An Adaptive Bernoulli-Gaussian Model Based Maximum-Likelihood Channel Equalizer for Detection of Binary Sequences

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Abstract—Based on a modified Bernoulli-Gaussian model, we propose an adaptive maximum-likelihood channel equalizer, which is a block signal processing algorithm, for the detection of binary sequences transmitted through an unknown slowly time-varying channel. Both computational load and storage required by the proposed adaptive channel equalizer are linearly rather than exponentially proportional to the size of signal processing block. A simulation example is provided to support that it can simultaneously track the variation of slowly time-varying channels and detect unknown binary sequences well.

I. Introduction

Estimating a desired signal $\mu(j)$ from a given set of noisy data $\{z(j),\ j=1,2,...,N\}$ based on the convolutional model

$$z(j)=\mu(j)*v(j)+n(j) \tag{1}$$

is a deconvolution problem where $n(j)$ is white Gaussian noise with variance σ_n^2 and $v(j)$ is the impulse response of a linear time-invariant signal distorting system which corresponds to such as the source wavelet in seismic deconvolution, the channel impulse response in channel equalization and the vocal-tract impulse response in speech analysis/synthesis.

Kormylo and Mendel [1-3] proposed a Bernoulli-Gaussian (B-G) model, which has been used in seismic deconvolution, for a sparse spike sequence with random amplitudes as

$$\mu(j)=r(j)\cdot q(j) \tag{2}$$

where $r(j)$ is a zero-mean white Gaussian random sequence with variance σ_r^2 and $q(j)$ is Bernoulli for which

$$P_r[q(j)]=\begin{cases}\lambda, & q(j)=1\\ 1-\lambda, & q(j)=0.\end{cases} \tag{3}$$

Quite many B-G model based maximum-likelihood deconvolution (MLD) off-line algorithms as well as adaptive algorithms such as [1-7] have been reported in the past decade.

Recently Chi and Chen [8] proposed an adaptive B-G model based MLD algorithm for estimating positive sparse spike trains, and it has been successfully applied to deconvolution of voiced speech signals because a positive sparse spike train can be modeled as a B-G signal by letting $E[r(j)]=m_r>0$ and $m_r/\sigma_r\gg 1$. Furthermore, they found that, by setting $m_r>0$ and $\sigma_r^2=0$, binary random sequences of $\{m_r,-m_r\}$ can also be modeled as

$$\mu(j)=r(j)\cdot q(j)=m_r q(j) \tag{4}$$

where $q(j)=-1$ or $q(j)=1$ with equal probability. Then they [9] proposed a B-G model based maximum-likelihood (ML) channel equalizer for the detection of binary sequences (modeled as (4)) assuming that the channel impulse response $v(j)$ is known a priori. It not only works as well as but also requires smaller computational load and storage than some well-known maximum-likelihood (ML) channel equalizers such as [10-12]. In this paper, we propose an adaptive B-G model based ML channel equalizer for the detection of binary sequences (modeled as (4)) transmitted through an unknown slowly time-varying channel.

II. An Adaptive B-G Model Based ML Channel Equalizer

The new adaptive B-G model based ML channel equalizer is a block signal processing algorithm. Let the size of signal processing block be $2L$ and the contiguous blocks have a 50% overlap. A block of $z(j)$, $j=k,k+1,...,k+2L-1$ is processed to yield $\hat{q}(k),\hat{q}(k+1),...,\hat{q}(k+L-1)$ and then the next block of $z(j)$, $j=k+L,k+L+1,...,k+3L-1$, is processed to yield $\hat{q}(k+1),...,\hat{q}(k+2L-1)$. $\hat{q}(j)$ for $j\geq k+2L$ are obtained so on and so forth.

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Assume that, within any signal processing block, $v(j)$ is a time-invariant pth-order autoregressive moving average (ARMA(p,p-1)) system with a system transfer function as follows:

$$V(z) = \frac{\beta_1 + \beta_2 z^{-1} + ... + \beta_p z^{-(p-1)}}{1 - \alpha_1 z^{-1} - ... - \alpha_p z^{-p}}. \quad (5)$$

The convolutional model (1) can also be represented in a pth-order state-variable form as

$$\underline{x}(j) = \Phi \underline{x}(j-1) + \underline{\gamma}\, m_r\, q_r(j) \quad (6)$$

$$z(j) = \underline{h}^t \underline{x}(j) + n(j) \quad (7)$$

where $\underline{x}(j)$, $\underline{\gamma}$ and $\underline{h}$ are $p \times 1$ vectors, Φ is a $p \times p$ matrix. Note that $v(j) = \underline{h}^t \Phi^j \underline{\gamma}$ and that given $V(z)$ there exist many $(\Phi, \underline{\gamma}, \underline{h})$'s.

