PERFORMANCE OF A CLASS OF BLIND DECONVOLUTION AND EQUALIZATION CRITERIA USING CUMULANT BASED INVERSE FILTERS

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ABSTRACT

Chi and Wu proposed a unified class of inverse filter criteria $J_{r,m}$ using an rth-order and an mth-order cumulants (where r is even and $m > r \geq 2$) which includes Wiggins' criterion, Shalvi and Weinstein's criterion and Tugnait's criteria as special cases, for blind deconvolution and equalization with only non-Gaussian output measurements of a nonminimum phase linear time-invariant (LTI) system (channel) $h(n)$. In this paper, we theoretically prove that for finite SNR, as Mendel's (nonblind) minimum-variance deconvolution (MVD) filter, the optimum inverse filter $v(n)$ associated with the criteria $J_{2,m}$ is a perfect phase equalizer but not a perfect amplitude equalizer, and the latter approaches the former as either m or SNR is increased or as the system $h(n)$ has wider bandwidth. For the other $J_{r,m}$ ($r \geq 4$), perfect equalization can be attained at the expense of SNR degradation. Finally, some simulation results are provided to support the proposed analytic results.

1. INTRODUCTION

Blind deconvolution and equalization is a signal processing procedure to estimate the input signal $u(n)$ which is distorted by a linear time-invariant (LTI) system (channel) $h(n)$ with only noisy output measurements of the system $h(n)$, *i.e.*,

$$x(n) = u(n) * h(n) + w(n) = \sum_{k=-\infty}^{\infty} h(k)u(n-k) + w(n) \tag{1}$$

where $w(n)$ is measurement noise. Chi and Wu [1] proposed a unified class of inverse filter criteria to estimate $u(n)$ with only a given set of non-Gaussian measurements $x(n)$. They estimate the optimum inverse filter estimate of $h(n)$, denoted $v(n)$, by maximizing the following objective function:

$$J_{r,m}(v(n)) = \frac{|C_m|^r}{|C_r|^m} \tag{2}$$

where r is even, $m > r \geq 2$, and C_r and C_m denote the rth-order and mth-order cumulants of the deconvolved signal

$$e(n) = x(n) * v(n) = u(n) * g(n) + w'(n) \tag{3}$$

respectively, in which $w'(n) = w(n) * v(n)$ corresponds to additive noise in $e(n)$ and

$$g(n) = h(n) * v(n) \tag{4}$$

is the overall system (channel) after equalization (deconvolution). The unified class of inverse filter criteria $J_{r,m}$ given by (2) includes Wiggins' criterion [2] (associated with $J_{2,4}$), Shalvi and Weinstein's criterion [3] (also associated with $J_{2,4}$), and Tugnait's inverse filter criteria $J_{2,3}$, $J_{2,4}$ and $J_{4,6}$ [4] as special cases. Assuming that the stable inverse filter of $h(n)$ exists, it has been proven in [1] that the optimum $g(n) = \alpha\delta(n-\tau)$ either when $r = 2$ and signal-to-noise ratio (SNR) equal to infinity or when $r > 2$, where $\alpha \neq 0$ is an unknown scale factor and τ is an unknown time delay. When SNR is finite, the inverse filter $v(n)$ associated with $J_{2,m}$ no longer leads $g(n)$ to a delta function. In general, the better $g(n)$ approximates to a delta function, the smaller is the amount of intersymbol interference (after equalization) defined as

$$\text{ISI}(g(n)) = \frac{\sum_n g^2(n) - max\{g^2(n)\}}{max\{g^2(n)\}} \tag{5}$$

Note that $\text{ISI}(\alpha g(n-\tau)) = \text{ISI}(g(n))$ and $\text{ISI}(\delta(n)) = 0$.

