A NEW CUMULANT BASED PHASE ESTIMATION METHOD FOR NONMINIMUM-PHASE SYSTEMS BY ALLPASS FILTERING

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ABSTRACT

This paper presents a new cumulant based phase estimation method for linear time-invariant (LTI) systems with only non-Gaussian measurements contaminated by Gaussian noise. An optimum allpass filter is designed to process the given measurements such that its output has a maximum Mth-order (≥ 3) cumulant in absolute value. It can be shown that the system phase is equivalent to the negative value of the optimum allpass filter phase except for a linear phase factor. Some simulation results are provided to support the proposed phase estimation method.

1. INTRODUCTION

The identification of a linear time-invariant (LTI) system $h(n)$ with only noisy output $x(n)$ based on the following convolutional model

$$x(n) = u(n) * h(n) + w(n) \tag{1}$$

is very important in many signal processing areas such as seismic deconvolution, channel equalization in communications, radar, sonar, speech processing and image processing. Recently, cumulant based system identification of nonminimum-phase LTI systems with only non-Gaussian output measurements has drawn extensive attention in the previous signal processing areas because cumulants, which are blind to any kind of a Gaussian process, can be used to not only extract the amplitude information but also the phase information of $h(n)$, meanwhile they are inherently immune from Gaussian measurement noise.

Let $H(z)$ be the transfer function of the unknown system $h(n)$ and $H(\omega) = H(z = exp\{j\omega\}) = |H(\omega)| \cdot exp\{j\theta(\omega)\}$. There exist quite many methods for estimating the system phase $\theta(\omega)$ using either higher-order cumulants or polyspectra of $x(n)$ and they basically form three categories. The first category is composed of phase estimation methods [1,2] which estimate $\theta(\omega)$ from the phase of polyspectra of $x(n)$. The second category includes parametric (model based) estimation methods [3-5] which estimate the coefficients of $H(z)$ from cumulants of $x(n)$ and then compute $\theta(\omega)$ from the estimated $H(z)$. Therefore, the amplitude estimation is not separable from the phase estimation. The third category consists of minimum-phase (MP) - allpass (AP) decomposition based methods [6-8] which preprocess $x(n)$ using a correlation based whitening filter to estimate the spectrally equivalent MP system $H_{MP}(z)$ and then estimate the allpass system $H_{AP}(z) = H(z)/H_{MP}(z)$.

In this paper, we propose a new phase estimation method by allpass filtering which only uses a single Mth-order ($M \geq 3$) cumulant. The proposed method differs from the existing phase estimation methods in that it never uses the phase of polyspectra of $x(n)$; it is a parametric estimation method without involving amplitude estimation and thus it is never an inverse filtering algorithm; it never needs any preprocessing with a correlation based whitening filter.

2. A NEW CUMULANT BASED PHASE ESTIMATION METHOD

Assume that data $x(n), n = 0, 1, \ldots, N-1$, were generated from the model given by (1) under the following assumptions:

(A1) $H(z)$ is a causal stable LTI system which can be minimum-phase or nonminimum-phase.

(A2) The input $u(n)$ is real, zero-mean, stationary, independent identically distributed (i.i.d.), non-Gaussian with Mth-order ($M \geq 3$) cumulant γ_M.

(A2) The measurement noise $w(n)$ is Gaussian which can be white or colored with unknown statistics.

(A2) The input $u(n)$ is statistically independent of $w(n)$.

The new phase estimation method is based on the following theorem:

__Theorem 1.__ Assume that $x(n)$ is the noisy signal generated from (1) under assumptions **(A1)** through **(A4)**. Let $\theta(\omega)$ denote the phase of the unknown system $H(z)$ and $y(n)$ be the output of an allpass filter $H_a(\omega) = exp\{j\varphi(\omega)\}$ with input $x(n)$. Then the Mth-order ($M \geq 3$) cumulant $C_{M,y}(k_1 = 0, \ldots, k_{M-1} = 0)$ of $y(n)$ is maximum in absolute value if and only if

$$\varphi(\omega) = -\theta(\omega) + \alpha\omega \tag{2}$$

where α is an unknown constant. Moreover, the maximum of $|C_{M,y}(0,\ldots,0)|$ is given by

$$max\ \{\ |C_{M,y}(0,\ldots,0)|\ \} = \left(\frac{1}{2\pi}\right)^{M-1} \cdot \int_0^{2\pi} \cdots \int_0^{2\pi} |S_{M,x}(\omega_1,\ldots,\omega_{M-1})| d\omega_1 \cdots d\omega_{M-1} \quad (3)$$

where $S_{M,x}(\omega_1,\ldots,\omega_{M-1})$ is the Mth-order polyspectrum of $x(n)$.