Let

$$\underline{\theta} = (\alpha_1, \alpha_2, ..., \alpha_p, \beta_1, \beta_2, ..., \beta_{p-1})^t, \quad (8)$$

$$\underline{z}_k = (z(1), z(2), ..., z(k+2L-1))^t, \quad (9)$$

$$\underline{q}_k = (q(1), q(2), ..., q(k+2L-1))^t \quad (10)$$

and

$$\underline{e}_k = (e(1), e(2), ..., e(k+2L-1))^t \quad (11)$$

where

$$e(j) = z(j) - m_r\, q(j) * v(j). \quad (12)$$

The new adaptive B-G model based ML channel equalizer tries to search for $\underline{q}_k = \hat{\underline{q}}_k$, $\underline{\theta} = \hat{\underline{\theta}}_k$ and $\sigma_n^2 = \hat{\sigma}_n^2(k)$ such that the likelihood function

$$S_k\{\underline{q}_k, \underline{\theta}, \sigma_n^2 | \underline{z}_k\} = p(\underline{z}_k | \underline{q}_k, \underline{\theta}, \sigma_n^2) \cdot P_r(\underline{q}_k | \lambda = 0.5)$$

$$= \frac{1}{(2\pi\sigma_n^2)^{(k+2L-1)/2}} \cdot exp\left(-\frac{\underline{e}_k^t \underline{e}_k}{2\sigma_n^2}\right) \cdot \left(\frac{1}{2}\right)^{k+2L-1} \quad (13)$$

is maximum under the "adaptiveness constraint":

(C1) $q(j) = \hat{q}(j)$ and $e(j) = \hat{e}(j) = z(j) - m_r \hat{q}(j) * \hat{v}(j)$ for $j \leq k-1$, where $\hat{q}(j)$ is the detected $q(j)$ prior to time k.

Our approach for finding a local maximum of S_k is an iterative block component method (BCM) [1,2] shown in Figure 1, where M is the allowed maximum number of iterations and is set ahead of time. Whenever a block of parameters, $\hat{\underline{q}}_k$ or $\hat{\underline{\theta}}_k$ or $\hat{\sigma}_n^2(k)$, is updated with the other parameters fixed, S_k is guaranteed to increase. We, next, present how to update $\hat{\underline{q}}_k$, $\hat{\underline{\theta}}_k$ and $\hat{\sigma}_n^2(k)$ by processing the measurement block of $z(k), z(k+1), ..., z(k+2L-1)$.

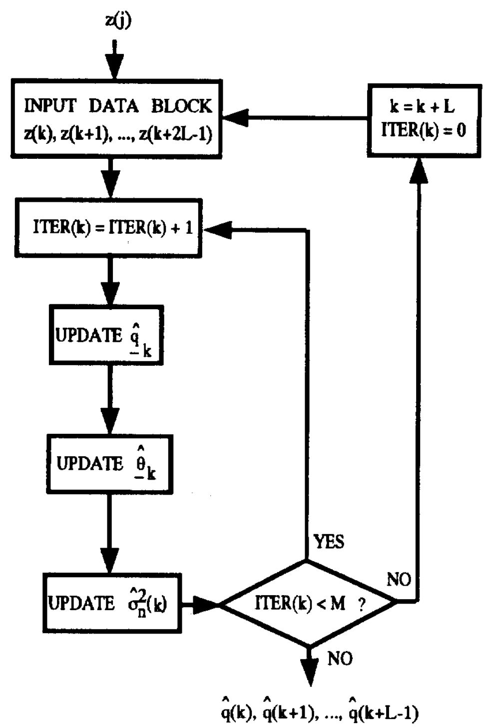

Figure 1. The signal processing procedure of the proposed adaptive B-G model based ML channel equalizer.