This paper proposes a performance analysis for the inverse filters associated with the unified class of inverse filter criteria $J_{r,m}$ when SNR is finite. This analysis includes the connection of these inverse filters for $r = 2$

This work is supported by the National Science Council under Grants NSC 85-2213-E007-012 and NSC 86-2213-E007-037.

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with the well-known Mendel's minimum-variance deconvolution (MVD) filter [5] (a nonblind equalizer) in terms of cumulant order m, SNR, the bandwidth of the system $h(n)$, and ISI$(g(n))$. Besides, the SNR degradation in $e(n)$ (after equalization) for the other $J_{r,m}$ ($r \geq 4$) is also presented followed by some simulation results.

2. MAJOR ANALYTIC RESULTS

Assume we are given a set of measurements $x(n)$ given by (1) where measurement noise $w(n)$ is white Gaussian with zero mean and variance σ_w^2, and the driving input $u(n)$ is zero-mean, independent identically distributed (i.i.d.) non-Gaussian with variance σ_u^2.

Let $H(\omega)$ and $G(\omega)$ denote the frequency responses of the system $h(n)$ and the overall system $g(n)$, respectively. Chi and Mendel [5] proved that the optimum $G(\omega)$ associated with the MVD filter, denoted $V_{\mathrm{MVD}}(\omega)$, is given by

$$G_{\mathrm{MVD}}(\omega) = H(\omega) \cdot V_{\mathrm{MVD}}(\omega) = \frac{(\sigma_u^2/\sigma_w^2)|H(\omega)|^2}{(\sigma_u^2/\sigma_w^2)|H(\omega)|^2 + 1} \quad (6)$$

which is zero phase with $H(\omega)$ and the ratio σ_u^2/σ_w^2 given *a priori* for the (nonblind) MVD filter. In other words, the MVD filter is a perfect phase equalizer, but not a perfect amplitude equalizer to the system $h(n)$.

2.1. Performance of $J_{2,m}$ ($r = 2$)

Four properties are proposed below regarding the inverse filter criteria $J_{2,m}$ (where $m > 2$) which use a second-order cumulant (correlation) and a higher-order cumulant.

Property 1. The optimum overall system $G(\omega)$ is a system with linear phase, *i.e.*,

$$\arg\{G(\omega)\} = -\omega\tau \quad (7)$$

which indicates that as the MVD filter, the optimum inverse filter $v(n)$ is also a perfect phase equalizer (except for an unknown time delay). □

Property 2. The optimum $G(\omega)$ satisfies

$$G(\omega) = \beta \cdot \underbrace{\{G(\omega) * G(\omega) \cdots * G(\omega)\}}_{m-1 \text{ terms}} \cdot G_{\mathrm{MVD}}(\omega) \quad (8)$$

or

$$g(n) = \beta \cdot \{g^{m-1}(n) * g_{\mathrm{MVD}}(n)\} \quad (9)$$

where β is an unknown constant and $g_{\mathrm{MVD}}(n)$ is the inverse Fourier transform of $G_{\mathrm{MVD}}(\omega)$. □

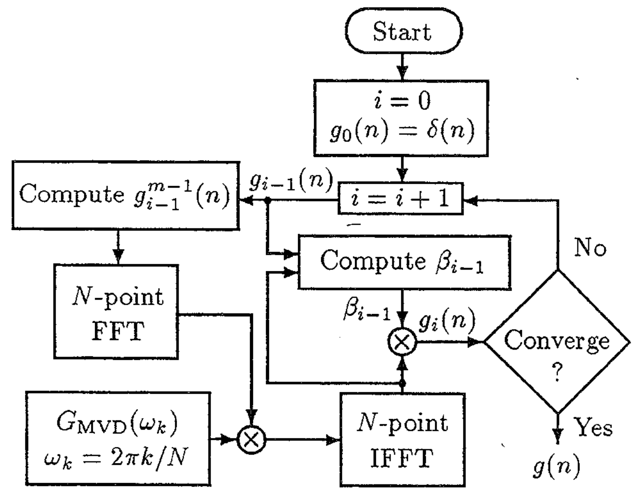

Figure 1. An iterative algorithm for obtaining the optimum $g(n)$ associated with $J_{2,m}$.

