# NEW INVERSE FILTER CRITERIA FOR IDENTIFICATION AND DECONVOLUTION OF NONMINIMUM-PHASE SYSTEMS BY SINGLE CUMULANT SLICE

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## ABSTRACT

In this paper, we propose a family of new cumulant based inverse filter criteria which only require a single slice of cumulants of the inverse filter output for the identification and deconvolution of linear time-invariant (LTI) nonminimum-phase systems with only non-Gaussian output measurements contaminated by Gaussian noise. Some simulation results and application to speech deconvolution are provided to demonstrate that inverse filtering algorithms based on the proposed new criteria work well.

## 1. INTRODUCTION

The identification of a linear time-invariant (LTI) system $h(k)$ with noisy output $x(k)$ based on the following convolutional model:

$$x(k) = u(k) * h(k) + n(k) \tag{1}$$

is very important in many signal processing areas such as seismic deconvolution, channel equalization, radar, sonar, speech processing and image processing. Recently, cumulant (higher order statistics (HOS)) based identification [1] of nonminimum-phase LTI systems with only non-Gaussian output measurements has drawn extensive attention in the previous signal processing areas because cumulants, which are blind to any kind of a Gaussian process, not only extract the amplitude information but also the phase information of $h(k)$, meanwhile they are inherently immune from Gaussian measurement noise $n(k)$. Higher order statistics based inverse filter criteria [2-6] have been used for identification and deconvolution of nonminimum-phase systems. In this paper, we propose a family of new cumulant based inverse filter criteria which only require a single slice of cumulants for the identification of $h(k)$ as well as the estimation of the desired signal $u(k)$.

## 2. NEW HOS BASED INVERSE FILTER CRITERIA

Assume that data $x(k), k = 0, 1, ..., N-1$ were generated from the model given by (1). Recently, Chi and Kung [6] proposed a new inverse filtering algorithm based on the following assumptions:

(A1) The system $h(k)$ is causal stable; it can be minimum-phase or nonminimum-phase.

(A2) The input $u(k)$ is real, zero-mean, stationary, independent identically distributed (*i.i.d.*), non-Gaussian with $M$th-order cumulant $\gamma_M$.

(A3) The measurement noise $n(k)$ is Gaussian which can be white or colored with unknown statistics.

(A4) The input $u(k)$ is statistically independent of $n(k)$.

Let $e(k)$ be the output of a stable LTI filter $v(k)$ with the input $x(k)$. Their inverse filtering algorithm is to find the optimum $\hat{v}(k)$ such that

$$\tilde{J}_M(v(k)) = \frac{1}{C_{M,e}^2(0,0,\cdots,0)} \cdot \sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_{M-1}=-\infty}^{\infty} C_{M,e}^2(k_1,k_2,\cdots,k_{M-1}) \geq \tilde{J}_M(\hat{v}(k)) \geq 1 \tag{2}$$

where $M \geq 3$, and $C_{M,e}(k_1,k_2,\cdots,k_{M-1})$ is the $M$th-order cumulant function of $e(k)$. It works well, but its computational load is quite heavy because the $(M-1)$-fold summation in $\tilde{J}_M(v(k))$ requires all $M$th-order cumulants of $e(k)$. They also showed that $h(k) * \hat{v}(k) = \alpha\delta(k-l)$ where $l$ is an unknown integer and $\alpha \neq 0$ is an unknown scale factor.

Next, we present a family of new inverse filter criteria which are described in Theorem 1 as follows:



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**Theorem 1.** Let $e(k)$ be the output of a stable LTI filter $v(k)$ with the input $x(k)$ generated from (1) under the previous assumptions (A1) through (A4). Let $\hat{v}(k)$ be the optimum $v(k)$ such that

$$J_{M,m}(v(k)) = \frac{1}{C_{M,e}^2(0,...,0)}$$

$$\cdot \sum_{k=-\infty}^{\infty} C_{M,e}^2(0,...,0,k_m = k,...,k_{M-1} = k)$$

$$\geq J_{M,m}(\hat{v}(k)) \geq 1 \quad (3)$$

where $M \geq 3$ and $1 \leq m \leq M-1$ are positive integers. Then $h(k) * \hat{v}(k) = \alpha\delta(k-l)$ as long as $\gamma_M \neq 0$ where $l$ is an unknown integer and $\alpha \neq 0$ is an unknown scale factor. (The proof of this theorem is omitted here.)

