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A Linear Fractional Semidefinite Relaxed ML Approach to Blind Detection of 16-QAM Orthogonal Space-Time Block Codes

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***Abstract*— The blind maximum-likelihood (ML) detection of orthogonal space-time block codes (OSTBCs) is a computationally challenging optimization problem. Fortunately, for BPSK and QPSK OSTBCs, it has been shown that the blind ML detection problem can be efficiently and accurately approximated by a semidefinite relaxation (SDR) approach [1]. This paper considers the situation where the 16-QAM signals are employed. Due to the nonconstant modulus nature of 16-QAM signals, the associated blind ML OSTBC detection problem has its objective function exhibiting a Rayleigh quotient structure, which makes the SDR approach not directly applicable. In the paper, a linear fractional SDR (LF-SDR) approach is proposed for efficient approximation of the optimum blind ML solution. In this approach, the blind ML 16-QAM OSTBC detection problem is first approximated by a quasi-convex relaxation problem. Generally quasi-convex problems may be computationally more complex to handle than convex problems, but we show that the optimum solution of our quasi-convex problem can be efficiently obtained by solving a convex problem, namely a semidefinite program. Simulation results demonstrate that the proposed LF-SDR based blind ML detector outperforms the norm relaxed blind ML detector and the blind subspace channel estimator [2], especially in the one-receive-antenna scenario.**

I. Introduction

The blind or noncoherent detection techniques for orthogonal space-time block codes (OSTBCs) have drawn a lot of interests because, compared to other space-time codes, the OSTBCs have a much simpler receiver structure. For instance, the differential OSTBC scheme [3] only requires symbol-by-symbol maximum-likelihood (ML) detection at the receiver, although this scheme suffers from a 3 dB performance loss in signal-to-noise ratio (SNR) compared to the coherent ML detector. By assuming that the channel is static for a large number of code blocks, the Shahbazpanahi's blind subspace channel estimator [2] has a simple closed-form structure and can achieve a near-coherent performance. However, the long-time channel quiescence assumption may be violated if the channel coherence time is short. The blind ML detector [1], [4]–[7] has been shown to be able to provide near-coherent performance even for small to moderate numbers of code blocks (say, 8-20 code blocks). For BPSK/QPSK constellations, it has been shown [1] that the blind ML OSTBC detection problem can be simplified to a Boolean quadratic program (BQP). The BQP is NP-hard in general, but fortunately it can be efficiently and accurately approximated by a semidefinite relaxation (SDR) approach [1], [8]. This successful endeavor has motivated some works [5], [6], [9] that extend the advantages of the BQP problem simplification to the case of M-ary PSK (MPSK) OSTBCs.

In this paper we consider the blind OSTBC detection techniques for 16-QAM signaling. The noncoherent detection problems in this case can be quite different compared with their BPSK/QPSK and MPSK counterparts. For example, the differential scheme [3] may not be applied in the 16-QAM case due to the nonconstant modulus nature of the signals. For the blind ML approach, we will show in Section II that the associated detection problem can be formulated as a discrete optimization problem with a Rayleigh quotient objective function. This problem is much more difficult to handle than the BQP encountered in the BPSK/QPSK case: Not only the former has more complex objective and constraint structures, but the standard SDR approach used in the previous works [1], [5] appears to be not directly applicable. In Section III we will present a *linear fractional SDR (LF-SDR) approach* to efficient approximation of the 16-QAM blind ML problem. In this approach, we first apply an SDR idea similar to that for 16-QAM coherent MIMO detection [10]. However, unlike [10], we will be faced with a relaxation problem that is quasi-convex. Specifically, the quasi-convexity of the problem arises from its linear fractional objective. Though a quasi-convex problem can be optimally solved (say, using the bisection method [11]), it is generally argued that solving a quasi-convex problem would be more complex than solving a convex problem. We will show that the optimum solution of our quasi-convex problem can be obtained by simply solving a convex semidefinite program (SDP). A proof is also presented to show that the proposed relaxation approach is better than the simple norm relaxation method [1]. Some simulation results are presented in Section IV to demonstrate that the proposed

LF-SDR based blind ML detector outperforms the existing suboptimal methods.

