathbf{R}$ were generated by i.i.d. complex Gaussian random variables with zero mean and unit variance. The 4 by 3 code in (13) was employed, and the blind ML detector in (10) was solved by a suboptimal method based on semidefinite relaxation [4]. The SNR was defined as

$$\mathrm{SNR} = \frac{\mathrm{E}\{\|\mathbf{C}(\boldsymbol{s}_p)\mathbf{R}\mathbf{H}\|_F^2\}/K_p}{\mathrm{E}\{\|\mathbf{W}\|_F^2\}},$$

and each simulation result was obtained from 10,000 trials. In the simulation, the first two relays were set disconnected from the network abruptly after $p = 32$ (as shown in Figure 1). The coherent ML detector did not have knowledge of this and used the virtual channel matrix information before the disconnection to detect all the symbols at all times. Since Shahbazpanahi's subspace method [9] and the differential OSTBC scheme [11] are applicable to the randomized DOSTBC systems, they were also simulated for comparison. It is worthwhile to mention that, for fairness of comparison, Shahbazpanahi's subspace method estimated the virtual channel matrix $\mathbf{H}_{\mathrm{v}}$ for every set of P consecutive block codes.

Figure 3 shows the average BER for the first 32 block codes, during which the virtual channel matrix $\mathbf{H}_{\mathrm{v}}$ was composed of the channel vectors of all the 9 cooperative relays, and the coherent ML detector has the perfect information of $\mathbf{H}_{\mathrm{v}}$. One can see from this figure that all the detectors under consideration perform consistently, and the proposed blind ML detector outperforms Shahbazpanahi's subspace method and the differential scheme. Figure 4 presents the average BER for the last 32 block codes, during which the virtual channel matrix $\mathbf{H}_{\mathrm{v}}$ was composed of the channel vectors of the 7 active cooperative relays, and thus the coherent ML detector has the mismatched channel information. Comparing this figure with Figure 3, the proposed blind ML detector and Shahbazpanahi's subspace method can still yield consistent BER performance since they performed detection in a P-block-by-P-block manner. By contrast, the differential scheme exhibits an error floor as the (mismatched) coherent ML detector since there are always errors in detecting $\mathbf{s}_{33}$ because $\mathbf{Y}_{32}$ is not an accurate channel estimate for $\mathbf{Y}_{33}$ for differential decoding.

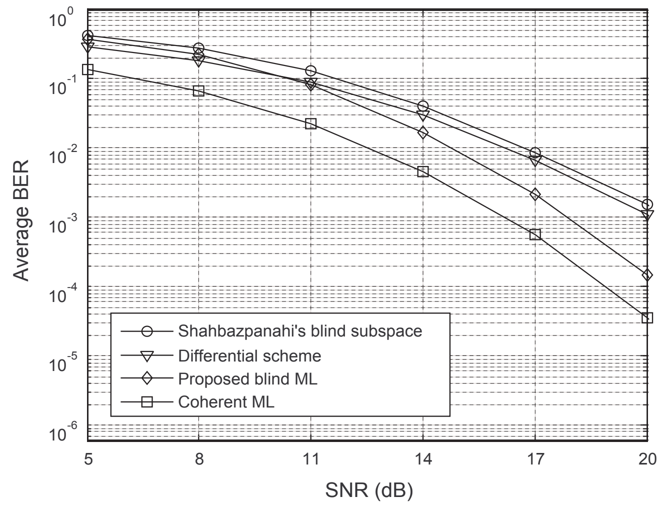

Figure 3. Performance comparison results with some existing methods when there are two relays disconnecting from the network for $p \geq 33$. The average BER was computed for $\mathbf{s}_1$ to $\mathbf{s}_{32}$.

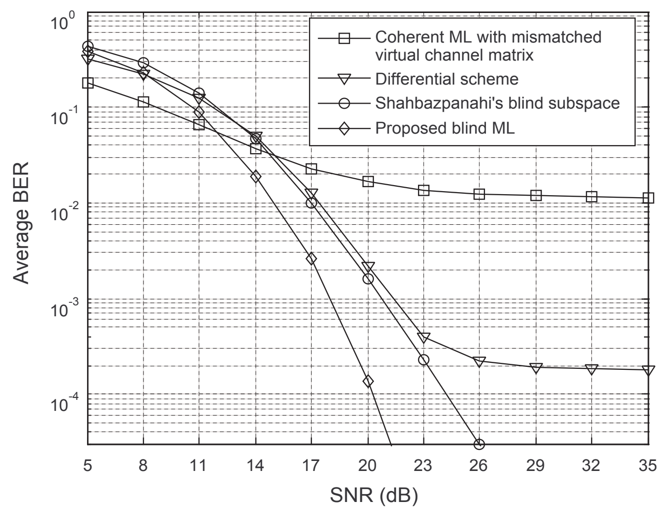

Figure 4. Performance comparison results with some existing methods when there are two relays disconnecting from the network for $p \geq 33$. The average BER was computed for $\mathbf{s}_{33}$ to $\mathbf{s}_{64}$.