Obviously, to estimate the inverse filter $v(k)$ of $h(k)$ by the proposed criteria given by (3) requires a much smaller computational load than by Chi-Kung's criterion given by (2). Assuming that $v(k)$ is an FIR filter of order $L$, i.e.,

$$e(k) = \sum_{j=0}^{L} v(j)x(k-j), \quad (4)$$

we obtain the desired $\hat{v}(k)$ by minimizing

$$\hat{J}_{M,m}(v(k)) = \frac{1}{\hat{C}_{M,e}^2(0,...,0)}$$

$$\cdot \sum_{k=-K}^{K} \hat{C}_{M,e}^2(0,...,0,k_m = k,...,k_{M-1} = k) \quad (5)$$

in which $\hat{C}_{M,e}(k_1, k_2, \cdots, k_{M-1})$ is the $M$th-order sample cumulant function of $e(k)$. A Newton-Raphson type iterative algorithm is used to find the desired $\hat{v}(k)$ because $\hat{J}_{M,m}$ is a highly nonlinear function of $v(k)$. After $\hat{v}(k)$ is obtained, it is trivial to obtain $\hat{h}(k)$ from $\hat{v}(k)$ except for a scale factor and a constant time-delay.

## 3. SIMULATION AND APPLICATION TO SPEECH DECONVOLUTION

In this section, we are to present some simulation results and some results for speech deconvolution by both inverse filtering algorithms based on (5) as well as (2) and the conventional minimum-phase linear prediction error (LPE) filter $v_b(k)$ obtained by the well-known Burg's algorithm. Both the order $L$ of inverse filters $v(k)$ and the order of $v_b(k)$ were equal to 30. The initial guesses $[v(0), v(1), ..., v(30)] = [0, ..., 0, 1, v_b(1), ..., v_b(15)]$, where $1, v_b(1), ..., v_b(15)$ are the coefficients of the conventional LPE filter of order equal to 15, were used to initialize inverse filtering algorithms.

### 3.1 Simulation Results

The driving input $u(k)$ used was a zero-mean Bernoulli-Gaussian sequence (a sparse spike sequence) [2] with skewness $\gamma_3 = 0$ and kurtosis $\gamma_4 = 0.27$. A third-order nonminimum-phase ARMA system $h(k)$ taken from [2] with transfer function

$$H(z) = \frac{1 + 0.1z^{-1} - 3.2725z^{-2} + 1.41125z^{-3}}{1 - 1.9z^{-1} + 1.1525z^{-2} - 0.1625z^{-3}} \quad (6)$$

was used. The synthetic data $x(k)$ ($N = 512$) shown in Fig. 1(a) were generated based on (1) for $SNR = 464$ (27 dB) where $n(k)$ was white Gaussian noise. The inverse filter criterion used was $\hat{J}_{4,1}$ with $K = 16$. The predictive deconvolved data $e_b(k)$ (dotted line) are shown in Fig. 1(b). The deconvolved data $e_1(k)$ (dotted line) obtained by the optimum inverse filter based on $\hat{J}_{4,1}$ are shown in Fig. 1(c). One can see, from Fig. 1(c), that $e_1(k)$ approximates $u(k)$ (solid line) very well except for a scale factor while the constant time-delay between $e_1(k)$ and $u(k)$ is artificially compensated for. Comparing Fig. 1(b) with Fig. 1(c), one can easily see that $e_1(k)$ is indeed a much better estimate of $u(k)$ than $e_b(k)$ since the LPE filter is inherently blind to the phase of $h(k)$. These simulation results support that the proposed criteria (see (3)) can be used for identification and deconvolution of LTI systems.

### 3.2 Some Results for Speech Deconvolution

The speech sound /a:/ uttered by a man was filtered by a lowpass filter with cutoff frequency set to 3 kHz and then sampled by a 12-bit A/D converter with sampling frequency 10 kHz. The speech data $x(k)$, shown in Fig. 2(a), can be viewed as output measurements based on (1) in which $h(k)$ is the impulse response of the vocal-tract filter and $u(k)$ is a pseudo-periodic pulse train (for voiced speech). The predictive deconvolved data $\hat{u}_b(k)$ are shown in Fig. 2(b). The inverse filter criteria used include $\hat{J}_{3,1}$, $\hat{J}_{4,1}$, $\hat{J}_{4,2}$ with $K = 45$ and Chi-Kung's criterion with $M = 3$ as follows

$$\tilde{J}_3 = \frac{1}{\hat{C}_{3,e}^2(0,0)} \sum_{(k_1,k_2) \in R_{3,q}} \hat{C}_{3,e}^2(k_1, k_2) \quad (7)$$

