PERFORMANCE OF CUMULANT BASED INVERSE FILTER CRITERIA FOR BLIND DECONVOLUTION OF MULTI-INPUT MULTI-OUTPUT LINEAR TIME-INVARIANT SYSTEMS

Chong-Yung Chi and Chii-Horng Chen

Department of Electrical Engineering,
National Tsing Hua University, Hsinchu, Taiwan, R.O.C.
Tel: 886-3-5731156, Fax: 886-3-5751787, E-mail: cychi@ee.nthu.edu.tw

ABSTRACT

Tugnait, and Chi and Chen proposed multi-input multi-output inverse filter criteria (MIMO-IFC) using higher-order statistics for blind deconvolution of multi-input multi-output (MIMO) linear time-invariant (LTI) systems. This paper proposes a performance analysis for the MIMO linear equalizer associated with MIMO-IFC for finite SNR, including (P1) perfect phase equalization property, (P2) a relation to MIMO minimum mean square error (MIMO-MMSE) equalizer, and (P3) a connection with the one obtained by Yeung and Yau's MIMO super-exponential algorithm (MIMO-SEA) that usually converges fast but no guarantee of convergence for finite data. Furthermore, based on (P3), a MIMO-IFC based algorithm with performance similar to that of the MIMO-SEA and with guaranteed convergence is proposed. Finally, some simulation results are presented to support the analytic results and the proposed algorithm.

1. INTRODUCTION

Blind deconvolution of a multi-input multi-output (MIMO) linear time-invariant system, denoted $\mathbf{H}[n]$ ($P\times K$ matrix), is a problem of estimating the vector input $\mathbf{u}[n] = (u_1[n], ..., u_K[n])^T$ (K inputs) with only a set of non-Gaussian vector output measurements $\mathbf{x}[n] = (x_1[n], ..., x_P[n])^T$ (P outputs) as follows [1-3]

$$\mathbf{x}[n] = \sum_{k=-\infty}^{\infty} \mathbf{H}[k]\mathbf{u}[n-k] + \mathbf{w}[n] \tag{1}$$

where $\mathbf{w}[n]$ ($P \times 1$ vector) is additive noise. Blind deconvolution of MIMO systems in multiuser detection of wireless communications includes suppression of multiple access interference (MAI) and removal of multiple transmission paths that are crucial to the receiver design of multiuser communications systems.

Let $\mathbf{v}[n] = (v_1[n], ..., v_P[n])^T$ denote a linear FIR equalizer of length $L = L_2 - L_1 + 1$ for which $\mathbf{v}[n] \neq \mathbf{0}$ for $n = L_1, L_1+1, ..., L_2$. Let $\mathrm{cum}\{y_1, y_2, ..., y_p\}$ denote the pth-order cumulant of random variables y_1, y_2, ..., y_p and $\mathcal{F}\{\bullet\}$ denote discrete-time Fourier transform operator. For ease of later use, let us define the following notations

$$\begin{aligned}
\mathrm{cum}\{y:p, ...\} &= \mathrm{cum}\{y_1 = y, ..., y_p = y, ...\} \\
C_{p,q}\{y\} &= \mathrm{cum}\{y:p,\ y^*:q\} \\
\boldsymbol{v}_j &= (v_j[L_1], ..., v_j[L_2])^T \\
\boldsymbol{\nu} &= (\boldsymbol{v}_1^T, \boldsymbol{v}_2^T, ..., \boldsymbol{v}_P^T)^T \\
\boldsymbol{x}_j[n] &= (x_j[n-L_1], ..., x_j[n-L_2])^T \\
\boldsymbol{R}_{i,j} &= E[\boldsymbol{x}_i^*[n]\boldsymbol{x}_j^T[n]] \ (L \times L \text{ matrix}) \\
\widetilde{\boldsymbol{R}} &= \{\boldsymbol{R}_{i,j}\} \ (P \times P \text{ block matrix})
\end{aligned}$$