II. Problem Statement

Consider an OSTBC system with N_t transmit antennas and N_r receive antennas. We assume that the channel is frequency flat and is static for a number of P code blocks. The respective signal model is given by

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{u}_p) + \mathbf{W}_p, \qquad p = 1, \ldots, P. \tag{1}$$

Here,

$\mathbf{Y}_p \in \mathbb{C}^{N_r \times T}$	received code matrix at block p, with T being the block length of the OSTBCs;
$\mathbf{u}_p \in \mathcal{U}^K$	transmitted symbol vector at block p, with $\mathcal{U} \subset \mathbb{C}$ being the symbol constellation set and K being the number of symbols per block;
$\mathbf{C} : \mathbb{C}^K \to \mathbb{C}^{N_t \times T}$	function that maps the given symbols to an orthogonal space-time code block;
$\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$	channel matrix;
$\mathbf{W}_p \in \mathbb{C}^{N_r \times T}$	additive white Gaussian noise matrix with the average power per entry given by σ_w^2.

An OSTBC mapping function $\mathbf{C}(\cdot)$ can always be expressed as [12]

$$\mathbf{C}(\mathbf{u}_p) = \sum_{k=1}^{K} \mathrm{Re}(u_{p,k})\mathbf{A}_k + j\sum_{k=1}^{K} \mathrm{Im}(u_{p,k})\mathbf{B}_k \tag{2}$$

for some basis matrices $\mathbf{A}_k \in \mathbb{R}^{N_t \times T}$ and $\mathbf{B}_k \in \mathbb{R}^{N_t \times T}$. More importantly, for any $\mathbf{u}_p \in \mathbb{C}^K$ OSTBCs satisfy the orthogonality condition

$$\mathbf{C}(\mathbf{u}_p)\mathbf{C}^H(\mathbf{u}_p) = \|\mathbf{u}_p\|^2 \mathbf{I}_{N_t} \tag{3}$$

where $\mathbf{I}_{N_t}$ is the $N_t \times N_t$ identity matrix.

Let us focus on the case of 16-QAM signaling, that is

$$\mathcal{U} = \mathcal{U}_{16\mathrm{qam}} := \{\ u = u_R + j\ u_I \mid u_R, u_I \in \{\pm 1, \pm 3\}\ \}.$$

Since a QAM symbol is composed of two independent pulse amplitude modulated (PAM) symbols, let us define

$$\begin{aligned}\mathbf{s}_p :&= [s_{p,1}, \ldots, s_{p,2K}]^T \\ &= [\mathrm{Re}(\mathbf{u}_p^T),\ \mathrm{Im}(\mathbf{u}_p^T)]^T \in \{\pm 1, \pm 3\}^{2K}\end{aligned}$$

as the real 4-PAM counterpart of $\mathbf{u}_p$. Then the OSTBC expression in (2) can be more conveniently represented by

$$\mathbf{C}(\mathbf{u}_p) = \mathbf{C}(\mathbf{s}_p) = \sum_{k=1}^{2K} \mathbf{X}_k s_{p,k}, \tag{4}$$

for some $\mathbf{X}_k \in \mathbb{C}^{N_t \times T}$ (depending on $\mathbf{A}_k$ and $\mathbf{B}_k$).

The problem of blind ML detection of $\mathbf{s} := [\mathbf{s}_1^T, \ldots, \mathbf{s}_P^T]^T \in \{\pm 1, \pm 3\}^{2PK}$ from $\mathbf{Y}_p$, $p = 1, \ldots, P$, can be written as the following optimization problem [7]

$$\{\mathbf{s}^\star, \mathbf{H}^\star\} = \arg \min_{\substack{\mathbf{s} \in \{\pm 1, \pm 3\}^{2PK} \\ \mathbf{H} \in \mathbb{C}^{N_r \times N_t}}} \sum_{p=1}^{P} \|\mathbf{Y}_p - \mathbf{H}\mathbf{C}(\mathbf{s}_p)\|^2. \tag{5}$$