where $R_{3,q}$ is the domain of support associated with the third-order cumulant function of non-Gaussian moving average (MA($q$)) processes with order $q = 30$. The deconvolved results are shown in Fig. 2(c) through Fig. 2(f). One can see, from Fig. 2(b) to Fig. 2(f), that all the deconvolved data based on criteria $\hat{J}_{3,1}$, $\hat{J}_{4,1}$, $\hat{J}_{4,2}$ and $\tilde{J}_3$ approximate a pseudo-periodic pulse train much better than the deconvolved data obtained by the conventional LPE filter since the vocal-tract filter associated with the speech data shown in Fig. 2(a) is nonminimum-phase. These successful experimental results in speech deconvolution also support that the proposed inverse filter criteria (see(3)) can be used for estimation of (nonminimum-phase) vocal-tract filter as well as pitch estimation.

## 4. CONCLUSIONS

In this paper, we proposed a family of new inverse filter criteria described in Theorem 1 for identification and deconvolution of LTI systems with only non-Gaussian output measurements $x(k)$ (see (1)). The proposed inverse filter criteria only require a single slice of $M$th-order cumulants (see (3)) rather than all $M$th-order cumulants (in Chi and Kung's criterion (see (2))) of the inverse filter output. Moreover, they are applicable for all $M \geq 3$ as long as the $M$th-order cumulant $\gamma_M$ of the driving input $u(k)$ of LTI systems is not equal to zero. We also provided some simulation results and some experimental results with real speech data to support the proposed inverse filter criteria.

## REFERENCES


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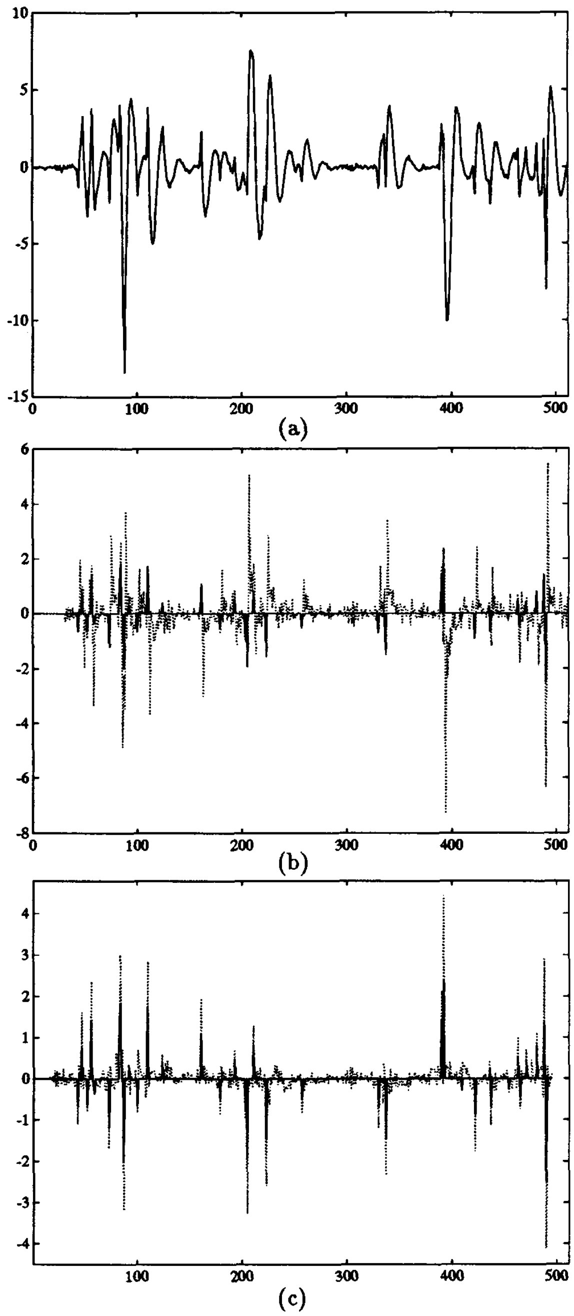


Fig. 1. Simulation results. (a) Synthetic noisy data for $N = 512$ and $SNR = 27$ dB, (b) true input signal $u(k)$ (solid line) and the predictive deconvolved data $e_b(k)$ (dotted line), and (c) true input signal $u(k)$ (solid line) and the deconvolved data $e_1(k)$ (dotted line) obtained by the optimum inverse filter based on $\hat{J}_{4,1}$.