where y^* denotes the complex conjugate of y. Then the output $e[n]$ of the FIR equalizer $\mathbf{v}[n]$ can be expressed as

$$e[n] = \sum_{j=1}^{P} v_j[n] * x_j[n] = \sum_{j=1}^{P} \boldsymbol{v}_j^T \boldsymbol{x}_j[n] \tag{2}$$

$$= \sum_{j=1}^{K} s_j[n] * u_j[n] + w[n] \quad \text{by (1)} \tag{3}$$

where $w[n]$ is the noise term due to $\mathbf{w}[n]$ and

$$s_j[n] = \sum_{m=1}^{P} \sum_{l=L_1}^{L_2} v_m[l] h_{m,j}[n-l] \tag{4}$$

where $h_{m,j}[n]$ is the (m,j)th component of $\mathbf{H}[n]$. The designed linear equalizer is usually evaluated by the amount of intersymbol interference (ISI) defined as [3,4]

$$\mathrm{ISI}(e[n]) = \frac{\{\sum_{j,n} |s_j[n]|^2\} - \max_{j,n}\{|s_j[n]|^2\}}{\max_{j,n}\{|s_j[n]|^2\}} \tag{5}$$

Note that $\mathrm{ISI}(e[n]) = 0$ as $s_\ell[n] = \alpha\delta[n-\tau]$ and $s_j[n] = 0$ for $j \neq \ell$.

Single-input single-output inverse filter criteria (SISO-IFC) [4-6] using higher-order cumulants have been widely used for blind deconvolution and their performance analyses for finite SNR have been reported by Feng and Chi [5,6]. In this paper, we propose performance analyses for cumulant based multi-input multi-output inverse filter criteria (MIMO-IFC) [1,2]. Furthermore, based on the analytic results, a MIMO-IFC based algorithm with performance similar to that of Yeung and Yau's MIMO super-exponential algorithm (MIMO-SEA) [3] and with guaranteed convergence is proposed.

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2. REVIEW OF MIMO-IFC AND MIMO-SEA

Assume that we are given a set of measurements $\mathbf{x}[n]$, $n = 0, 1, ..., N-1$, modeled by (1) with the following assumptions:

(A1) $u_j[n]$ is zero-mean, independent identically distributed (i.i.d.) non-Guassian with variance $\sigma_{u_j}^2$ and $(p+q)$th-order cumulant $C_{p,q}\{u_j[n]\}$, and statistically independent of $u_k[n]$ for all $k \neq j$.

(A2) The MIMO system $\mathbf{H}[n]$ is exponentially stable.

(A3) The noise $\mathbf{w}[n]$ is zero-mean Gaussian and statistically independent of $\mathbf{u}[n]$.

Chi and Chen [2] find the optimum $\boldsymbol{\nu}$ by maximizing the following MIMO-IFC

$$J_{p,q}(\boldsymbol{\nu}) = \frac{|\mathrm{cum}\{e[n] : p, e^*[n] : q\}|}{[\mathrm{cum}\{e[n], e^*[n]\}]^{(p+q)/2}} \quad (6)$$

where p and q are nonnegative integers and $p+q \geq 3$ through using iterative optimization algorithms because all MIMO-IFC $J_{p,q}$ are a highly nonlinear function of $\boldsymbol{\nu}$. Note that the MIMO-IFC given by (6) include Tugnait's MIMO-IFC [1] for $(p,q) = (2,1)$ and $(p,q) = (2,2)$ as special cases.

The MIMO-SEA proposed by Yeung and Yau [3] iteratively updates $\boldsymbol{\nu}$ at the Ith iteration by solving the following linear equations

$$\widetilde{\boldsymbol{R}}\boldsymbol{\nu}_I = \frac{1}{\left\|\widetilde{\boldsymbol{R}}^{-1}\widetilde{\boldsymbol{d}}^{[I-1]}\right\|} \cdot \widetilde{\boldsymbol{d}}^{[I-1]} \quad (7)$$

where $\widetilde{\boldsymbol{d}}^{[I-1]} = (\boldsymbol{d}_1^T, \boldsymbol{d}_2^T, ..., \boldsymbol{d}_P^T)^T$ in which

$$\boldsymbol{d}_i = \mathrm{cum}\{e^{[I-1]}[n] : r, (e^{[I-1]}[n])^* : s-1, \boldsymbol{x}_i^*[n]\} \quad (8)$$

in which $r+s \geq 3$ and $e^{[I-1]}[n]$ is the equalizer output obtained at the $(I-1)$th iteration.