A. Detection of $q(j)$ *for* $j = k, k+1, ..., k+L-1$*:*

The well-known iterative single-most-likely-replacement (SMLR) algorithm [2,9] with some necessary modifications is suited for the detection of $q(j)$. Let $\Lambda(j)$ denote the likelihood ratio

$$\Lambda(j) = \frac{S_k\{\underline{q}_j^*, \underline{\theta}, \sigma_n^2 | \underline{z}_k\}}{S_k\{\underline{q}_r^*, \underline{\theta}, \sigma_n^2 | \underline{z}_k\}} \quad (14)$$

where $\underline{q}_r^* = (q_r(1) = \hat{q}(1), q_r(2) = \hat{q}(2), ..., q_r(k-1) = \hat{q}(k-1), q_r(k), q_r(k+1), ..., q_r(k+2L-1))^t$ is a reference sequence (due to (C1)) and $\underline{q}_j^* = (q_r(1), q_r(2), ..., q_r(j-1), q_r(j) = -q_r(j), q_r(j+1), ..., q_r(k+2L-1))^t$ is a test sequence which differs from $\underline{q}_r^*$ only at a single time location j. During the recursion k, the iterative detection algorithm searches for the optimum $\hat{\underline{q}}_k$ as follows:

(A) Compute $ln\Lambda(j)$ for $j = k, k+1, ..., k+2L-1$.

(B) Assume that $ln\Lambda(j') = max\{ln\Lambda(j), k \leq j \leq k+2L-1\}$; if $ln\Lambda(j') > 0$, update $q_r(j')$ by $-q_r(j')$ and go to (A).

When $ln\Lambda(j) \leq 0$ for all $k \leq j \leq k + 2L - 1$, the detection procedure is finished and the first L elements of the obtained $\underline{q}^*$ are the desired estimates $\hat{q}(k), \hat{q}(k+1), ..., \hat{q}(k + L - 1)$. It has been shown in [9] that

$$ln\Lambda(j) = -2\, m_r \,\{q_r(j) f_j + m_r a_j\} \quad (15)$$

where

$$f_j = \underline{\gamma}^t \underline{w}(j), \quad (16)$$

$$a_j = \underline{\gamma}^t C_w(j) \underline{\gamma} \quad (17)$$

in which the $p \times 1$ vector $\underline{w}(j)$ and the $p \times p$ matrix $C_w(j)$ can be obtained by running

$$\hat{\underline{x}}(j) = \Phi\, \hat{\underline{x}}(j-1) + \underline{\gamma}\, m_r q_r(j) \quad (18)$$

$$\tilde{z}(j) = z(j) - \underline{h}^t\, \hat{\underline{x}}(j) \quad (19)$$

forwards from $j = k$ to $k + 2L - 1$ and then running

$$\underline{w}(j) = \Phi^t \underline{w}(j+1) + \underline{h}\, \tilde{z}(j)/\sigma_n^2 \quad (20)$$

$$C_w(j) = \Phi^t C_w(j+1)\Phi + \underline{h}\, \underline{h}^t/\sigma_n^2 \quad (21)$$

backwards from $j = k + 2L - 1$ to k. The initial condition $\hat{\underline{x}}(k-1)$ for (18) is associated with S_{k-L}, and thus is available priori to time point k. The initial conditions for (20) and (21) are $\underline{w}(k + 2L) = \underline{0}$ (zero vector) and $C_w(k + 2L) = [0]$ (zero matrix), respectively.

B. Estimation of $\underline{\theta}$

Maximizing S_k given by (13) with respect to $\underline{\theta}$ under the constraint **(C1)** is equivalent to minimizing the following highly nonlinear objection function

$$J(\underline{\theta}) = \sum_{j=k}^{k+2L-1} \frac{1}{2} e^2(j). \quad (22)$$

Estimating the system parameter $\underline{\theta}$ with the system input $\hat{\mu}(j) = m_r \hat{q}(j)$ and the output $z(j)$ based on $J(\underline{\theta})$ is nothing but the well-known prediction error identification method [13]. We use a Newton-Raphson type iterative algorithm to search for a local minimum of $J(\underline{\theta})$ and the associated $\hat{\underline{\theta}}$.

C. Estimation of σ_n^2

Setting the partial derivative of S_k with respect to σ_n^2 equal to zero, one can obtain

$$\hat{\sigma}_n^2 = \frac{1}{k + 2L - 1} \left(\sum_{j=1}^{k-1} \hat{e}(j)^2 + \sum_{j=k}^{k+2L-1} e(j)^2 \right) \quad (23)$$

where we have used $e(j) = \hat{e}(j)$ for $j \leq k - 1$ (see **(C1)**).