By following the reformulation idea in [1][1], but assuming nonconstant modulus symbols, one can show that (5) can be simplified to a discrete Rayleigh quotient maximization problem

$$\mathbf{s}^\star = \arg \max_{\mathbf{s} \in \{\pm 1, \pm 3\}^{2PK}} \frac{\mathbf{s}^T\mathbf{F}\mathbf{s}}{\mathbf{s}^T\mathbf{s}}, \tag{6}$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{1,1} & \cdots & \mathbf{F}_{1,P} \\ \vdots & \ddots & \vdots \\ \mathbf{F}_{P,1} & \cdots & \mathbf{F}_{P,P} \end{bmatrix} \in \mathbb{R}^{2PK \times 2PK},$$
$$[\mathbf{F}_{p,q}]_{k,\ell} = \mathrm{Re}\{\mathrm{Tr}\{\mathbf{Y}_p \mathbf{X}_k^H \mathbf{X}_\ell \mathbf{Y}_q^H\}\},$$

in which $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix. Problem (6) can be solved by enumerating all possible points in the set $\{\pm 1, \pm 3\}^{2PK}$. In that case 2^{4PK} trials would be required, which is impractical for large values of $2PK$. One simple suboptimal approach for (6) is to relax the discrete set $\{\pm 1, \pm 3\}$ to the real space $\mathbb{R}$, leading to the norm relaxed blind ML problem [1]

$$f_{\mathrm{NR}} := \max_{\mathbf{s} \in \mathbb{R}^{2PK}} \frac{\mathbf{s}^T\mathbf{F}\mathbf{s}}{\mathbf{s}^T\mathbf{s}}, \tag{7}$$

of which the principal eigenvector of $\mathbf{F}$ is the optimum solution. An approximate solution of (6) can be obtained by quantizing the relaxation solution in the set $\{\pm 1, \pm 3\}^{2PK}$. Specifically, let $\boldsymbol{v}^\star \in \mathbb{R}^{2PK}$ denote the principal eigenvector of $\mathbf{F}$, and assume that $s_{1,1}$ is known to the receiver. Then an approximate solution of (6) by norm relaxation is given by

$$\hat{\mathbf{s}}_{\mathrm{NR}} = \sigma_{4\mathrm{PAM}}\left(\frac{s_{1,1}}{v_1^\star}\boldsymbol{v}^\star\right), \tag{8}$$

where $\sigma_{4\mathrm{PAM}} : \mathbb{R}^{2PK} \to \{\pm 1, \pm 3\}^{2PK}$ is a function in which the ith element of $\sigma_{4\mathrm{PAM}}(\boldsymbol{x})$ is the rounding of x_i to the set $\{\pm 1, \pm 3\}$.

We will show in our simulation results that this simple norm relaxation method is far from optimal when P is small to moderate (which is the case for short channel coherence time), and when only one receive antenna is available. To obtain a better approximate solution of problem (6), we present in the next section an LF-SDR approach.

III. Linear Fractional SDR Approach

This section presents the main results, namely the LF-SDR approach to approximation of the 16-QAM blind ML OSTBC detection problem.

A. Homogeneous Reformulation of the Blind ML Problem

In the 16-QAM blind ML OSTBC detection problem in (6), one can see that the optimal symbol decision suffers from ambiguity up to a scalar of $\{\pm 1, \pm 3\}$. To fix this problem,

[1]We should add that this reformulation idea is the key reason why blind ML OSTBC detection with constant modulus constellations can be effectively handled in the preceding studies [1], [5], [6].

we assume without loss of generality that $s_{1,1}$ is known to the receiver. Let us partition

$$\mathbf{F} = \begin{bmatrix} u & \mathbf{v}^T \\ \mathbf{v} & \mathbf{R} \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} s_{1,1} \\ \tilde{\boldsymbol{x}} \end{bmatrix}, \tag{9}$$

where $u \in \mathbb{R}$, $\mathbf{v} \in \mathbb{R}^{2PK-1}$, $\mathbf{R} \in \mathbb{R}^{(2PK-1)\times(2PK-1)}$ and $\tilde{\boldsymbol{x}} \in \{\pm 1, \pm 3\}^{2PK-1}$. With $s_{1,1}$ being known, the blind ML problem [in (6)] should be modified as

$$f_{\rm ML} := \max_{\tilde{\boldsymbol{x}} \in \{\pm 1, \pm 3\}^{2PK-1}} \frac{\tilde{\boldsymbol{x}}^T \mathbf{R} \tilde{\boldsymbol{x}} + 2(s_{1,1}\mathbf{v}^T)\tilde{\boldsymbol{x}} + s_{1,1}^2 u}{\tilde{\boldsymbol{x}}^T \tilde{\boldsymbol{x}} + s_{1,1}^2}. \tag{10}$$

