CUMULANT BASED PHASE ESTIMATION FOR 1-D AND 2-D NONMINIMUM PHASE SYSTEMS BY FOURIER SERIES BASED ALLPASS MODEL

Horng-Ming Chien *Chong-Yung Chi*

Department of Electrical Engineering
National Tsing Hua University, Hsinchu, Taiwan, Republic of China
Tel: 886-35-731156, Fax: 886-35-715971, E-mail: cychi@ee.nthu.edu.tw

ABSTRACT

Yang and Chi proposed a family of 1-D criteria for estimating the phase of a 1-D nonminimum phase linear time-invariant (LTI) system with only non-Gaussian measurements corrupted by additive Gaussian noise. The phase of the LTI system is obtained through an iterative algorithm which processes the given measurements by an ARMA allpass model such that a single absolute Mth-order ($M \geq 3$) cumulant of the allpass model output is maximum. In this paper, a family of 1-D and 2-D criteria, in which Yang and Chi's 1-D criteria are included, is proposed for phase estimation using a Fourier series based allpass model. The optimum allpass models for 1-D and 2-D LTI systems are obtained by a 1-D and a 2-D iterative algorithms, respectively. The paper concludes with some simulation results followed by some conclusions.

1. INTRODUCTION

Identification of an unknown real linear time-invariant (LTI) system $h(n)$ with Gaussian noise corrupted measurements $x(n)$, i.e.,

$$x(n) = u(n) * h(n) + w(n) \qquad (1)$$

plays an important role in various engineering applications such as seismic deconvolution, channel equalization, speech deconvolution and image restoration. Estimation of amplitude response of $h(n)$ is the kernel of correlation (second-order statistics) based parametric spectral estimation. On the other hand, phase estimation must resort to higher-order statistics, known as cumulants, simply because correlations are phase blind.

A number of phase estimation methods have been reported such as polyspectrum phase based methods [1,2] which estimate the phase of $h(n)$ from the phase of polyspectra of $x(n)$ without involving amplitude estimation of $h(n)$, and minimum-phase (MP) - allpass (AP) decomposition based methods [3-5] which estimate the amplitude of $h(n)$ using a correlation based method and then estimate the phase of $h(n)$ using a cumulant based method. Recently, Yang and Chi [6] proposed a new parametric method which estimates the phase response $\theta(\omega) = arg\{H(z = e^{j\omega})\}$ of $h(n)$ neither involving the amplitude estimation nor using the phase of polyspectra of $x(n)$ This method is implemented by an iterative optimization algorithm which processes $x(n)$ with an optimum ARMA allpass filter such that a single absolute Mth-order ($M \geq 3$) cumulant of the allpass filter output is maximum. Yang and Chi's method performs well and is free from the phase unwrapping problem of polyspectrum phase based methods, and meanwhile is more insensitive to additive Gaussian noise than MP-AP decomposition based methods because it never resorts to correlations throughout their algorithm.

Dianat and Raghuveer [7] proposed a Fourier series based paramteric model for both the phase and magnitude of non-Gaussian signals with the model parameters estimated from bispectra of data. In this paper, the Fourier series based model [7] is used for estimating the phase of $H(z)$ following the previous Yang and Chi's estimation procedure. Moreover, sharing the same advantages of Yang and Chi's method mentioned above, the proposed phase estimation method is applicable for both 1-D and 2-D LTI systems which can have zeros on the unit circle for the former and the unit bi-circle for the latter.

2. NEW CUMULANT BASED PHASE ESTIMATION METHOD

Let us define some notations for ease of later use:

$$\mathbf{n} = (n_1, n_2),\ \mathbf{k} = (k_1, k_2)$$

$$\mathbf{z} = (z_1, z_2),\ \boldsymbol{\omega} = (\omega_1, \omega_2)$$

$$H(\omega) = H(z = e^{j\omega}),\ H(\boldsymbol{\omega}) = H(\mathbf{z} = (e^{j\omega_1}, e^{j\omega_2}))$$

$$\sum_{\mathbf{k}=-\infty}^{\infty} = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty}$$

$$\int_{-\pi}^{\pi} f(\boldsymbol{\omega}) d\boldsymbol{\omega} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(\omega_1, \omega_2) d\omega_1 d\omega_2$$