A known fact and two observations regarding MIMO-IFC and MIMO-SEA are as follows:

(F1) In the absence of noise (i.e., SNR $= \infty$), the optimum $e[n] = \alpha_\ell u_\ell[n-\tau_\ell]$ (*perfect equalization*) (i.e., ISI$(e[n]) = 0$) for both MIMO-IFC and MIMO-SEA as $L_1 \to -\infty$ and $L_2 \to \infty$ where $\ell \in \{1, 2, ..., K\}$ is unknown. For finite SNR and L, $\widehat{u}_\ell[n] = e[n]$ is an estimate of $u_\ell[n]$ up to a scale factor and a time delay, and $h_{i,\ell}[k]$ can also be estimated as

$$\widehat{h}_{i,\ell}[k] = \frac{E[x_i[n+k]\widehat{u}_\ell^*[n]]}{E[|\widehat{u}_\ell[n]|^2]}, \quad i = 1, 2, ..., P \quad (9)$$

(O1) The computationally efficient MIMO-SEA converges at a super-exponential rate for SNR $= \infty$ and sufficiently large N, but it may diverge for finite SNR and N.

(O2) With larger computational load than solving the linear equations given by (7), gradient type iterative MIMO-IFC algorithms (such as Fletcher-Powell algorithm [7]) always spend more iterations (lower convergence speed) than MIMO-SEA.

Estimates $\widehat{u}_1[n]$, $\widehat{u}_2[n]$, ..., $\widehat{u}_K[n]$ can be obtained by the MIMO-IFC or MIMO-SEA in a non-sequential order through a multistage successive cancellation (MSC) procedure [1] that includes the following two steps at each stage:

(S1) Find an input estimate, said $\widehat{u}_\ell[n]$ (where ℓ is unknown), and the associated channel estimates $\widehat{h}_{i,\ell}[n]$, $i = 1, 2, ..., P$ using MIMO-IFC or MIMO-SEA.

(S2) Update $x_i[n]$ by $x_i[n] - \widehat{u}_\ell[n] * \widehat{h}_{i,\ell}[n]$, $i = 1, 2, ..., P$.

3. PERFORMANCE ANALYSIS FOR MIMO-IFC

Prior to presenting analytical results for the performance of the FIR equalizer $\mathbf{v}[n]$ associated with MIMO-IFC, let us present the nonblind MIMO minimum mean square error (MIMO-MMSE) equalizer, denoted $\mathcal{V}_{\mathrm{MMSE}}(\omega)$ ($K \times P$ matrix), that has some relation to $\mathbf{v}[n]$. It can be shown by orthogonality principle [8] that

$$\mathcal{V}_{\mathrm{MMSE}}^T(\omega) = \left[\mathcal{R}^T(\omega)\right]^{-1} \cdot \mathcal{H}^*(\omega) \cdot \mathbf{S} \quad (10)$$

where $\mathcal{R}(\omega) = \mathcal{F}\{\mathbf{R}[k]\} = \mathcal{F}\{E[\mathbf{x}[n]\mathbf{x}^H[n-k]]\}$, $\mathcal{H}(\omega) = \mathcal{F}\{\mathbf{H}[n]\}$ and

$$\mathbf{S} = \mathrm{diag}\{\sigma_{u_1}^2, ..., \sigma_{u_K}^2\}. \quad (11)$$

Some analytical results regarding the optimum $\mathbf{v}[n]$ for finite SNR are summarized as follows:

Property 1. *The optimum overall impulse response $s_j[n]$ given by (4), $j = 1, ..., K$, are linear phase for finite L, i.e., their phase responses are given by*

$$\arg[S_j(\omega)] = \omega\tau_j + \xi_j, \quad \forall\omega \in [-\pi, \pi) \quad (12)$$

where $S_j(\omega) = \mathcal{F}\{s_j[n]\}$, τ_j and ξ_j are real constants. □

Property 2. *The optimum $\boldsymbol{V}(\omega) = \mathcal{F}\{\mathbf{v}[n]\}$ for $L_1 \to -\infty$ and $L_2 \to \infty$ is related to $\mathcal{V}_{\mathrm{MMSE}}(\omega)$ by*

$$\boldsymbol{V}(\omega) = \mathcal{V}_{\mathrm{MMSE}}^T(\omega) \cdot \left(\alpha_{p,q}\widetilde{\mathbf{S}}_{p,q}\boldsymbol{D}_{p,q}(\omega) + \alpha_{q,p}\widetilde{\mathbf{S}}_{q,p}\boldsymbol{D}_{q,p}(\omega)\right) \quad (13)$$

where

$$\alpha_{p,q} = \frac{p \cdot C_{1,1}\{e[n]\}}{(p+q) \cdot C_{q,p}\{e[n]\}}, \quad (14)$$

$$\widetilde{\mathbf{S}}_{p,q} = \mathrm{diag}\{C_{q,p}\{u_1[n]\}/\sigma_{u_1}^2, ..., C_{q,p}\{u_K[n]\}/\sigma_{u_K}^2\} \quad (15)$$

and

$$\boldsymbol{D}_{p,q}(\omega) = [D_1(\omega), ..., D_K(\omega)]^T \quad (16)$$

in which

$$D_j(\omega) = \mathcal{F}\{s_j^q[n](s_j^*[n])^{p-1}\}. \quad (17)$$

□

Property 3. *The optimum $\mathbf{v}[n]$ and the one obtained by the MIMO-SEA are the same for $p = q = r = s \geq 2$ and finite L.* □

Furthermore, based on Property 3 and the observations (O1) and (O2), a fast iterative algorithm is proposed for finding the optimum $\mathbf{v}[n]$ associated with MIMO-IFC for $p = q$ as follows:

Algorithm 1. Given $\boldsymbol{\nu}_{I-1}$ and $e^{[I-1]}[n]$ obtained at the $(I-1)$th iteration, $\boldsymbol{\nu}_I$ at the Ith iteration is obtained by the following two steps.

(T1) As the MIMO-SEA, obtain $\boldsymbol{\nu}_I$ by solving (7) with $r = s = p = q$ and obtain the associated $e^{[I]}[n]$.

(T2) If $J_{p,p}(\boldsymbol{\nu}_I) > J_{p,p}(\boldsymbol{\nu}_{I-1})$, go to the next iteration, otherwise update $\boldsymbol{\nu}_I$ through a gradient type optimization algorithm and obtain the associated $e^{[I]}[n]$.

It can be easily shown that

$$\left.\frac{\partial J_{p,p}(\boldsymbol{\nu})}{\partial \boldsymbol{\nu}}\right|_{\boldsymbol{\nu}=\boldsymbol{\nu}_{I-1}} \propto \frac{1}{C_{p,p}\{e^{[I-1]}[n]\}} \cdot \left(\widetilde{\boldsymbol{d}}^{[I-1]}\right)^* - \frac{1}{C_{1,1}\{e^{[I-1]}[n]\}} \cdot \left(\widetilde{\boldsymbol{R}}\boldsymbol{\nu}_{I-1}\right)^* \quad (18)$$

where $\widetilde{\boldsymbol{d}}^{[I-1]}$ has been obtained in (T1) (see (7)) and $\widetilde{\boldsymbol{R}}$ is the same at each iteration, indicating simple and straightforward computation for obtaining $\partial J_{p,p}(\boldsymbol{\nu})/\partial \boldsymbol{\nu}$ in (T2). Let us conclude this section with the following remark:

(R1) Algorithm 1 performs as a fast gradient type MIMO-IFC algorithm with convergence speed, computational load, and ISI similar to those of MIMO-SEA (due to the step (T1)) and with guaranteed convergence (due to the step (T2)).