Assume that $H_p(z)$ is a pth-order causal stable allpass filter with transfer function

$$H_p(z) = \frac{a_p + a_{p-1}z^{-1} + \cdots + a_1 z^{-p+1} + z^{-p}}{1 + a_1 z^{-1} + \cdots + a_p z^{-p}} \quad (4)$$

and $y(n)$ be the output of $H_p(z)$ with input $x(n)$ as follows:

$$y(n) = -\sum_{k=1}^{p} a_k y(n-k) + x(n-p) + \sum_{k=1}^{p} a_k x(n+k-p). \quad (5)$$

Based on Theorem 1, the new phase estimation method searches for the desired $\hat{H}_p(z)$, whose phase $\hat{\varphi}(\omega) = \angle\hat{H}_p(\omega)$ is used as an approximation to the $\varphi(\omega)$ described in Theorem 1, by maximizing

$$J(\underline{a}_p) = \widehat{C}^2_{M,y}(0,\cdots,0) \quad (6)$$

with respect to the coefficients $\underline{a}_p = (a_1,\ldots,a_p)^T$, where $\widehat{C}_{M,y}(0,\ldots,0)$ is the Mth-order sample cumulant of $y(n)$. Because the objective function $J(\underline{a}_p)$ is a highly nonlinear function of $\underline{a}_p$, a Newton-Raphson type iterative algorithm is used to search for the desired $\underline{\hat{a}}_p$. After $\hat{H}_p(z)$ is obtained, $\hat{\varphi}(\omega)$ is obtained and $\hat{\theta}(\omega)$ can then be obtained by (2) except for a linear phase factor (a constant time delay). The procedure of the proposed phase estimation method is summarized as follows:

(S1) Set the allpass filter order $p = 0$ (i.e., $H_0(z) = 1$) and compute $J(\underline{a}_0)$.

(S2) Set $p = p + 1$.

(S3) Search for the $J(\underline{\hat{a}}_p)$ (maximum of $J(\underline{a}_p)$) by a Newton-Raphson type iterative algorithm with the initial conditions $\underline{a}_p(0) = (\underline{\hat{a}}^T_{p-1}, 0)^T$.

(S4) If $p \leq p_{max}$ and

$$\frac{J(\underline{\hat{a}}_p) - J(\underline{\hat{a}}_{p-1})}{J(\underline{\hat{a}}_{p-1})} > \xi$$

where p_{max} is the maximum allowed order for the allpass filter and ξ is a preassigned small positive constant, then go to **(S2)**, otherwise stop.

Some worthy remarks regarding the proposed phase estimation method are as follows:

(R1) The iterative search algorithm used in **(S3)** guarantees the increase of $J(\underline{a}_p)$ whenever $\underline{\hat{a}}_p$ is updated. On the other hand, $|\widehat{C}_{M,y}(0,\ldots,0)|$ is bounded by Theorem 1. Therefore, the convergence of the proposed method is guaranteed.

(R2) The proposed phase estimation method only uses a single Mth-order cumulant $C_{M,y}(0,\ldots,0)$ for any $M \geq 3$ as long as the Mth-order cumulant γ_M of the driving input $u(n)$ is not equal to zero.

(R3) In addition to the causal stable allpass filter $H_p(z)$, the anticausal stable allpass filter with transfer function $H'_p(z) = H_p(z^{-1})$ can also be used and then $y(n)$ must be computed backwards from $x(n)$.

(R4) Chi and Kung [9,10] proposed a cumulant based allpass system identification algorithm which estimates the phase of an unknown (nonminimum-phase) allpass system by maximizing $J(\underline{a}_p)$ given by (6) as well. The proposed phase estimation method generalizes their algorithm to the case of nonminimum-phase LTI systems.

(R5) The amplitude response estimate $|\hat{H}(\omega)|$, which can be obtained using existing spectral estimation methods or cumulant based methods, and the phase response estimate $\hat{\theta}(\omega)$ obtained by the proposed method suggest the deconvolution by the inverse filter $(1/|\hat{H}(\omega)|) \cdot exp\{-j\hat{\theta}(\omega)\}$ of the estimated LTI system.

