Nonminimum-Phase Complex Fourier Series Based Model for Statistical Signal Processing

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Abstract

In this paper, a parametric complex Fourier series based model (FSBM), an extension of the real FSBM proposed by Chi, for or as an approximation to an arbitrary nonminimum-phase complex linear time-invariant (LTI) system is proposed for statistical signal processing applications where signals and LTI systems of interest are complex. Based on the proposed complex FSBM, a complex linear prediction error (LPE) filter is presented along with the Cramer Rao (CR) bound for the case of finite Gaussian measurements. Then an MP-AP algorithm is presented for the estimation of the complex FSBM parameters followed by some simulation results for channel identification and equalization in communications.

1. Introduction

Chi [1,2] proposed a real parametric nonminimum-phase Fourier series based model (FSBM) for or as an approximation to any arbitrary nonminimum-phase linear time-invariant (LTI) systems. This model is applicable in a variety of statistical signal processing areas such as system identification, deconvolution and equalization, and spectral estimation as long as the signal of interest is real. However, in some applications both signals and LTI systems of interest are complex. For instance, in digital communications, symbol streams of complex finite alphabet are transmitted through complex Rayleigh fading multipath channels, and both the identification of complex channels and the detection of complex symbol streams are crucial to the design of the optimal receiver. This paper extends Chi's real nonminimum-phase FSBM to the case of complex LTI systems so that the proposed complex FSBM can be applied in communication signal processing (e.g., equalization, channel identification and multiuser detection).

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2. Nonminimum-phase complex FSBM

The proposed complex nonminimum-phase FSBM for a complex LTI system $h(n)$ is defined as the following frequency response

$$H(\omega) = H_{\rm MP}(\omega) \cdot H_{\rm AP}(\omega) \tag{1}$$

where $H_{\rm MP}(\omega)$ is a causal minimum-phase complex FSBM given by

$$H_{\rm MP}(\omega) = \exp\left\{ \sum_{k=1}^{p} [a_k + j\widetilde{a}_k]\, e^{-j\omega k} \right\} \tag{2}$$

and $H_{\rm AP}(\omega)$ is an allpass complex FSBM given by

$$H_{\rm AP}(\omega) = \exp\left\{ j \sum_{k=1}^{q} \left[b_k \sin(k\omega) + \widetilde{b}_k \cos(k\omega) \right] \right\} \tag{3}$$

where a_k, $\widetilde{a}_k$, b_k and $\widetilde{b}_k$ are real. Note that $h_{\rm MP}(0) = 1$ and that the proposed complex FSBM given by (1) reduces to the real FSBM [1,2] when $\widetilde{a}_k = 0$ in (2) and $\widetilde{b}_k = 0$ in (3) for all k.

The complex cepstrum of the complex FSBM (inverse Fourier transform of $\ln H(\omega)$), that can be used in deconvolution [3], can be easily shown to be

$$\widetilde{h}(n) = \widetilde{h}_{\rm MP}(n) + \widetilde{h}_{\rm AP}(n) \tag{4}$$

where $\widetilde{h}_{\rm MP}(n)$ and $\widetilde{h}_{\rm AP}(n)$ are the complex cepstra of $H_{\rm MP}(\omega)$ and $H_{\rm AP}(\omega)$, respectively, as follows:

$$\widetilde{h}_{\rm MP}(n) = \begin{cases} a_n + j\widetilde{a}_n, & 1 \le n \le p \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$\widetilde{h}_{\rm AP}(n) = \begin{cases} (-b_n + j\widetilde{b}_n)/2, & 1 \le n \le q \\ (b_{-n} + j\widetilde{b}_{-n})/2, & -q \le n \le -1 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

The proposed complex FSBM given by (1) shares the following characteristics of the real FSBM summarized as follows:

(C1) Both the complex FSBM $H(\omega)$ and the inverse system $1/H(\omega)$, that can be nonminimum phase or noncausal, are guaranteed stable since they are continuous functions of ω with period of 2π.