We consider a homogeneous reformulation of (10) which is an essential procedure in applying SDR [1], [8], [10]. Let us introduce a new variable $t \in \{\pm 1\}$, and define $\tilde{\boldsymbol{x}} = t\boldsymbol{x}$ where $\boldsymbol{x} \in \{\pm 1, \pm 3\}^{2PK-1}$. Problem (10) is equivalent to

$$\max_{\substack{\boldsymbol{x} \in \{\pm 1, \pm 3\}^{2PK-1} \\ t \in \{\pm 1\}}} \frac{\boldsymbol{x}^T \mathbf{R} \boldsymbol{x} + 2t(s_{1,1}\mathbf{v}^T)\boldsymbol{x} + s_{1,1}^2 u}{\boldsymbol{x}^T \boldsymbol{x} + s_{1,1}^2}. \tag{11}$$

By further denoting $n = 2PK$, $\boldsymbol{y} = [\boldsymbol{x}^T, t]^T$, and

$$\mathbf{G} = \begin{bmatrix} \mathbf{R} & s_{1,1}\mathbf{v} \\ s_{1,1}\mathbf{v}^T & s_{1,1}^2 u \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{I}_{n-1} & \mathbf{0} \\ \mathbf{0}^T & s_{1,1}^2 \end{bmatrix}, \tag{12}$$

problem (11) can be homogenized as

$$\begin{aligned} &\max_{\boldsymbol{y} \in \mathbb{R}^n} \frac{\boldsymbol{y}^T \mathbf{G} \boldsymbol{y}}{\boldsymbol{y}^T \mathbf{D} \boldsymbol{y}} && \text{(13a)} \\ &\text{subject to (s.t.)} \ \ y_k \in \{\pm 1, \pm 3\}, && \text{(13b)} \\ &\qquad k = 1, \ldots, n-1, \\ &\qquad y_n \in \{\pm 1\}. && \text{(13c)} \end{aligned}$$

It is easy to see that if $\boldsymbol{y}^\star$ is a solution of (13) and it is partitioned as $\boldsymbol{y}^\star = [(\boldsymbol{x}^\star)^T, t^\star]^T$, then $t^\star \boldsymbol{x}^\star$ is a solution of (10).

B. Linear Fractional Semidefinite Relaxation

We can now introduce the LF-SDR approach for the approximation of (13). To illustrate this, we define $\mathbf{Y} = \boldsymbol{y}\boldsymbol{y}^T$ and rewrite (13) as

$$\begin{aligned} &\max_{\mathbf{Y} \in \mathbb{R}^{n \times n}} \frac{{\rm Tr}(\mathbf{G}\mathbf{Y})}{{\rm Tr}(\mathbf{D}\mathbf{Y})} && \text{(14a)} \\ &\text{s.t.} \ [\mathbf{Y}]_{k,k} \in \{1, 9\}, \ k = 1, \ldots, n-1, && \text{(14b)} \\ &\quad [\mathbf{Y}]_{n,n} = 1, && \text{(14c)} \\ &\quad \mathbf{Y} \succeq \mathbf{0} \ \ \text{(positive semidefinite)}, && \text{(14d)} \\ &\quad {\rm rank}(\mathbf{Y}) = 1, && \text{(14e)} \end{aligned}$$

where (14b) and (14c) are due to (13b) and (13c), respectively. By dropping the constraint ${\rm rank}(\mathbf{Y}) = 1$ and by relaxing the discrete set $\{1, 9\}$ to the interval $[1, 9]$, we end up with the following *linear fractional semidefinite relaxation (LF-SDR)*