5. CONCLUSIONS

In the paper, we have presented an NIS-DOSTBC scheme and a blind ML detector for the randomized DOSTBC system with achievable maximum transmit diversity. Under the transmission protocol **A2**), we have shown that the proposed blind ML detection scheme outperforms some existing methods, and is particularly effective for the situation where there are cooperative relays unexpectedly disconnecting from the network during the data frame transmission.

6. APPENDIX: PROOF OF THEOREM 2

For simplicity, let us assume that $N_r = 1$ and rewrite (7) as

$$\mathbf{y}_m = \boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)\mathbf{R}\mathbf{h} + \mathbf{w}_m. \tag{16}$$

We use $\boldsymbol{\mathcal{G}}_1$ and $\boldsymbol{\mathcal{G}}_2$ to represent the two distinct codewords $\boldsymbol{\mathcal{G}}(\boldsymbol{s}_m)$ and $\boldsymbol{\mathcal{G}}(\boldsymbol{s}'_m)$ ($\boldsymbol{s}_m \neq \pm\boldsymbol{s}'_m$), respectively. By (10), we show in the proof that the pair-wise error probability (PEP)

$$\mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \rightarrow \boldsymbol{\mathcal{G}}_2 | \, \boldsymbol{\mathcal{G}}_1) = \mathrm{P_b}\left((\frac{1}{P\sigma_w^2})(\mathbf{y}_m^H \boldsymbol{\mathcal{G}}_1 \boldsymbol{\mathcal{G}}_1^H \mathbf{y}_m - \mathbf{y}_m^H \boldsymbol{\mathcal{G}}_2 \boldsymbol{\mathcal{G}}_2^H \mathbf{y}_m) \leq 0 \right) \tag{17}$$

satisfies [15]

$$\lim_{\sigma_w^2 \rightarrow 0} \frac{-\log \mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \rightarrow \boldsymbol{\mathcal{G}}_2 | \, \boldsymbol{\mathcal{G}}_1)}{\log(1/\sigma_w^2)} \geq N_t, \tag{18}$$

therefore achieving the maximum transmit diversity order N_t.

Since $\mathbf{y}_m$ is complex Gaussian distributed with zero mean and covariance matrix equal to

$$\mathbf{C}_y = \boldsymbol{\mathcal{G}}_1 \mathbf{R}\mathbf{R}^H \boldsymbol{\mathcal{G}}_1^H + \sigma_w^2 \mathbf{I}_T, \tag{19}$$

one can show by Chernoff bound [16] that the PEP conditioned on $\mathbf{R}$ is upper bounded by

$$\begin{aligned}&\mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \to \boldsymbol{\mathcal{G}}_2 |\, \boldsymbol{\mathcal{G}}_1, \mathbf{R}) \\ &\leq \min_{0\leq\eta\leq\alpha} \det{}^{-1}\!\left(\mathbf{I}_T - \left(\frac{\eta}{P\sigma_w^2}\right)\begin{bmatrix}-\boldsymbol{\mathcal{G}}_1^H \\ \boldsymbol{\mathcal{G}}_2^H\end{bmatrix}\mathbf{C}_y\big[\boldsymbol{\mathcal{G}}_1\ \boldsymbol{\mathcal{G}}_2\big]\right), \qquad (20)\end{aligned}$$

where α is chosen such that the right-hand side (R.H.S.) of (20) for all $0 \leq \eta \leq \alpha$ is finite and bounded. By Schur complement and some tedious derivations, the R.H.S. of (20) can be further shown to be

$$\begin{aligned}\det{}^{-1}\!\Bigg(\mathbf{I}_{N_t} + \mathbf{V}\cdot\left(\frac{\eta P}{\sigma_w^2}\right)\mathrm{diag}\{\lambda_i - \eta(\lambda_i + \sigma_w^2/P)\} \\ \cdot \mathbf{V}^H(\mathbf{I}_{N_t} - (1/P^2)\boldsymbol{\mathcal{G}}_1^H\boldsymbol{\mathcal{G}}_2\boldsymbol{\mathcal{G}}_2^H\boldsymbol{\mathcal{G}}_1)\Bigg), \quad (21)\end{aligned}$$