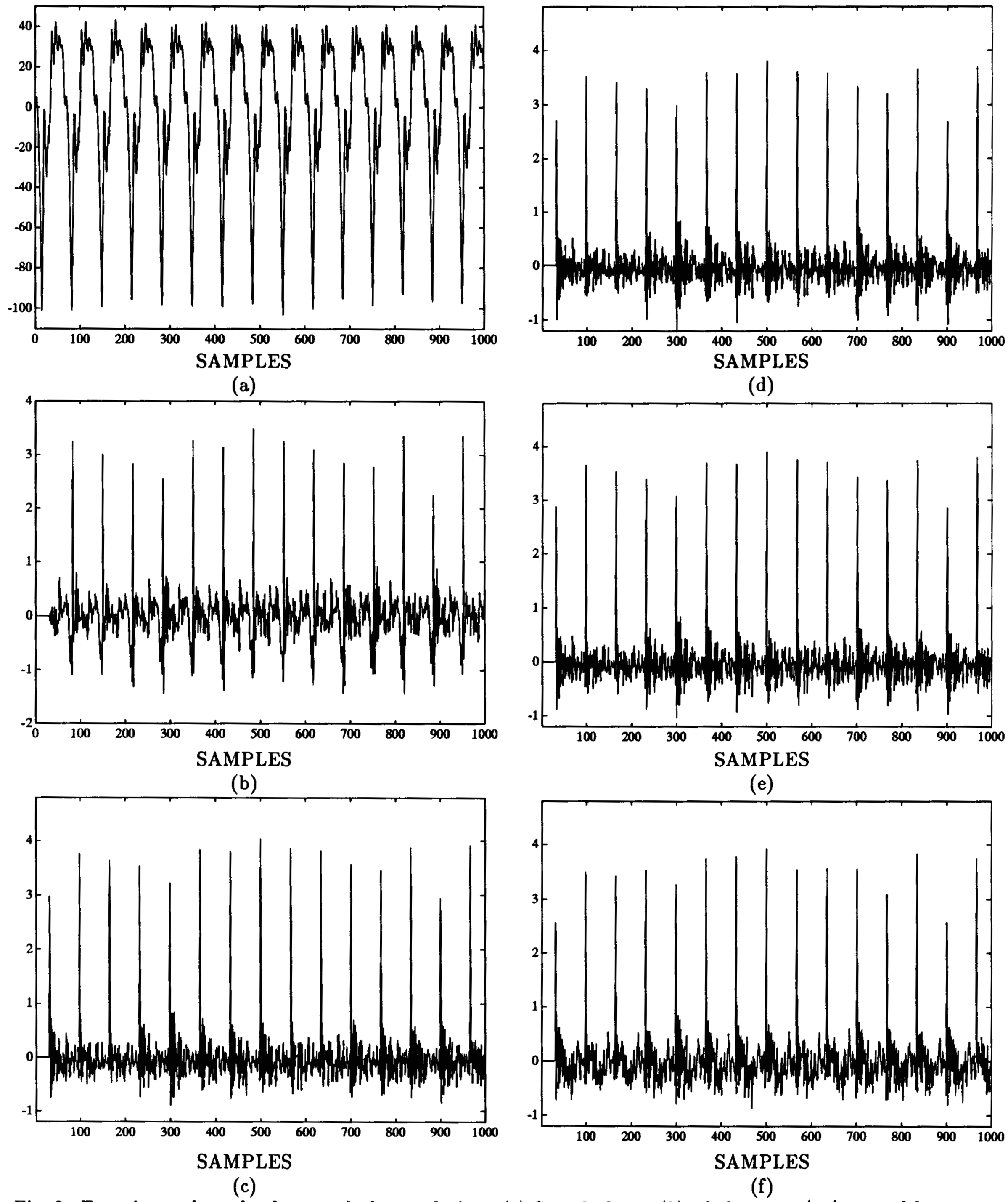


Fig. 2. Experimental results for speech deconvolution. (a) Speech data $x(k)$ of phoneme $/a{:}/$ uttered by a man (sampling rate equal to 10 kHz); (b) the predictive deconvolved data $\hat{u}_b(k)$; the deconvolved data obtained by the optimum inverse filter based on (c) $\tilde{J}_3$ given by (7), (d) $\hat{J}_{3,1}$, (e) $\hat{J}_{4,1}$, and (f) $\hat{J}_{4,2}$, respectively.