Note that (9) is a highly nonlinear equation of $g(n)$. The optimum $g(n)$ can be obtained using the iterative algorithm as shown in Figure 1. At the ith iteration, $g(n)$ is updated by (see (9))

$$g_i(n) = \beta_{i-1} \cdot \{g_{i-1}^{m-1}(n) * g_{\mathrm{MVD}}(n)\} \quad (10)$$

where the value for the scale factor β_{i-1} is chosen such that $\sum_n g_i(n) g_{i-1}(n) = \sum_n g_{i-1}^2(n)$.

Property 3. The optimum $G(\omega)$ approaches $G_{\mathrm{MVD}}(\omega)$ as either m or SNR increases or as the system $h(n)$ has wider bandwidth. As the MVD filter, the optimum $v(n)$ is a perfect amplitude equalizer only when SNR $= \infty$ or when $h(n)$ is an allpass system. □

Property 4. Either the larger SNR or the wider the bandwidth of the system $h(n)$, the smaller are the values of both ISI$(g(n))$ and ISI$(g_{\mathrm{MVD}}(n))$. Moreover, ISI$(g(n))$ is either close to or larger than ISI$(g_{\mathrm{MVD}}(n))$. □

2.2. Performance of $J_{r,m}$ for $r > 2$

For the inverse filter criteria $J_{r,m}$ where r is even and $m > r \geq 4$, as shown in [1] the optimum inverse filter $v(n)$ is a perfect equalizer for finite SNR. Let ρ denote the ratio of SNR associated with $e(n)$ given by (3) to that associated with $x(n)$ given by (1), *i.e.*,

$$\rho = \frac{E\{[e(n) - w'(n)]^2\}/E\{w'(n)^2\}}{E\{[x(n) - w(n)]^2\}/E\{w^2(n)\}} \quad (11)$$

Property 5. It can be shown that

$$\rho = \frac{1}{\frac{1}{2\pi}\int_{-\pi}^{\pi}|H(\omega)|^2 d\omega \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{|H(\omega)|^2}d\omega} \leq 1 \quad (12)$$

In other words, SNR always decreases in the deconvolved signal $e(n)$ except when $h(n)$ is an allpass system, although the optimum $v(n)$ is a perfect equalizer. □

3. SIMULATION RESULTS

In the simulation, a nonminimum phase broadband system (taken from [6]) and a minimum phase narrowband system (taken from [5]) whose impulse responses $h(n)$'s are shown in Figure 2(a) (dotted line for the former and solid line for the latter), were used to demonstrate the proposed performance analysis for the inverse filter criteria $J_{2,m}$.

The iterative algorithm shown in Figure 1 was used to obtain the overall system $g(n)$. For the broadband system, Figure 2(b) shows the frequency response $G(\omega)$ of the overall system $g(n)$ associated with $J_{2,3}$ (short-dash line and short-dash dot line for SNR = 5 dB and 0 dB, respectively) and the associated $G_{\mathrm{MVD}}(\omega)$ (long-dash line and long-dash dot line for SNR = 5 dB and 0 dB, respectively) together with the amplitude response $|H(\omega)|$ of the broadband system (solid line), where linear phase terms and scale factors were artificially removed. The corresponding results associated with $J_{2,4}$ are shown in Figure 2(c). Note that in Figure 2(c) the short-dash line and the long-dash line almost overlap each other. For the narrowband system, the results corresponding to those shown in Figures 2(b) and (2c) are depicted in Figures 2(d) and 2(e), respectively, for SNR = 40 dB and 20 dB instead. One can see, from Figures 2(b) through 2(e), that both $G(\omega)$'s of the broadband and narrowband systems are better approximations to the associated $G_{\mathrm{MVD}}(\omega)$ for either larger m or higher SNR. These results justify Property 3. Furthermore, Figure 2(a) also shows the obtained $g(n)$ associated with $J_{2,4}$ for SNR = 0 dB (dash-dotted line for the broadband system and dashed line for the narrowband system). One can see, from this figure, that both $g(n)$'s are zero phase which justifies Property 1, and that $g(n)$ is a better approximation to a delta function for the broad system which justifies Property 4.