$$\begin{aligned} \mathbf{Y}^\star = \arg &\max_{\mathbf{Y} \in \mathbb{R}^{n \times n}} \frac{{\rm Tr}(\mathbf{G}\mathbf{Y})}{{\rm Tr}(\mathbf{D}\mathbf{Y})} && \text{(15a)} \\ &\text{s.t.} \ 1 \le [\mathbf{Y}]_{k,k} \le 9, k = 1, \ldots, n-1, && \text{(15b)} \\ &\quad [\mathbf{Y}]_{n,n} = 1, && \text{(15c)} \\ &\quad \mathbf{Y} \succeq \mathbf{0}. && \text{(15d)} \end{aligned}$$

Note that the above relaxation idea is similar to those in [10], where the SDR method were developed for the 16-QAM coherent MIMO detection problem. In the latter, the relaxation problems are convex SDPs and can be efficiently solved by available interior point algorithms [13]. The problem we are dealing with is quite different. In essence, problem (15) is a quasi-convex problem. In general, this problem can be solved by the bisection method [11] in which a sequence of SDP feasibility problems need to be solved. Fortunately, we will show in the next subsection that problem (15) is equivalent to an SDP. In other words, we can obtain the optimum solution of (15) by solving one SDP, instead of solving many SDPs in the bisection treatment.

C. SDP Reformulation of LF-SDR, and Implications

In this subsection, we show that problem (15) is equivalent to an SDP. This investigation will ease the computational efforts for obtaining an optimum solution of (15). Consider the following SDP

$$\begin{aligned} \mathbf{Z}^\star = \arg &\max_{\mathbf{Z} \in \mathbb{R}^{n \times n}} \ {\rm Tr}(\mathbf{G}\mathbf{Z}) && \text{(16a)} \\ &\text{s.t.} \ {\rm Tr}(\mathbf{D}\mathbf{Z}) = 1, && \text{(16b)} \\ &\quad [\mathbf{Z}]_{n,n} \le [\mathbf{Z}]_{k,k} \le 9[\mathbf{Z}]_{n,n}, && \text{(16c)} \\ &\quad k = 1, \ldots, n-1, \\ &\quad \mathbf{Z} \succeq \mathbf{0}. && \text{(16d)} \end{aligned}$$

The following proposition states the equivalence between (15) and (16).

Proposition 1 *The linear fractional quasi-convex problem* (15) *has the same optimum objective value as the SDP in* (16). *Moreover, an optimum solution of* (15) *can be obtained through the relation* $\mathbf{Y}^\star = \mathbf{Z}^\star / [\mathbf{Z}^\star]_{n,n}$.

Proof: We first show that for any feasible $\mathbf{Z}$ of problem (16), $[\mathbf{Z}]_{n,n} \neq 0$. Suppose that $[\mathbf{Z}]_{n,n} = 0$. Then by (16c) and (16d), $\mathbf{Z} = \mathbf{0}$, which however violates (16b). Therefore, we can always define a point $\bar{\mathbf{Y}} = \mathbf{Z}/[\mathbf{Z}]_{n,n}$. It is easy to show that $\bar{\mathbf{Y}}$ is feasible for problem (15) and has the same objective value ${\rm Tr}(\mathbf{G}\bar{\mathbf{Y}})/{\rm Tr}(\mathbf{D}\bar{\mathbf{Y}}) = {\rm Tr}(\mathbf{G}\mathbf{Z})$. On the other hand, it can be seen from (12), (15c) and (15d) that for any feasible $\mathbf{Y}$ of problem (15), ${\rm Tr}(\mathbf{D}\mathbf{Y}) = \sum_{k=1}^{n-1} [\mathbf{Y}]_{k,k} + s_{1,1}^2 > 0$. Let $\bar{\mathbf{Z}} = \mathbf{Y}/{\rm Tr}(\mathbf{D}\mathbf{Y})$. Then it is also easy to show that $\bar{\mathbf{Z}}$ is feasible for problem (16) and has the same objective value ${\rm Tr}(\mathbf{G}\bar{\mathbf{Z}}) = {\rm Tr}(\mathbf{G}\mathbf{Y})/{\rm Tr}(\mathbf{D}\mathbf{Y})$. Therefore, we conclude that problems (15) and (16) are equivalent and $\mathbf{Y}^\star = \mathbf{Z}^\star / [\mathbf{Z}^\star]_{n,n}$. ■

Proposition 1 implies that the optimum solution $\mathbf{Y}^\star$ of (15) can be simply obtained by solving the SDP (16) in lieu of the bisection method. The SDP (16) can be solved in polynomial time using an interior-point method [13], with a worst-case complexity of $\mathcal{O}(n^{3.5})$.