3. SIMULATION RESULTS

Two simulation examples are presented to demonstrate that the proposed phase estimation method is effective. Example 1 includes some performance tests to the proposed method and Example 2 is seismic deconvolution with an inverse filter mentioned in **(R5)**. Next, let us turn to Example 1.

Example 1. (Performance test)
The driving input $u(n)$ used was a zero-mean Exponentially distributed i.i.d. sequence with variance $\sigma_u^2 = 1$ and skewness $\gamma_3 = 2$. An ARMA(3,2) nonminimum-phase system (taken from [5]) with transfer function

$$H(z) = \frac{1 - 2.95\ z^{-1} + 1.9\ z^{-2}}{1 - 1.3\ z^{-1} + 1.05\ z^{-2} - 0.32\ z^{-3}} \quad (7)$$

was used. The synthetic data $x(n)$ of length $N = 1024$ were generated for signal-to-noise ratio (SNR) equal to 10. Thirty independent runs were performed with the cumulant order $M = 3$ and the (causal stable) allpass filter order $p_{max} = 5$ in the proposed method. The obtained thirty system phase estimates with unknown linear phase factors artificially removed are shown in Figure 1(a) and their average (dashed line) is shown in Figure 1(b) together with the true system phase (solid line). The results shown in these two figures indicate that the proposed method is unbiased with a small variance.

Example 2. (Seismic deconvolution)
The driving input $u(n)$ used was a zero-mean Bernoulli-Gaussian (B-G) random sequence with variance $\sigma_u^2 = 1$, skewness $\gamma_3 = 0$ and kurtosis $\gamma_4 = 0.27$. An ARMA(3,3) nonminimum-phase system (taken from [9,10]) with transfer function

$$H(z) = \frac{1 + 0.1\ z^{-1} - 3.2725\ z^{-2} + 1.41125\ z^{-3}}{1 - 1.9\ z^{-1} + 1.1525\ z^{-2} - 0.1625\ z^{-3}} \quad (8)$$

was used. The synthetic data $x(n)$ for $N = 512$ and $SNR = 100$ are shown in Figure 2(a). The system phase was estimated using the proposed method with the cumulant order $M = 4$ and the (anticausal stable) allpass filter order $p_{max} = 7$. The obtained system phase estimate $\hat{\theta}(\omega)$ (dashed line) with the unknown linear phase factor artificially removed is shown in Figure 2(b) together with the true system phase (solid line). Data $x(n)$ were then processed by a tenth-order cumulant based (minimum-phase) linear prediction error (LPE) filter $V(z)$ [11,12] to get the deconvolved signal $\tilde{u}(n)$ (dashed line) shown in Figure 2(c). Finally, data $x(n)$ were processed by the inverse filter $|V(\omega)|exp\{-j\hat{\theta}(\omega)\}$ to get the deconvolved signal $\hat{u}(n)$ (dashed line) shown in Figure 2(d). One can see, from Figures 2(c) and 2(d), that the deconvolved signal shown in the latter approximates the true input signal $u(n)$ (solid line) much better than that shown in the former except for a scale factor. The reason for this is simply that a phase distortion remains in the deconvolved signal shown in Figure 2(c) because the (nonminimum) phase of the system can not be equalized by the (minimum) phase of the LPE filter.

4. CONCLUSIONS

A new cumulant based phase estimation method with non-Gaussian measurements based on Theorem 1 has been presented which estimates the phase response of an unknown LTI system from the phase of an optimum pth-order ARMA allpass filter (see (2)). The optimum allpass filter was obtained by maximizing a single output cumulant in absolute value with input being the given non-Gaussian measurements. The proposed method is implemented by a nonlinear optimization algorithm whose convergence is guaranteed (see **(R1)**). It is also an "amplitude - phase" decomposition based method without amplitude estimation throughout the algorithm. It is applicable for all $M \geq 3$ as long as the Mth-order cumulant γ_M of the driving input $u(n)$ is not equal to zero (see **(R2)**). Finally, some simulation results were provided to justify the good performance of the proposed phase estimation method.

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REFERENCES

[1] T. Matsuoka and T. J. Ulrych, "Phase estimation using the bispectrum," Proc. IEEE, vol. 72, no. 10, pp. 1403-1411, Oct. 1984.

[2] R. Pan and C. L. Nikias, "Phase reconstruction in the trispectrum domain," IEEE Trans. Acoustics, Speech, and Signal Processing, vol. ASSP-35, no. 6, pp. 895-897, June 1987.