Table 1 shows the obtained ISI($g(n)$)'s associated with $J_{2,3}$ and $J_{2,4}$ and ISI($g_{\mathrm{MVD}}(n)$) (in dB) (after equalization) together with ISI($h(n)$) (before equalization) for the broadband system. Table 2 shows the results corresponding to those shown in Table 1 for the narrowband system. One can see, from Tables 1 and 2, that both ISI($h(n)$)'s (before equalization) are significantly reduced by all the inverse filters. Furthermore, the ISI($g(n)$)'s and ISI($g_{\mathrm{MVD}}(n)$) shown in these tables decrease as SNR increases, and the former are either close to or larger than the latter. Also note that those shown in Table 1 for SNR = 0 dB are close to those shown in Table 2 for SNR = 40 dB which indicates that for the same ISI, higher SNR is required for a system with narrower bandwidth. These results also justify Property 4.

4. CONCLUSIONS

We have presented a performance analysis for Chi and Wu's unified class of inverse filter criteria $J_{r,m}$ where r is even and $m > r \geq 2$. The proposed analysis includes four properties relating these inverse filters associated with $J_{2,m}$ to the (nonblind) MVD filter, and one property about SNR degradation in $e(n)$ associated with the other $J_{r,m}$ ($r \geq 4$). The analytic results were then justified through simulation.

5. REFERENCES

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Table 1. ISI($g(n)$) and ISI($g_{\mathrm{MVD}}(n)$) (in dB) (after equalization) for the broadband system.

ISI($h(n)$) = 2.9229 dB (before equalization)			
SNR	$J_{2,3}$	$J_{2,4}$	MVD
0 dB	-1.7339	-3.1179	-3.8440
5 dB	-7.8739	-8.3597	-8.4871
10 dB	-12.9506	-12.8908	-12.9009
20 dB	-18.2710	-18.2177	-18.2189
40 dB	-41.5448	-41.5379	-41.5379

Table 2. ISI($g(n)$) and ISI($g_{\mathrm{MVD}}(n)$) (in dB) (after equalization) for the narrowband system.

ISI($h(n)$) = 9.3033 dB (before equalization)			
SNR	$J_{2,3}$	$J_{2,4}$	MVD
0 dB	6.4182	6.6020	6.2023
5 dB	5.7917	5.6351	5.2008
10 dB	5.1403	4.8484	4.2803
20 dB	3.3977	2.9749	2.1965
40 dB	-2.0096	-3.1099	-3.8829