where we have applied eigenvalue decomposition to

$$\mathbf{R}\mathbf{R}^H = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^H \qquad (22)$$

where $\mathbf{V} \in \mathbb{C}^{N_t\times N_t}$ is a unitary matrix and the diagonal matrix $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_{N_t}\} \succ 0$ (positive definite) under $\mathbf{A1}$). In (21), the matrix $\mathrm{diag}\{\lambda_i - \eta(\lambda_i + \sigma_w^2/P)\} \in \mathbb{R}^{N_t\times N_t}$ is a diagonal matrix with the ith diagonal element equal to $\lambda_i - \eta(\lambda_i + \sigma_w^2/P)$. By taking $\eta = 1/2$ and substituting (21) into (20), we have the upper bound for the conditioned PEP as

$$\begin{aligned}\mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \to \boldsymbol{\mathcal{G}}_2 |\, \boldsymbol{\mathcal{G}}_1, \mathbf{R}) \leq \det{}^{-1}\!\Bigg(\mathbf{I}_{N_t} + \left(\frac{P}{4\sigma_w^2}\right)\mathbf{V}\cdot \\ \mathrm{diag}\{\lambda_i - \sigma_w^2/P\}\cdot\mathbf{V}^H\cdot(\mathbf{I}_{N_t} - (1/P^2)\boldsymbol{\mathcal{G}}_1^H\boldsymbol{\mathcal{G}}_2\boldsymbol{\mathcal{G}}_2^H\boldsymbol{\mathcal{G}}_1)\Bigg). \quad (23)\end{aligned}$$

It is noticed from (23) that the matrix

$$\begin{aligned}\bar{\mathbf{R}}(\sigma_w^2) &\triangleq \mathbf{V}\cdot\mathrm{diag}\{\lambda_i - \sigma_w^2/p\}\cdot\mathbf{V}^H \\ &= \mathbf{R}\mathbf{R}^H - (\sigma_w^2/P)\mathbf{I}_{N_t} \qquad (24)\end{aligned}$$

is positive definite for $\sigma_w^2 < P\min\{\lambda_1, \ldots, \lambda_{N_t}\}$ (sufficiently high SNR). Besides, since $\boldsymbol{\mathcal{G}}_1$ and $\boldsymbol{\mathcal{G}}_2$ are NIS codes,

$$\mathbf{B} \triangleq \mathbf{I}_{N_t} - (1/P^2)\boldsymbol{\mathcal{G}}_1^H\boldsymbol{\mathcal{G}}_2\boldsymbol{\mathcal{G}}_2^H\boldsymbol{\mathcal{G}}_1 \qquad (25)$$

has full rank N_t [15]. By Theorem 7.6.3 in [17], we see that $\bar{\mathbf{R}}(\sigma_w^2)\mathbf{B}$ has N_t positive eigenvalues and therefore

$$\begin{aligned}\det\!\left(\mathbf{I}_{N_t} + \left(\frac{P}{4\sigma_w^2}\right)\bar{\mathbf{R}}(\sigma_w^2)\mathbf{B}\right) \geq \\ \left(\frac{P}{4\sigma_w^2}\right)^{N_t}\det\left(\bar{\mathbf{R}}(\sigma_w^2)\right)\cdot\det(\mathbf{B}). \qquad (26)\end{aligned}$$

Hence, the PEP $\mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \to \boldsymbol{\mathcal{G}}_2 |\, \boldsymbol{\mathcal{G}}_1)$ is upper bounded by

$$\begin{aligned}\mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \to \boldsymbol{\mathcal{G}}_2 |\, \boldsymbol{\mathcal{G}}_1) &= \mathrm{E}_{\mathbf{R}}\{\mathrm{P_b}(\boldsymbol{\mathcal{G}}_1 \to \boldsymbol{\mathcal{G}}_2 |\, \boldsymbol{\mathcal{G}}_1, \mathbf{R})\} \\ &\leq \left(\frac{P}{4\sigma_w^2}\right)^{-N_t}\mathrm{E}_{\mathbf{R}}\{\det{}^{-1}(\bar{\mathbf{R}}(\sigma_w^2))\}\cdot\det{}^{-1}(\mathbf{B}). \qquad (27)\end{aligned}$$

With (27), we can then conclude that (18) is true since

$$\lim_{\sigma_w^2\to 0}\mathrm{E}_{\mathbf{R}}\{\det{}^{-1}(\bar{\mathbf{R}}(\sigma_w^2))\} = \mathrm{E}_{\mathbf{R}}\{\det{}^{-1}(\mathbf{R}\mathbf{R}^H)\} \qquad (28)$$

which is finite according to $\mathbf{A1}$). ■

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