Let $x(\mathbf{n})$ be the noisy output of a 2-D unknown LTI system $h(\mathbf{n})$ driven by a non-Gaussian input $u(\mathbf{n})$

$$x(\mathbf{n}) = u(\mathbf{n}) * h(\mathbf{n}) + w(\mathbf{n})$$

*This work is supported by the National Science Council under Grant NSC 84-2213-E-007-074

$$= \sum_{\mathbf{k}=-\infty}^{\infty} u(\mathbf{k})h(\mathbf{n}-\mathbf{k}) + w(\mathbf{n}) \quad (2)$$

with the following assumptions:

(A1) $u(\mathbf{n})$ is real, zero-mean, i.i.d., non-Gaussian with Mth-order cumulant $C_{M,u} = \gamma_M \neq 0$.
(A2) $w(\mathbf{n})$ is zero-mean Gaussian with unknown statistics.
(A3) $u(\mathbf{n})$ and $w(\mathbf{n})$ are statistically independent.
(A4) $h(\mathbf{n})$ is a real stable LTI system which can be nonminimum phase.

Let $H_p(z)$ be a 1-D allpass model whose phase is modeled as [7]

$$\phi_p(\omega) = arg\{H_p(\omega)\} = \sum_{k=1}^{p} a_k \cdot \sin(k\omega) \quad (3)$$

and $H_{p1,p2}(\mathbf{z})$ be a 2-D allpass model whose phase is modeled as [7]

$$\phi_{p1,p2}(\boldsymbol{\omega}) = arg\{H_{p1,p2}(\boldsymbol{\omega})\} = \sum_{k=1}^{p1} \sum_{l=-p2}^{p2} a_{k,l} \cdot \sin(k\omega_1 + l\omega_2) + \sum_{l=1}^{p2} a_{0,l} \cdot \sin(l\omega_2) \quad (4)$$

The new phase estimation method to be presented below is based on the allpass model associated with (3) and (4) and the following theorem.

Theorem 1. Assume that $x(\mathbf{n})$ was generated from (2) under the assumptions (A1) through (A4). Let $y(\mathbf{n})$ be the output of a 2-D allpass system $H_{AP}(\mathbf{z})$ with input $x(\mathbf{n})$. Then the absolute Mth-order ($M \geq 3$) cumulant $|C_{M,y}|$ of $y(\mathbf{n})$ is maximum if and only if

$$\phi(\boldsymbol{\omega}) = arg\{H_{AP}(\boldsymbol{\omega})\} = -\theta(\boldsymbol{\omega}) + \boldsymbol{\alpha} \cdot \boldsymbol{\omega}^T \quad (5)$$

where $\theta(\boldsymbol{\omega}) = arg\{H(\boldsymbol{\omega})\}$ and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ is an unknown constant row vector. □

Note that Theorem 1 is a 2-D extension of the corresponding theorem reported in [6] for the 1-D case. The former reduces to the latter with $\mathbf{n}, \boldsymbol{\omega}, \boldsymbol{\pi}$ and $\boldsymbol{\alpha}$ replaced by n and ω, π and α, respectively.

Let

$$y(\mathbf{n}) = \begin{cases} x(n) * h_p(n) & \text{1-D case} \\ x(\mathbf{n}) * h_{p1,p2}(\mathbf{n}) & \text{2-D case} \end{cases}$$