(C2) The complex FSBM given by (1) is called MP-AP decomposition. Other equivalent complex FSBMs can be obtained from either (1) or (4). For instance, with $\widetilde{h}(n) = \widetilde{h}_1(n) + \widetilde{h}_2(n)$ where $\widetilde{h}_1(n)$ and $\widetilde{h}_2(n)$ are the causal part and anticausal part of $\widetilde{h}(n)$, respectively, one can obtain minimum-phase and maximum-phase (MN-MX) decomposition for the complex FSBM that may be suitable for certain needs [3].

3. Estimation of FSBM parameters

Assume that we are given a set of data $x(n)$ modeled as

$$x(n) = u(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k)u(n-k) \quad (7)$$

where $u(n)$ is a complex independent identically distributed (i.i.d) random sequence with zero mean and $h(n)$ is a complex FSBM given by (1). Only with $x(n)$, we desire to estimate the FSBM parameters including minimum phase FSBM parameters a_k and $\widetilde{a}_k$ and allpass FSBM parameters b_k and $\widetilde{b}_k$.

3.1. Estimation of minimum phase FSBM parameters

The estimation of a_k and $\widetilde{a}_k$ is through a complex linear prediction error (LPE) filter (a complex Wiener filter) $V_p(\omega)$ using the causal minimum-phase FSBM as follows:

$$V_p(\omega) = \exp\left\{ \sum_{k=1}^{p} \left[\alpha_k + j\widetilde{\alpha}_k\right] e^{-j\omega k} \right\} \quad (8)$$

where α_k and $\widetilde{\alpha}_k$ are real. The optimum $\widehat{V}_p(\omega)$ is obtained by minimizing

$$\varepsilon(p) = E[|e(n)|^2] \quad (9)$$

where

$$e(n) = x(n) * v_p(n) = x(n) + \sum_{k=1}^{\infty} v_p(k)x(n-k) \quad (10)$$

It can be shown that

$$\widehat{V}_p(\omega) = \frac{1}{H_{\rm MP}(\omega)} \quad (11)$$

or $\widehat{\alpha}_k = -a_k$ and $\widehat{\widetilde{\alpha}}_k = -\widetilde{a}_k$ for all $1 \le k \le p$, and therefore the optimum

$$e(n) = u(n) * h_{\rm AP}(n) \quad (12)$$

It can be shown that when data $x(n)$ are Gaussian and finite, both $\widehat{\alpha}_k$ and $\widehat{\widetilde{\alpha}}_k$ are approximate maximum-likelihood estimates with uniform CR bounds as follows:

$$\text{var}\{\widehat{\alpha}_k\} \ \ge \ 1/(2N) \quad (13)$$
$$\text{var}\{\widehat{\widetilde{\alpha}}_k\} \ \ge \ 1/(2N) \quad (14)$$

where N is the number of measurements $x(n)$. Moreover, the Akaike Information Criteria (AIC)

$$\text{AIC}(k) = 2N[\ln\varepsilon(k)] + 2k \quad (15)$$

can be used to estimate the order p. The optimum $\widehat{p}$ is the integer such that AIC(k) is minimum for $k = \widehat{p}$.

3.2. Estimation of allpass FSBM parameters

Assuming that $x(n)$ is non-Gaussian, the allpass FSBM parameter b_k and $\widetilde{b}_k$ can be estimated by further processing the optimum $e(n)$ by an allpass FSBM as follows:

$$G_q(\omega) = \exp\left\{ j\sum_{k=1}^{q} \left[\beta_k \sin(k\omega) + \widetilde{\beta}_k \cos(k\omega)\right] \right\} \quad (16)$$

The optimum $G_q(\omega)$ is obtained by maximizing the objective function [4]

$$\eta(q) = |C_M\{y(n)\}| \quad (17)$$

where $C_M\{y(n)\}$ is the Mth-order ($M \ge 3$) cumulant of $y(n)$ and

$$y(n) = e(n) * g_q(n) \quad (18)$$

It can be shown that the optimum

$$\widehat{G}_q(\omega) = \frac{1}{H_{\rm AP}(\omega)} \cdot e^{j(\omega\tau + c)} \quad (19)$$

where τ and c are unknown constants.