From the above SDP reformulation, we can also prove that the proposed LF-SDR approach has a better approximation accuracy than the simple norm relaxation method in (7), as the following proposition states.

Proposition 2 *Let $f_{\rm LF-SDR} := {\rm Tr}(\mathbf{G}\mathbf{Z}^\star)$ be the optimum value of the SDR problem* (16)*, and recall that $f_{\rm ML}$ and $f_{\rm NR}$ are the optimum values of the original blind problem [in* (10)*] and the norm relaxation problem [in* (7)*], respectively. Then*

$$|f_{\rm ML} - f_{\rm LF-SDR}| \leq |f_{\rm ML} - f_{\rm NR}|.$$

Proof: The idea of this proof follows that for Theorem 1 in [1]. Since $f_{\rm LF-SDR} \geq f_{\rm ML}$ and $f_{\rm NR} \geq f_{\rm ML}$ (a basic result in relaxation), it suffices to show that $f_{\rm LF-SDR} \leq f_{\rm NR}$. Suppose that

$$\mathbf{Z}^\star = \begin{bmatrix} \mathbf{P} & \mathbf{q} \\ \mathbf{q}^T & r \end{bmatrix},$$

where $\mathbf{P} \in \mathbb{R}^{(n-1)\times(n-1)}$, $\mathbf{q} \in \mathbb{R}^{n-1}$ and $r \in \mathbb{R}$. Let

$$\tilde{\mathbf{Z}} = \begin{bmatrix} s_{1,1}^2 r & s_{1,1}\mathbf{q}^T \\ s_{1,1}\mathbf{q} & \mathbf{P} \end{bmatrix} \succeq \mathbf{0}.$$

Then one can readily show from (9), (12) and (16) that

$$\mathrm{Tr}(\mathbf{G}\mathbf{Z}^\star) = \mathrm{Tr}(\mathbf{F}\tilde{\mathbf{Z}}), \tag{17}$$

$$\mathrm{Tr}(\mathbf{D}\mathbf{Z}^\star) = \mathrm{Tr}(\tilde{\mathbf{Z}}) = 1. \tag{18}$$

Consider the eigenvalue decomposition of $\tilde{\mathbf{Z}} = \sum_{k=1}^n \lambda_k \mathbf{g}_k \mathbf{g}_k^T$, where $\lambda_k \geq 0$ is the kth eigenvalue of $\tilde{\mathbf{Z}}$, and $\mathbf{g}_k \in \mathbb{R}^n$ is the associated unit-norm eigenvector. Then

$$\mathrm{Tr}(\mathbf{F}\tilde{\mathbf{Z}}) = \sum_{k=1}^n \lambda_k \mathbf{g}_k^T \mathbf{F} \mathbf{g}_k \leq \sum_{k=1}^n \lambda_k \max_{\|\mathbf{g}\|_2=1} \mathbf{g}^T\mathbf{F}\mathbf{g} = \mathrm{Tr}(\tilde{\mathbf{Z}}) f_{\rm NR}. \tag{19}$$

By (17), (18) and (19), we obtain $f_{\rm LF-SDR} = \mathrm{Tr}(\mathbf{G}\mathbf{Z}^\star) \leq f_{\rm NR}$. ■