[3] J. M. Mendel, "Tutorial on higher-order statistics (spectra) in signal processing and system theory; theoretical results and some applications," Proc. IEEE, vol. 79, no. 3, pp. 278-305, March 1991.

[4] C. L. Nikias and A. P. Petropulu, *Higher-Order Spectra Analysis,* Prentice-Hall, 1993.

[5] G. B. Giannakis, "Wavelet parameter and phase estimation using cumulant slices," IEEE Trans. Geoscience and Remote Sensing, vol. 27, no. 4, pp. 452-455, July 1989.

[6] J. K. Tugnait, "Identification of nonminimum phase linear stochastic systems," Automatica, vol. 22, no. 4, pp. 457-464, 1986.

[7] G. B. Giannakis and J. M. Mendel, "Stochastic realization of nonminimum phase systems," Proc. 1986 American Control Conf., vol. 2, pp. 1254-1259, Seattle, WA.

[8] C.-Y. Chi and J.-Y. Kung, "A phase determination method for nonminimum phase ARMA systems by a single cumulant sample," IEEE Trans. Signal Processing, vol. 41, no. 2, pp. 981-985, Feb. 1993.

[9] C.-Y. Chi and J.-Y. Kung, "A new identification algorithm for allpass systems by higher-order statistics," Proc. Sixth European Signal Processing Conference (EUSIPCO-92), vol. 2, pp. 779-782, Brussels, Belgium, Aug. 24-27, 1992.

[10] C.-Y. Chi and W.-T. Chen, "The deconvolution of speech signals by the utilization of higher-order statistics based algorithm," a Chapter in Control and Dynamic Systems, C. T. Leondes, ed., Academic Press, 1994.

[11] C.-Y. Chi and W.-J. Chang and C. -C. Feng, "A new algorithm for the design of linear prediction error filters using cumulant-based MSE criteria," to appear in IEEE Trans. Signal Processing.

[12] C.-Y. Chi and W.-J. Chang, "New higher-order statistics based criteria for the design of linear prediction error filters," Proc. IEEE Signal Processing Workshop on Higher-Order Statistics, pp. 106-110, South Lake Tahoe, CA, June 1993.

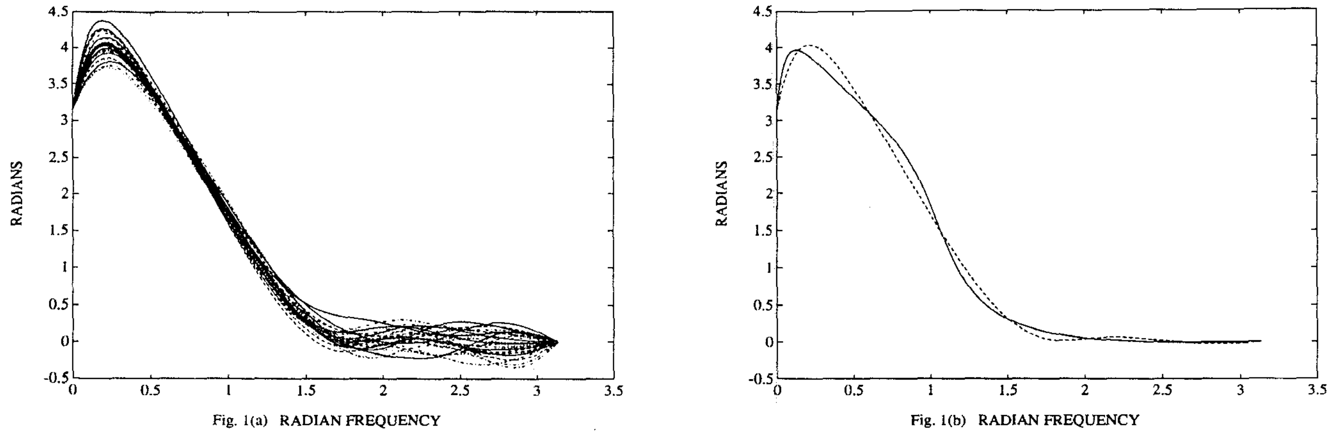

Fig. 1. Simulation results associated with Example 1 for SNR $= 10$ and $N = 1024$. (a) Thirty system phase estimates $\hat{\theta}(\omega)$ with the unknown linear phase factor artificially removed for each estimate; (b) the average (dashed line) of the thirty estimates shown in Part (a) and the true system phase response (solid line).