By Theorem 1, except for an unknown linear phase term (an unknown time delay), the phase $\theta(\boldsymbol{\omega})$ of the unknown system $H(\mathbf{z})$ can be estimated using (5) by maximizing the following highly nonlinear objective function

$$J(\mathbf{a}) = |\hat{C}_{M,y}|^2 \quad (6)$$

where $\mathbf{a}$ is a column vector containing all the coefficients of the allpass model used and $\hat{C}_{M,y}$ is the Mth-order sample cumulant of $y(\mathbf{n})$. To find the maximum of $J(\mathbf{a})$, a gradient type iterative optimization algorithm (IOA) is used which updates $\hat{\mathbf{a}}$ at ith-iteration by

$$\hat{\mathbf{a}}(i) = \hat{\mathbf{a}}(i-1) + \rho \frac{\partial J(\mathbf{a})}{\partial \mathbf{a}}\bigg|_{\mathbf{a}=\hat{\mathbf{a}}(i-1)} \quad (7)$$

where $0 < \rho \leq 1$ is a constant, and $\partial J(\mathbf{a})/\partial \mathbf{a}$ depends only on $y(\mathbf{n})$ and $\partial y(\mathbf{n})/\partial \mathbf{a}$.

The new 1-D phase estimation algorithm using the allpass model $H_p(z)$ (see (3)) is described as follows:

<u>*Algorithm 1*</u> : (1-D phase estimation algorithm)

(S1) Set p_{max} (maximum of p), cumulant order M and integer increment parameter $s \geq 1$.
(S2) Set $t = 1, p = s$ and $\mathbf{a}_p = (a_1, ..., a_p)^T$ which contains all the coefficients of $\phi_p(\omega)$ defined by (3). Search for the maximum of $J(\mathbf{a}_p)$ by the above IOA with $\hat{\mathbf{a}}_p(0) = \mathbf{0}$.
(S3) Set $t = t + 1$ and $p = s \cdot t$. Search for the maximum of $J(\mathbf{a}_p)$ by the above IOA with $\hat{\mathbf{a}}_p(0) = (\hat{\mathbf{a}}_{p-s}^T, \mathbf{0}_s^T)^T$ where $\mathbf{0}_s$ is an $s \times 1$ column vector containing only zeros.
(S4) If $p < p_{max}$, go to (S3), otherwise stop.

The optimum $\hat{\theta}(\omega)$, is then obtained as (see (5))

$$\hat{\theta}(\omega) = -arg\{\hat{H}_p(\omega)\} = -\hat{\phi}_p(\omega) \quad (8)$$

Regarding the computation of $y(n)$ and $\partial y(n)/\partial \mathbf{a}_p$ required for computing $J(\mathbf{a}_p)$ and $\partial J(\mathbf{a}_p)/\partial \mathbf{a}_p$ in (S2) and (S3), the former can be obtained by computing $y(n) = x(n) * \hat{h}_p(n)$ where $\hat{h}_p(n)$ is the inverse FFT of $\hat{H}_p(\omega)$, and the latter can be shown to be

$$\frac{\partial y(n)}{\partial a_m} = \frac{1}{2}\{y(n+m) - y(n-m)\} \quad (9)$$

The new 2-D phase estimation algorithm using the allpass model $H_{p1,p2}(\mathbf{z})$ (see (4)) is described as follows:

<u>*Algorithm 2*</u> : (2-D phase estimation algorithm)

(S1) Set $p1, p2$, cumulant order M, and let $\mathbf{a}$ be a column vector containing all the coefficients of $\phi_{p1,p2}(\boldsymbol{\omega})$ defined by (4).
(S2) Search for the maximum of $J(\mathbf{a})$ by the above IOA with $\hat{\mathbf{a}}(0) = \mathbf{0}$.

The optimum $\hat{\theta}(\boldsymbol{\omega})$, is again obtained as (see (5))

$$\hat{\theta}(\boldsymbol{\omega}) = -arg\{\hat{H}_{p1,p2}(\boldsymbol{\omega})\} = -\hat{\phi}_{p1,p2}(\boldsymbol{\omega}) \quad (10)$$

For computing $J(\mathbf{a})$ and $\partial J(\mathbf{a})/\partial \mathbf{a}$ required in (S2), $y(\mathbf{n})$ can be obtained by taking the inverse

FFT of $Y(\omega) = X(\omega) \cdot \hat{H}_{p1,p2}(\omega)$ and $\partial y(\mathbf{n})/\partial a_{r,s}$ can be shown to be

$$\frac{\partial y(\mathbf{n})}{\partial a_{r,s}} = \frac{1}{2}\{y(n_1+r, n_2+s) - y(n_1-r, n_2-s)\} \tag{11}$$

Some remarks for the proposed two phase estimation algorithms are worthwhile to be summarized as follows:

(R1) For *Algorithm 1*, s allpass model parameters are increased at each iteration (t) which often leads to much faster convergence for $s > 1$ than for $s = 1$ with almost the same performance by our experience.