D. Gaussian Randomization

Since the optimum solution $\mathbf{Y}^\star$ of problem (15) does not necessarily have rank one, a feasible rank-1 approximation solution of problem (13) obtained from $\mathbf{Y}^\star$ is needed. One straightforward method is to compute the principal eigenvector of $\mathbf{Y}^\star$ (thereby performing rank-1 approximation), and then quantize the principal eigenvector in the set $\{\pm 1, \pm 3\}^{n-1} \times \{\pm 1\}$. Another method practically proven to be effective is the Gaussian randomization [10]. In this method, we first generate L random vectors $\boldsymbol{\xi}^{(\ell)} \in \mathbb{R}^n$, $\ell = 1, \ldots, L$, following the Gaussian distribution $\mathcal{N}(\mathbf{0}, \mathbf{Y}^\star)$ [i.e., $\boldsymbol{\xi}^{(\ell)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Y}^\star)$], then quantize $\boldsymbol{\xi}^{(\ell)}$ in the set $\{\pm 1, \pm 3\}^{n-1} \times \{\pm 1\}$. Denote by $\hat{\boldsymbol{y}}^{(\ell)} \in \{\pm 1, \pm 3\}^{n-1} \times \{\pm 1\}$ the quantized vector of $\boldsymbol{\xi}^{(\ell)}$; viz.

$$\hat{\boldsymbol{y}}^{(\ell)} = [\ \sigma_{\rm 4PAM}(\xi_1^{(\ell)}), \ldots, \sigma_{\rm 4PAM}(\xi_{n-1}^{(\ell)}), \mathrm{sgn}(\xi_n^{(\ell)})\ ]^T,$$

where $\mathrm{sgn} : \mathbb{R} \rightarrow \{\pm 1\}$ is the sign function. We pick the quantized vector that yields the largest objective value

$$\ell^\star = \arg \max_{\ell=1,\ldots,L} \frac{(\hat{\boldsymbol{y}}^{(\ell)})^T \mathbf{G} \hat{\boldsymbol{y}}^{(\ell)}}{(\hat{\boldsymbol{y}}^{(\ell)})^T \mathbf{D} \hat{\boldsymbol{y}}^{(\ell)}}.$$

and choose $\hat{\boldsymbol{y}}^{(\ell^\star)}$ as our approximate solution of problem (13). Typically, $L = 50 \sim 100$ suffices to obtain a good approximation performance.

IV. Simulation Results

In this section, we present some simulation results to demonstrate the efficacy of the proposed LF-SDR based blind ML detector. The detector performance was evaluated using symbol error rate (SER), and there were at least $10,000$ trials performed for each simulation result. The SDP in (16) was solved by a specialized interior point algorithm [14] that is particularly suitable for (16) and runs faster than the general-purpose SDP solver SeDuMi [15]. An approximate solution of problem (6) was obtained either by quantizing the principal eigenvector of $\mathbf{Y}^\star$ or by the Gaussian randomization procedure with $L = 50$ random vectors generated for each trial. The coefficients of $\mathbf{H}$ are zero-mean i.i.d. complex Gaussian distributed with variance equal to 1. The SNR used in the performance plots is the SNR per receive antenna, defined as

$$\mathrm{SNR} = \frac{\mathrm{E}\{\|\mathbf{H}\mathbf{C}(\mathbf{s})\|_F^2\}}{\mathrm{E}\{\|\mathbf{W}\|_F^2\}} = \frac{10N_t}{T\sigma_w^2}.$$

We compared the proposed detector with the norm relaxed blind ML detector (i.e., Eqn. (8)), the Shahbazpanahi's blind subspace channel estimator [2], the cyclic ML method [7] (initialized by the norm relaxed blind ML detector), and the coherent ML detector (which has perfect channel state information (CSI)). Note that for white Gaussian noises, the Shahbazpanahi's blind subspace channel estimator is equivalent to the norm relaxed blind ML detector [1], [2] from a theoretical viewpoint. However, the former employed a different method of using the pilot to fix the channel ambiguity (Please refer to [2] for the details). As a result, the two methods will be seen to exhibit different simulation performances.