Chi's cumulant variation rate (CVR) [2] defined as

$$\mathrm{CVR}(k) = \frac{|\eta(k) - \eta(k-1)|}{|\eta(k-1)|} \times 100\% \qquad (20)$$

can also be used for the estimation of the order q. The optimum $\widehat{q}$ is the smallest integer such that $\mathrm{CVR}(k)$ is below a threshold for all $k > \widehat{q}$.

3.3. FSBM parameter estimation algorithm

The preceding estimation of $H_{\mathrm{MP}}(\omega)$ and $H_{\mathrm{AP}}(\omega)$ leads to an MP-AP algorithm for the estimation of the FSBM $H(\omega)$ with finite data $x(n)$ as follows:

MP-AP ALGORITHM:

(S1) Find the LPE filter $V_p(\omega)$ (a minimum-phase FSBM) by minimizing $\varepsilon(p)$ given by (9). Then $H_{\mathrm{MP}}(\omega) = 1/\widehat{V}_p(\omega)$ and save the optimum $e(n)$ that is also an amplitude equalized signal.

(S2) Find the optimum $G_q(\omega)$ (an allpass FSBM) by minimizing $\eta(q)$ given by (17). Then $H_{\mathrm{AP}}(\omega) = 1/\widehat{G}_q(\omega)$ and the optimum $y(n)$ is the deconvolved signal.

Two worthy remarks in using the proposed MP-AP algorithm are as follows:

(R1) Second-order and higher-order cumulants used in $\varepsilon(p)$ and $\eta(q)$, respectively, can be replaced by the associated sample cumulants. When the FSBM is known to be minimum-phase, the (S2) is redundant; when the FSBM is known to be allpass, the (S1) is redundant.

(R2) When the FSBM order (p, q) is unknown, the estimation of p using $\mathrm{AIC}(k)$ and the estimation of q using $\mathrm{CVR}(k)$ can be included in (S1) and (S2), respectively.

Iterative gradient type optimization algorithms are needed in both (S1) and (S2) for finding the minimum and maximum of the highly nonlinear functions $\varepsilon(p)$ and $\eta(q)$, respectively. At each iteration, the computations of $e(n)$ and $y(n)$ can be efficiently performed using FFT, and it can be easily shown that

$$\partial e(n)/\partial \alpha_k = e(n-k) \qquad (21)$$

$$\partial e(n)/\partial \widetilde{\alpha}_k = je(n-k) \qquad (22)$$

$$\partial y(n)/\partial \beta_k = \frac{1}{2}(y(n+k) - y(n-k)) \qquad (23)$$

$$\partial y(n)/\partial \widetilde{\beta}_k = \frac{j}{2}(y(n+k) + y(n-k)) \qquad (24)$$

that lead to a computationally efficient parallel structure in computing gradients of $\varepsilon(p)$ and $\eta(q)$ with respect to FSBM parameters.

4. Simulation results

Figures 1(a) through 1(f) show some simulation results for blind channel identification and equalization using the proposed MP-AP algorithm using the proposed FSBM, Hatzinakos and Nikias' tricepstrum-based algorithm [3] as well as Shalvi and Weinstein's super-exponential algorithm [5], , for the case that the driving input $u(n)$ is a 16-QAM signal and the channel $h(n)$ is a complex noncausal FIR system taken from [3]. One can see from these figures that the results (thirty independent estimates of $H(\omega)$) for $N = 4096$ and SNR= 20 dB (additive white Gaussian noise) associated with the proposed MP-AP algorithm ($p = q = 4$ and $M = 4$) are better than the other two algorithms due to smaller bias and variance for both magnitude and phase estimates of $H(\omega)$.

5. Conclusions

We have extended the results reported in [1,2] (real nonminimum phase FSBM) to the case of complex nonminimum phase FSBM including theoretical proofs for the identifiability of FSBM parameters, channel identification and equalization algorithms, simulation results, and comparison with Hatzinakos and Nikias' algorithm [3] as well as Shalvi and Weinstein's algorithm [5]. The proposed complex FSBM and associated computationally efficient MP-AP algorithm can be applied to blind channel identification and equalization in digital communications.