(R2) The optimum allpass filter ($\hat{H}_p(z)$ or $\hat{H}_{p1,p2}(\mathbf{z})$) is actually an optimum phase equalizer to remove the phase distortion of the unknown LTI system ($H(z)$ or $H(\mathbf{z})$) which itself can also be an allpass system.

(R3) *Algorithm 1* is computationally faster than Yang and Chi's 1-D phase estimation algorithm because the former and *Algorithm 2* have the same parallel structure in computing $\partial y(n)/\partial a_m$ and $\partial y(\mathbf{n})/\partial a_{r,s}$, which are nothing but the output of a two-point ($-1/2$ and $1/2$) FIR filter (see (9) and (11)) driven by $y(n)$ and $y(\mathbf{n})$, respectively.

(R4) The optimum phase estimate $\hat{\theta}(\omega)$ is a continuous approximation to the true system phase $\theta(\omega)$ which itself can have discontinuities. Moreover, $\hat{\theta}(\omega)$ is blind to a constant π since $|\hat{C}_{M,y}|$ is invariant for either of $y(\mathbf{n})$ and $-y(\mathbf{n})$.

(R5) It can be shown that the phase estimation error

$$e(\omega) = |\theta(\omega) - \hat{\theta}(\omega)| \tag{12}$$

is smaller for those ω where $|H(\omega)|$ is larger. Therefore, when $\theta(\omega)$ has discontinuities due to zeros on the unit circle (1-D case) or unit bi-circle (2-D case), $e(\omega)$ is always large in the vicinity of discontinuities of $\theta(\omega)$ even if $SNR = \infty$. This is also consistent with the well-known property of Fourier series expansion that $e(\omega) \neq 0$ for those ω where $\theta(\omega)$ is not continuous even if $p = \infty$ (1-D case) or $p1 = p2 = \infty$ (2-D case).

3. SIMULATION RESULTS

Two simulation examples are presented below to justify the good performance of the proposed two phase estimation algorithms. The driving input sequence $u(n)$ or $u(\mathbf{n})$ used was a zero-mean i.i.d. exponential random sequence and the phase $\theta(\omega)$ or $\theta(\boldsymbol{\omega})$ of the unknown system $h(n)$ or $h(\mathbf{n})$ used has discontinuities.

Example 1. 1-D phase estimation

A nonminimum phase ARMA(3,4) 1-D system $H(z)$ with a pair of zeros at $e^{\pm j0.9237}$ was used in this example. The amplitude $|H(\omega)|$ (solid line) and phase $\theta(\omega)$ (solid line) responses of $H(z)$ are shown in Figures 1(a) and 1(b), respectively. One can see a spectral null at $\omega = 0.9237$ in Figure 1(a). The synthetic data $x(n)$ were generated for $N = 1024$ (data length) and $SNR = 20$dB (white Gaussian noise). The estimated continuous system phase $\hat{\theta}(\omega)$ (dashed line) (see (R4)) obtained by *Algorithm 1* with $M = 3$ (cumulant order), $p_{max} = 16$ and $s = 8$ is also shown in Figure 1(b). As predicted (see (R5)), the phase estimation error $e(\omega)$ is small for all ω except for the vicinity of $\omega = 0.9237$.