Figure 1 shows the results for complex 3×4 OSTBC ($N_t = 3$, $T = 4$), $P = 8$ and (a) $N_r = 1$, (b) $N_r = 4$. One can see from Figure 1(a) that the Shahbazpanahi's subspace method, the norm relaxed blind ML detector, and the cyclic ML method cannot properly decode the transmitted OSTBCs. However, the proposed blind ML detector exhibits consistent SER performance. For the multiple-receive-antenna case as shown in Figure 1(b), the Shahbazpanahi's subspace method can properly identify the transmitted symbols (some theoretical reasoning for the significant performance difference of the subspace method in the one-receive-antenna and multi-receive-antenna cases has been provided in [16]). Nevertheless, one can see from Figure 1 that the proposed blind ML detector outperforms the subspace method as well as the norm relaxed blind ML detector. This result provides a numerical support for Proposition 2. One can also observe from Figure 1(b) that the performance difference between the LF-SDR based blind ML detector and the coherent ML detector is less than 1.5 dB. It implies that the proposed LF-SDR approach can provide reasonable approximation quality to the blind ML OSTBC detection problem (6). It is interesting to see that these results have the same trends as those for BPSK/QPSK OSTBCs studied in [1]. Figures 1 (a) and (b) also indicate that the Gaussian randomization procedure is a better approximation method than the principal eigenvector method.

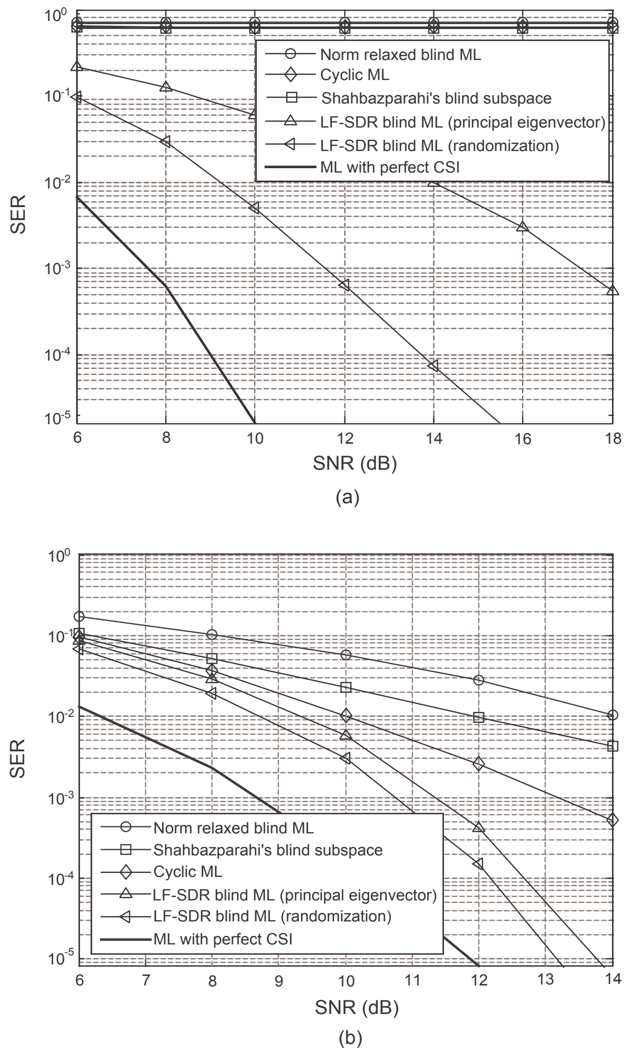

Figure 1. Performance (SER) of the proposed LF-SDR based blind ML detector for complex 3×4 OSTBC, $P = 8$ and (a) $N_r = 1$, (b) $N_r = 4$.

V. Conclusions

In the paper, we have presented a suboptimal LF-SDR approach for blind ML detection of 16-QAM OSTBCs. While the associated blind ML detection problem is a computationally difficult optimization problem, we have shown that the proposed approach can be efficiently implemented by solving an SDP followed by a simple randomization procedure. We have proved that the presented LF-SDR approach has a better approximation accuracy than the norm relaxation method, and our simulation results have further justified the efficacy of the proposed detector.

While the technique in [10] is adopted in this paper to relax the discrete set $\{1, 9\}$ in (14b) to the interval $[1, 9]$, we have noticed that there exist some other methods dealing with this discrete constraint in the context of coherent MIMO detection, e.g., [17] and [18]. Future work will consider the possibility of extending these techniques to the fractional quadratic case and comparing their approximation performances and complexities.

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