III. A Simulation Example

In this section, we present a simulation example to support the proposed adaptive B-G model based ML channel equalizer. A time-varying channel with $V(z) = \beta/(1 - \alpha(j)\, z^{-1})$ represented by the following state-variable model

$$x(j) = \alpha(j) x(j-1) + m_r q(j) \quad (24)$$

$$z(j) = \beta x(j) + n(j) \quad (25)$$

was used in our simulation where $\beta = 1$, $\alpha(j) = 0.3 + 0.5\, sin(j/6000)$, $m_r = 1$ and $\sigma_n^2 = 0.1582$. The parameters L and M used in the proposed adaptive equalizer were $L = 128$ and $M = 1$, respectively. The simulation results are shown in Figure 2, from which one can see that estimate $\hat{\alpha}(j)$ (dashdot line) shown in Figure 2(a) tracks $\alpha(j)$ (solid line) very well and that all estimates $\hat{\beta}$ (dot's) shown in Figure 2(a) are quite close to $\beta = 1$. The cumulative symbol error rate (SER(k)) shown in Figure 2(b), defined as

$$\text{Cumulative SER}(k) = \frac{\text{number of correct detections of } q(j) \text{ up to } j = k}{k}, \quad (26)$$

converges toward 0.007 after the initial transient overshoot. These simulation results manifest the good performance of the proposed adaptive B-G model based ML channel equalizer for this time-varying channel.

IV. Conclusions

We have presented a new adaptive ML channel equalizer based on the modified B-G model given by (4) for the detection of binary sequences transmitted through an unknown slowly time-varying channel. It is also an adaptive block signal processing algorithm with 50% overlap based on the likelihood function S_k (see (13)) under the constraint **(C1)**, and it is implemented by a block component method shown in Figure 1. We also provided a simulation example to support that it can track the variation of slowly time-varying channels well and detect unknown binary sequences well in the meantime. On the other hand, both computational load and storage required by the proposed adaptive channel equalizer are linearly rather than exponentially proportional to the size of signal processing block.

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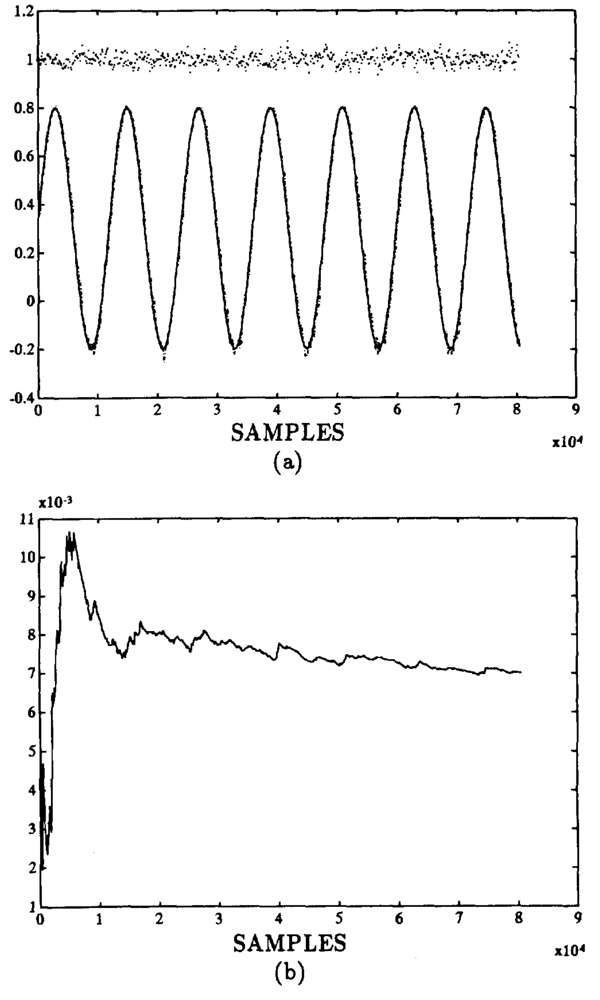

Figure 2. Simulation results for a single-pole channel with $V(z) = \beta/(1-\alpha(j)z^{-1})$. (a) Dashdot line denotes estimate $\hat{\alpha}(j)$, solid line denotes true $\alpha(j)$ and dot's denote estimate $\hat{\beta}$; (b) cumulative SER(k).