2-D BLIND DECONVOLUTION USING FOURIER SERIES BASED MODEL AND HIGHER-ORDER STATISTICS WITH APPLICATION TO TEXTURE SYNTHESIS

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ABSTRACT

With a given set of non-Gaussian output measurements of a 2-D linear shift-invariant (LSI) system, a 2-D blind deconvolution algorithm is proposed that uses Chi's Fourier series based model (FSBM) for the unknown system and the cumulant based inverse filter criteria proposed by Chi and Wu, and Tuganit. The proposed algorithm is an iterative optimization algorithm that is computationally efficient with a parallel structure. The estimated FSBM for the unknown system that can be nonseparable or noncausal, is guaranteed to be stable. Then application of the proposed algorithm to texture synthesis with real texture images is also presented, in addition to some simulation results. Finally, we draw some conclusions.

1. INTRODUCTION

The two-dimensional (2-D) blind deconvolution is a widely known problem of estimating the desired signal $u(m,n)$ and the unknown linear shift-invariant (LSI) system $h(m,n)$ with only a set of non-Gaussian measurements

$$x(m,n) = u(m,n) * h(m,n) + w(m,n) \qquad (1)$$

where $w(m,n)$ is measurement noise. Recently, higher-order (≥ 3) cumulants, that contain both amplitude and phase information of the unknown system $h(m,n)$, have been used for 2-D blind deconvolution as well as image modeling [1], texture image analysis and classification [2,3] and texture image synthesis [4,5].

Let $v(m,n)$ denote a stable deconvolution filter (an inverse filter) and

$$e(m,n) = x(m,n) * v(m,n) \qquad (2)$$

be the output signal by processing $x(m,n)$ with this filter. Chi and Wu's 2-D inverse filter criteria $J_{r,k}(v(m,n))$ defined as [1]

$$J_{r,k}(v(m,n)) = \frac{|C_k\{e(m,n)\}|^r}{|C_r\{e(m,n)\}|^k} \qquad (3)$$

where r is even and $k > r$, and $C_l\{y\}$ denote the lth-order cumulant of random variable y, have been used for 2-D deconvolution [1] and texture image synthesis [4,5]. However, the model used for both the 2-D system $H(z_1,z_2)$ (2-D z-transform of $h(m,n)$) and the 2-D inverse filter $V(z_1,z_2)$ in [1,4,5] is the rational model, i.e., autoregressive (AR), moving average (MA) or autoregressive moving average (ARMA) model. Because the objective function $J_{r,k}(v(m,n))$ to be maximized is a highly nonlinear function of model parameters, one has to resort to iterative gradient type optimization algorithms for finding the optimum inverse filter $V(z_1,z_2)$. This leads to the following issue during the iterative search procedure:

(A) The stability of nonseparable ARMA systems $H(z_1,z_2)$ and $V(z_1,z_2)$ can hardly be guaranteed, and meanwhile, the computational load and complexity is significant, especially for computing the gradient of $J_{r,k}(v(m,n))$ with respect to model parameters.

Recently, Chien, Yang and Chi [6] proposed a Fourier series based model (FSBM) for 1-D and 2-D allpass filters. Chi [7] further proposed a 1-D FSBM for arbitrary linear time-invariant systems that can be useful in statistical signal processing due to the following characteristics:

(B) The stability of 1-D FSBM (that can be nonminimum phase or noncausal) is guaranteed, and meanwhile the associated iterative gradient type algorithm is computationally efficient with a simple parallel structure.

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In this paper, motivated by (A) and (B), a 2-D blind deconvolution algorithm is proposed that also maximizes Chi and Wu's inverse filter criteria $J_{r,k}(v(m,n))$ with the 2-D FSBM used for the unknown 2-D system $h(m,n)$ and its inverse system $v(m,n)$. Then the proposed algorithm is applied to texture synthesis since texture images can be modeled as (1) as reported in [3-5].

2. A 2-D DECONVOLUTION ALGORITHM

Assume that measurements $x(m,n)$ can be modeled as (1) where $h(m,n)$ is a real stable LSI system, $u(m,n)$ is real, zero-mean, stationary, non-Gaussian, and $w(m,n)$ is zero-mean Gaussian and statistically independent of $u(m,n)$.

A. 2-D FSBM

By extending Chi's FSBM for the 1-D case [7], the frequency response $V(\omega_1,\omega_2)$ of 2-D FSBM for the real stable inverse filter $v(m,n)$ can be expressed as

$$\begin{aligned} V(\omega_1,\omega_2) &= V^*(-\omega_1,-\omega_2) \\ &= \exp\left\{ \sum_{i_1=-p_1}^{p_1} \sum_{i_2=-p_2}^{p_2} \gamma_{i_1,i_2} e^{j(i_1\omega_1+i_2\omega_2)} \right\} \end{aligned} \quad (4)$$

where $\gamma_{0,0} = 0$ and γ_{i_1,i_2}'s are real. The sensitivity of the inverse filter output of $e(m,n)$ with respect to γ_{i_1,i_2}, $\partial e(m,n)/\partial \gamma_{i_1,i_2}$, is crucial to the computational complexity of the 2-D blind deconvolution algorithm to be presented below. It can be easily shown that

$$\frac{\partial e(m,n)}{\partial \gamma_{i_1,i_2}} = e(m+i_1, n+i_2) \quad (5)$$