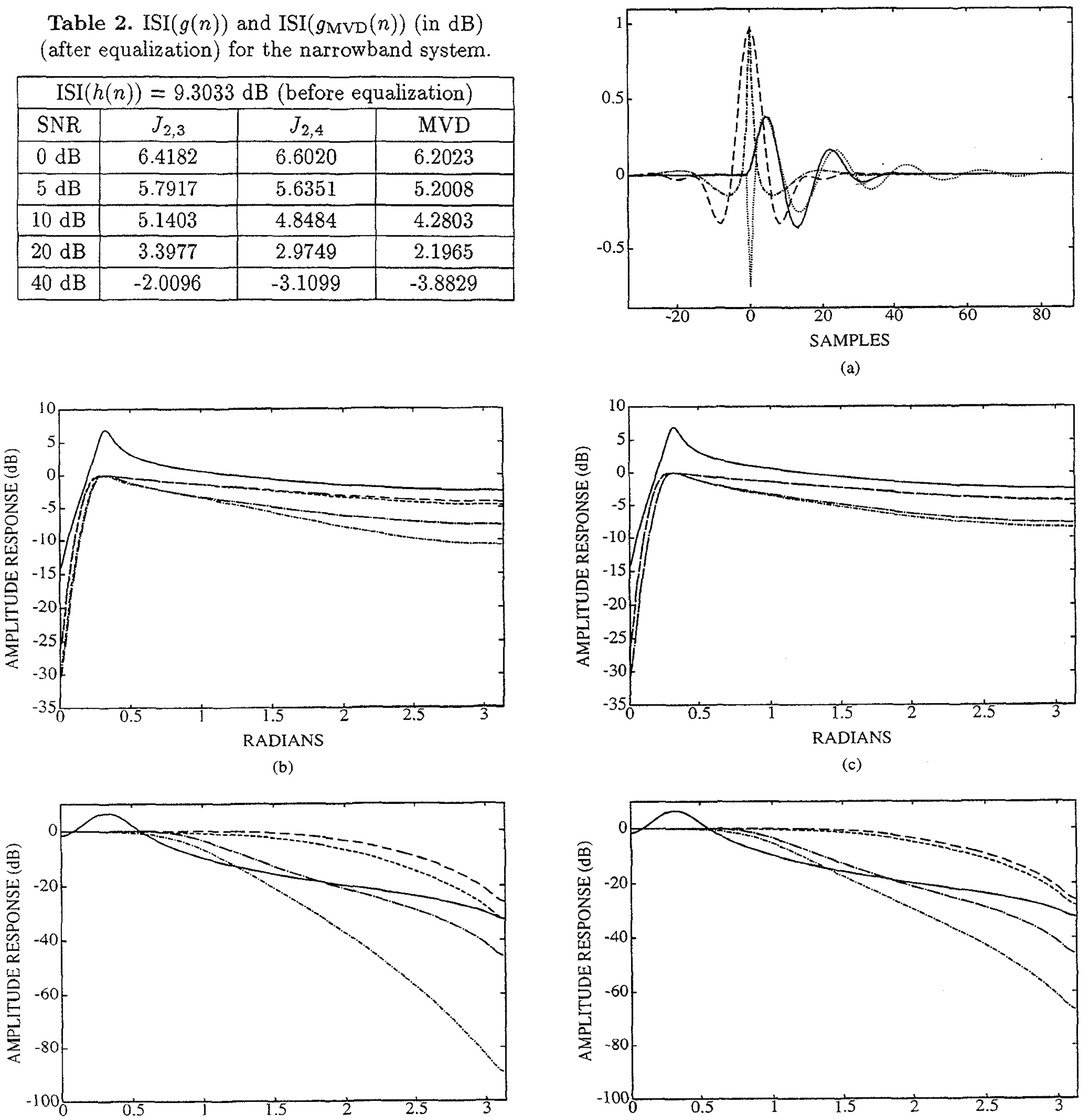

Figure 2. (a) The impulse responses $h(n)$'s of the broadband system (dotted line) and the narrowband system (solid line), respectively, and the obtained overall system $g(n)$ associated with $J_{2,4}$ for SNR = 0 dB (dash-dotted line for the former and dashed line for the latter); (b) the frequency response $G(\omega)$ associated with $J_{2,3}$ (short-dash line and short-dash dot line for SNR = 5 dB and 0 dB, respectively) and the associated $G_{\mathrm{MVD}}(\omega)$ (long-dash line and long-dash dot line for SNR = 5 dB and 0 dB, respectively) together with the amplitude response $|H(\omega)|$ (solid line) of the broadband system and (c) the corresponding results associated with $J_{2,4}$; (d) and (e) show the results corresponding to parts (b) and (c), respectively, for the narrowband system and SNR = 40 dB and 20 dB, instead.