4. SIMULATION RESULTS

A two-input two-output system taken from [1] was considered with the two inputs $u_1[n]$ and $u_2[n]$ assumed to be equally probable binary random sequences of $\{+1, -1\}$. The synthetic data $\mathbf{x}[n]$ for $N = 900$ and SNR $= 15$ dB (spatially independent and temporally white Gaussian noise) were processed by the inverse filter $\mathbf{v}[n]$ of length $L = 30$ ($L_1 = 0$ and $L_2 = 29$) associated with MIMO-IFC using the iterative Fletcher-Powell algorithm [7], MIMO-SEA and Algorithm 1, respectively, with $p = q = r = s = 2$. The initial condition associated with $\boldsymbol{\nu}_0$ was $v_1[n] = v_2[n] = \delta[n-14]$ for the first stage and $v_1[n] = \delta[n-14]$ and $v_2[n] = 0$ for the second stage of the MSC procedure.

Thirty independent realizations of the optimum $s_1[n]$ (associated with $u_1[n]$) and the associated thirty ISI versus iteration number obtained at the first stage of the MSC procedure are shown in Figures 1(a) through 1(f) using the three algorithms, respectively. One can see, from Figure 1, that the resultant $s_1[n]$'s are linear phase and they are similar for the three algorithms thus verifying Properties 1 and 3, while the convergence speed for the proposed Algorithm 1 is basically the same as that of MIMO-SEA and faster than the MIMO-IFC using Fletcher-Powell algorithm, thus verifying (O2). The corresponding results for $s_2[n]$ and ISI obtained at the second stage of the MSC procedure are shown in Figures 2(a) through 2(f). These results also support Properties 1 and 3, and (O2), but the MIMO-SEA failed to converge in one realization (see Figure 2(d)) and the associated $s_2[n]$ failed to approximate a delta function (see Figure 2(c)) thus verifying (O1). Algorithm 1 outperforms the other two algorithms in the second stage of the MSC procedure because the former converges as fast as the MIMO-SEA in all the thirty realizations (without any divergence) and converges faster than MIMO-IFC using the Fletcher-Powell algorithm.

5. CONCLUSIONS

We have presented a performance analysis for the MIMO linear equalizer $\mathbf{v}[n]$ associated with Chi and Chen's MIMO-IFC for finite SNR, including perfect phase equalization, a relation to the nonblind MIMO-MMSE equalizer, and equivalence to the one associated with MIMO-SEA for $p = q = r = s$, as presented in Properties 1, 2 and 3 respectively. Based on Property 3, a MIMO-IFC based algorithm, Algorithm 1, was presented that performs as the MIMO-SEA (in terms of ISI, computational load and convergence speed) with guaranteed convergence (see (R1)) while the latter may not converge for finite SNR and data (see (O1)). Some simulation results were also presented that support the proposed analytical results and Algorithm 1. The application of MIMO-IFC to multiuser detection of CDMA systems using Algorithm 1 is under study.