References

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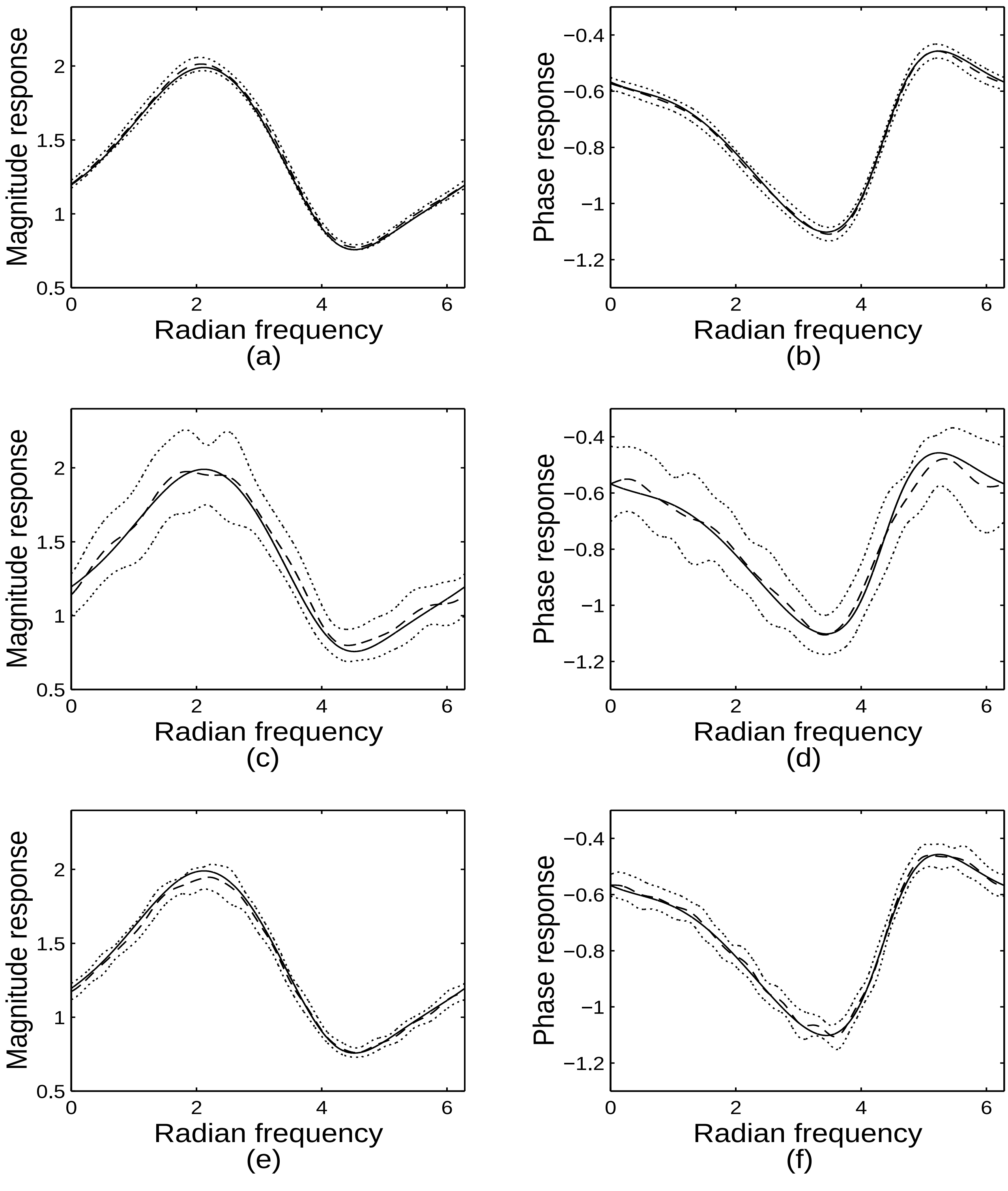

Figure 1. (a) Mean (dashed lines) and mean±one standard deviation (dotted lines) of thirty magnitude estimates and (b) those of thirty phase estimates associated with the proposed MP-AP algorithm using the proposed FSBM, together with the magnitude and phase (solid lines) of the system $H(\omega)$; (c) & (d) are the corresponding results for the tricepstrum-based algorithm; (e) & (f) are the corresponding results for the super-exponential algorithm.