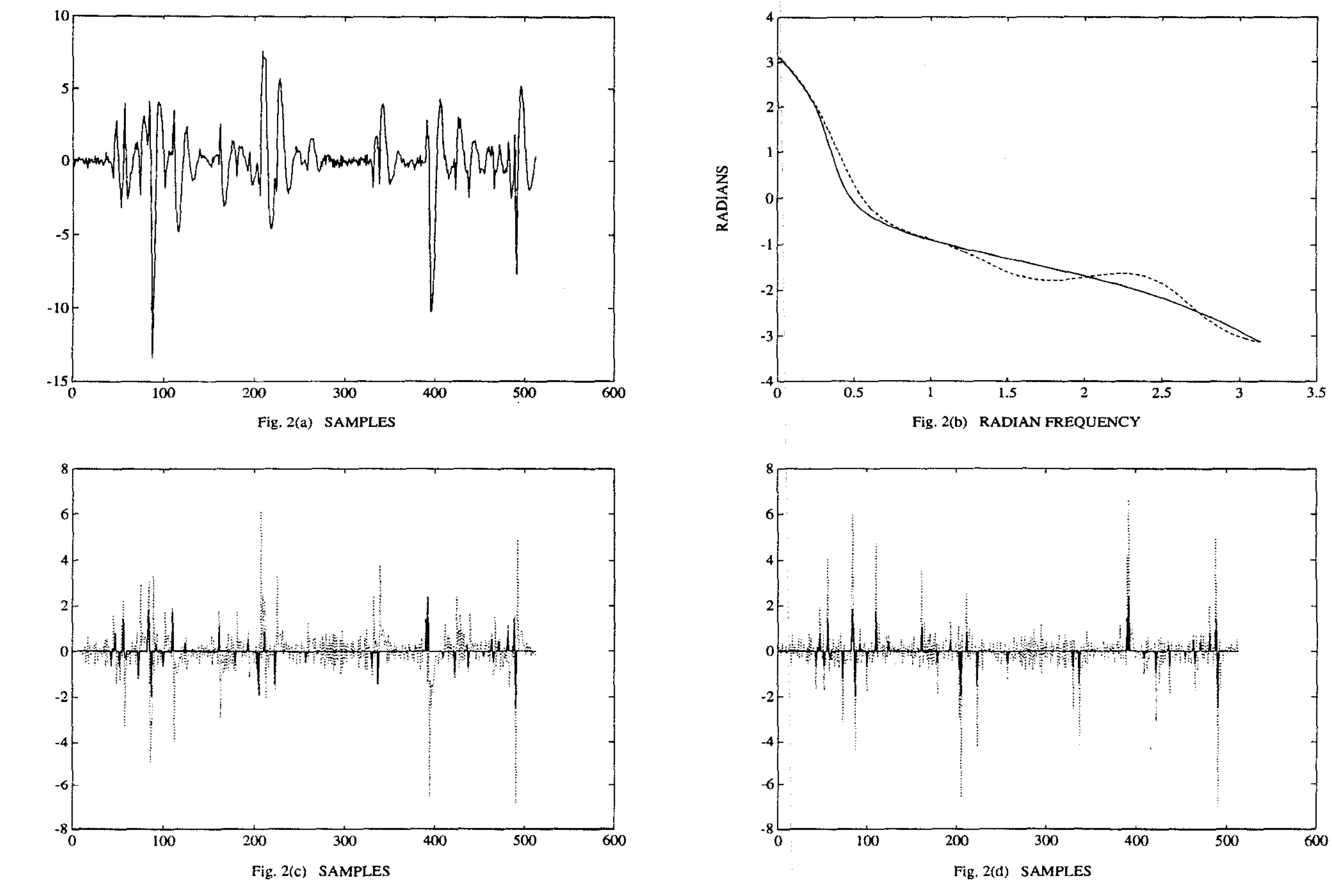

Fig. 2. Simulation results associated with Example 2. (a) Synthetic seismic data $x(n)$ of length $N = 512$ for SNR $= 100$; (b) the estimated phase response $\hat{\theta}(\omega)$ (dashed line) of the system with the unknown linear phase factor artificially removed and the true system phase response (solid line); (c) the deconvolved signal $\tilde{u}(n)$ (dotted line) obtained by a tenth-order LPE filter $V(z)$ and the true input signal $u(n)$ (solid line); (d) the deconvolved signal $\hat{u}(n)$ (dotted line) obtained by the inverse filter $|V(\omega)|exp\{-j\hat{\theta}(\omega)\}$ and the true input signal $u(n)$ (solid line).