6. REFERENCES

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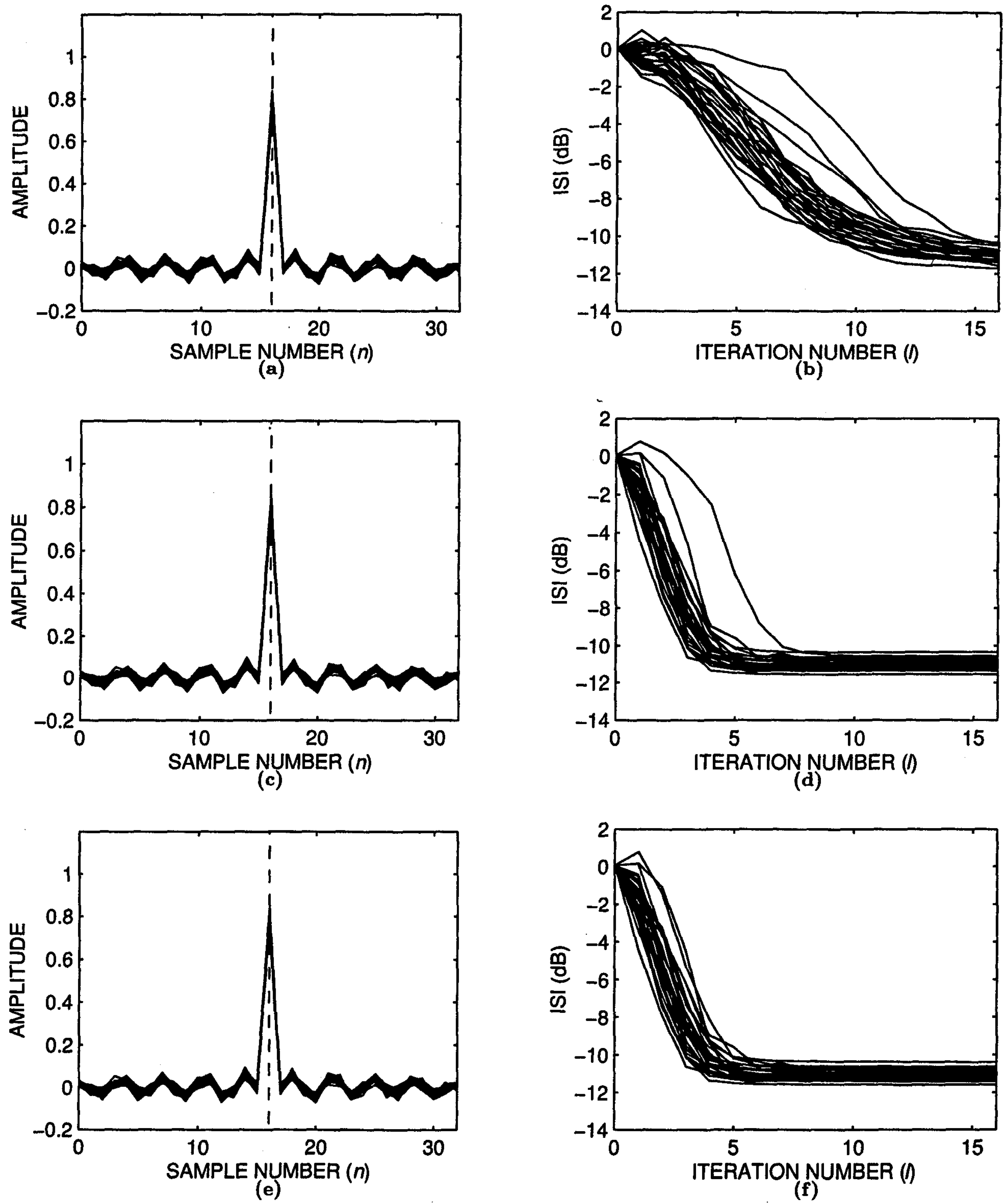

ig. 1. Thirty simulation results of $s_1[n]$ and ISI versus iteration number I at the first stage of the MSC procedure. (a) $_1[n]$ and (b) ISI associated with MIMO-IFC for $p = q = 2$ using Fletcher-Powell Algorithm, (c) $s_1[n]$ and (d) ISI associated ith MIMO-SEA for $r = s = 2$, and (e) $s_1[n]$ and (f) ISI associated with Algorithm 1 for $p = q = r = s = 2$.