Example 2. 2-D phase estimation

A nonseparable 3×3 MA 2-D system $H(\boldsymbol{\omega})$ was used whose amplitude $|H(\boldsymbol{\omega})|$ and phase $\theta(\boldsymbol{\omega})$ responses are shown in Figures 2(a) and 2(b), respectively. Spectral nulls in $|H(\boldsymbol{\omega})|$ and discontinuities (jumps of 2π or π) in $\theta(\boldsymbol{\omega})$ can be seen from these two figures, respectively. A 256×256 synthetic field $x(\mathbf{n})$ was generated for $SNR = \infty$ and then processed by *Algorithm 2* with $M = 3$ and $p1 = p2 = 5$. The estimated continuous system phase $\hat{\theta}(\boldsymbol{\omega})$ (see (R4)) is shown in Figure 2(c). Again, as predicted (see (R5)), the phase estimation error $e(\boldsymbol{\omega})$ is small for all $\boldsymbol{\omega}$ where $\theta(\boldsymbol{\omega})$ is continuous.

4. CONCLUSIONS

Based on the Fourier series based allpass model (see (3) and (4)) and Theorem 1, we have presented a 1-D and a 2-D iterative phase estimation algorithms (*Algorithms 1* and *2*) by maximizing a single absolute Mth-order cumulant of the allpass model output (see (6)). The optimum allpass filters obtained by these two algorithms can be regarded as an optimum phase equalizer with a computationally efficient parallel structure suitable for both software and hardware implementation (see (R2) and (R3)). Two simulation examples with $|H(\omega)|$ having spectral nulls were provided which support the proposed phase estimation algorithms and meanwhile are consistent with the predicted performance described in (R4) and (R5).

5. REFERENCES

[1] K.S. Lii and M. Rosenblatt, "Deconvolution and estimation of transfer function phase and coefficients for non-Gaussian linear processes," Ann. Statist., vol. 10, no. 4, pp. 1195-1208, 1982.

[2] T. Matsuoka and T.J. Ulrych, "Phase estimation using the bispectrum," Proc. IEEE, vol. 72, pp. 1403-1411, Oct. 1984.

[3] G.B. Giannakis and J.M. Mendel, "Stochastic realization of nonminimum phase systems," Proc. 1986 Amer. Control Conf., Seattle, WA, vol. 2, pp. 1254-1259.

[4] J.K. Tugnait, "Identification of nonminimum phase linear stochastic systems," Automatica, vol. 22, no. 4, pp. 457-464, 1986.

[5] C.-Y. Chi and J.-Y. Kung, "A phase determination method for nonminimum phase ARMA systems by a single cumulant sample," IEEE Trans. Signal Processing, vol. 41, no. 2, pp. 981-985, Feb. 1993.

[6] H.-L. Yang and C.-Y. Chi, "A new cumulant based phase estimation method for nonminimum phase systems by allpass filtering," Proc. IEEE Seventh SP Workshop on Statistical Signal and Array Processing, Quebec City, Canada, June 1994, pp. 231-234.

[7] S.A. Dianat and M.R. Raghuveer, "Fast algorithms for phase and magnitude reconstruction from bispectra," Optical Engineering, vol. 29, no. 5, pp. 504-512, May 1990.