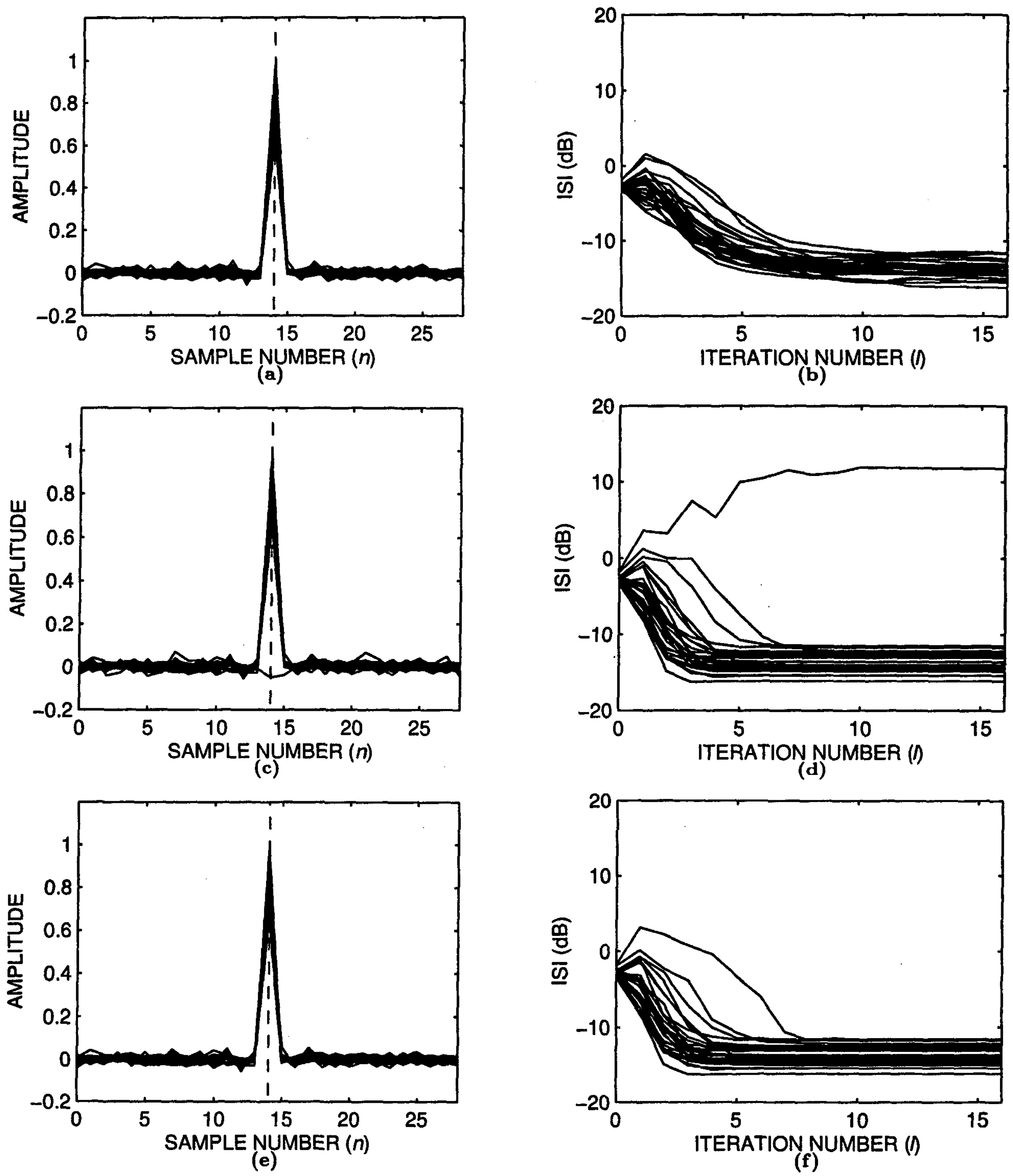

Fig. 2. Thirty simulation results of $s_2[n]$ and ISI versus iteration number I at the second stage of the MSC procedure. (a) $s_2[n]$ and (b) ISI associated with MIMO-IFC for $p = q = 2$ using Fletcher-Powell Algorithm, (c) $s_2[n]$ and (d) ISI associated with MIMO-SEA for $r = s = 2$, and (e) $s_2[n]$ and (f) ISI associated with Algorithm 1 for $p = q = r = s = 2$.