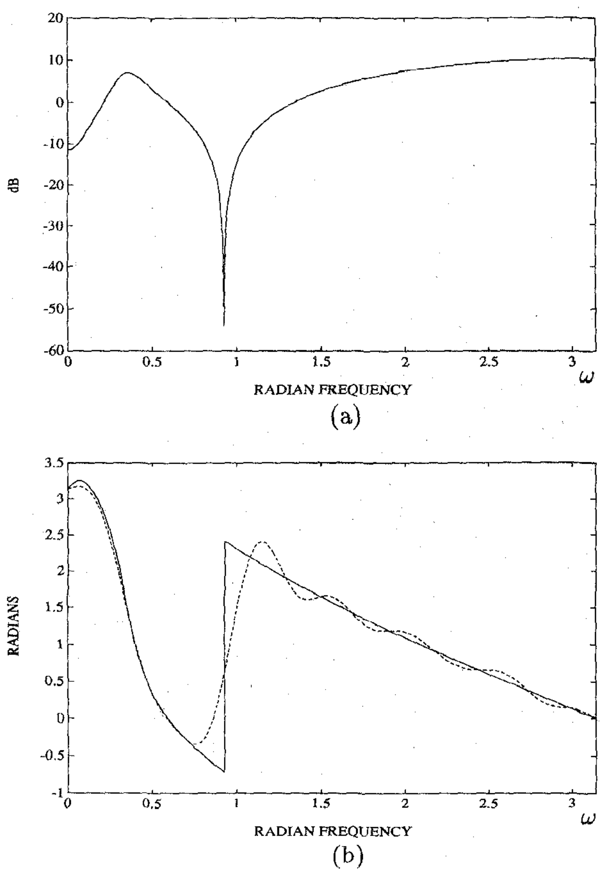

Figure 1. Simulation results for 1-D phase estimation. (a) Amplitude response $|H(\omega)|$ of the true system and (b) estimated phase response $\hat{\theta}(\omega)$ (dashed line) and the true system phase $\theta(\omega)$ (solid line).

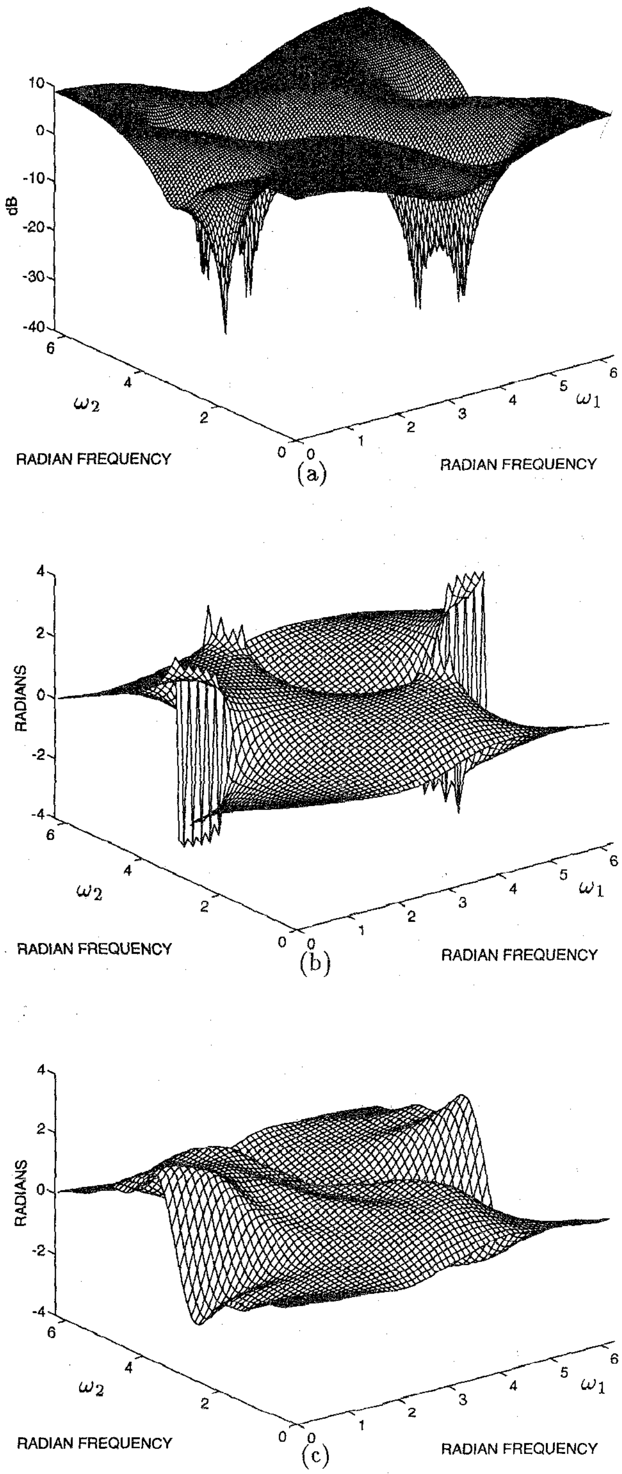

Figure 2. Simulation results for 2-D phase estimation. (a) Amplitude response $|H(\omega)|$ and (b) phase response $\theta(\omega)$ of the true system, respectively; (c) estimated phase response $\hat{\theta}(\omega)$.