B. Objective Function

The rth-order cumulant $C_r\{e(m,n)\}$ and the kth-order cumulant $C_k\{e(m,n)\}$ used in $J_{r,k}(v(m,n))$ have to be estimated from $x(m,n)$, or simply replaced with the associated sample cumulants, $\widehat{C}_r\{e(m,n)\}$ and $\widehat{C}_k\{e(m,n)\}$, respectively. In other words, the objective function to be minimized is given by

$$J(\gamma) = -J_{r,k}(\gamma) = -\frac{|\widehat{C}_k\{e(m,n)\}|^r}{|\widehat{C}_r\{e(m,n)\}|^k} \quad (6)$$

where r is even and $k > r$, and γ is a $p \times 1$ ($p = (2p_1+1)(2p_2+1) - 1$) column vector containing all the unknown parameters γ_{i_1,i_2} of the 2-D inverse filter $V(\omega_1,\omega_2)$, i.e.,

$$\gamma = [\gamma_{-p_1,-p_2}, \gamma_{-p_1+1,-p_2}, \ldots, \gamma_{p_1,p_2-1}, \gamma_{p_1,p_2}]^T \quad (7)$$

C. Algorithm

The proposed iterative 2-D blind deconvolution algorithm, that uses the iterative Fletcher-Powell algorithm [8] to find the minimum of $J(\gamma)$ and the optimum γ, is shown in Figure 1. Next, let us illuminate this 2-D deconvolution algorithm.

The parameter $\lambda_{\min}$ is the minimum for the step size λ and the parameter ε is for convergence tolerance. The initial $p \times p$ matrix $\mathbf{R}^{(0)}$ can be any positive definite matrix (e.g., identity matrix) which always leads to a positive $\mathbf{R}^{(i)}$ for $i > 0$ provided $J(\gamma^{(i)}) < J(\gamma^{(i-1)})$.

At each iteration, the proposed algorithm updates $\gamma^{(i)}$ by

$$\gamma^{(i)} = \gamma^{(i-1)} - \lambda \mathbf{R}^{(i-1)} \mathbf{g}^{(i-1)} \quad (8)$$

where

$$\mathbf{g}^{(i)} = \left.\frac{\partial J(\gamma)}{\partial \gamma}\right|_{\gamma=\gamma^{(i)}} = -\left.\frac{\partial J_{r,k}(\gamma)}{\partial \gamma}\right|_{\gamma=\gamma^{(i)}} \quad (9)$$

and

$$\begin{aligned} \mathbf{R}^{(i)} &= \mathbf{R}^{(i-1)} + \frac{1}{\mathbf{r}^T\mathbf{s}} \Big[(1 + \frac{\mathbf{s}^T\mathbf{R}^{(i-1)}\mathbf{s}}{\mathbf{r}^T\mathbf{s}}) \mathbf{r}\mathbf{r}^T \\ &\quad - \mathbf{r}\mathbf{s}^T\mathbf{R}^{(i-1)} - \mathbf{R}^{(i-1)}\mathbf{s}\mathbf{r}^T \Big] \end{aligned} \quad (10)$$

where $\mathbf{r} = \gamma^{(i)} - \gamma^{(i-1)}$ and $\mathbf{s} = \mathbf{g}^{(i)} - \mathbf{g}^{(i-1)}$. When the algorithm converges ($|[J(\gamma^{(i-1)}) - J(\gamma^{(i)})]/J(\gamma^{(i)})| < \varepsilon$),

$$\widehat{\gamma} = \gamma^{(i)} \quad (11)$$

D. Signal Processing Procedure

The signal processing procedure for obtaining the inverse filter output $e(m,n)$, the cost function $J(\gamma^{(i)})$ and the gradient $\mathbf{g}^{(i)}$ is shown in Figure 2. It can be easily seen that the signal $e(m,n)$ can be obtained using FFT. Then $J(\gamma^{(i)})$ can be computed using (6). To compute $\mathbf{g}^{(i)}$, one has to compute

$$\begin{aligned} \frac{\partial J(\gamma)}{\partial \gamma_{i_1,i_2}} = -\frac{(\widehat{C}_k\{e(m,n)\})^r}{(\widehat{C}_r\{e(m,n)\})^k} \Bigg\{ \frac{r}{\widehat{C}_k\{e(m,n)\}} \cdot \\ \frac{\partial \widehat{C}_k\{e(m,n)\}}{\partial \gamma_{i_1,i_2}} - \frac{k}{\widehat{C}_r\{e(m,n)\}} \cdot \frac{\partial \widehat{C}_r\{e(m,n)\}}{\partial \gamma_{i_1,i_2}} \Bigg\} \end{aligned} \quad (12)$$

which requires to compute $\partial \widehat{C}_k\{e(m,n)\}/\partial \gamma_{i_1,i_2}$. For instance, it can be easily shown that

$$\frac{\partial \widehat{C}_2\{e(m,n)\}}{\partial \gamma_{i_1,i_2}} = \frac{2}{N^2} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} e(m,n) \frac{\partial e(m,n)}{\partial \gamma_{i_1,i_2}} \quad (13)$$

$$\frac{\partial \widehat{C}_3\{e(m,n)\}}{\partial \gamma_{i_1,i_2}} = \frac{3}{N^2} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} e^2(m,n) \frac{\partial e(m,n)}{\partial \gamma_{i_1,i_2}} \quad (14)$$

$$\frac{\partial \widehat{C}_4\{e(m,n)\}}{\partial \gamma_{i_1,i_2}} = \frac{4}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} e^3(m,n)\frac{\partial e(m,n)}{\partial \gamma_{i_1,i_2}} - \frac{12}{N^2}\widehat{C}_2\{e(m,n)\}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} e(m,n)\frac{\partial e(m,n)}{\partial \gamma_{i_1,i_2}} \quad (15)$$

where $\partial e(m,n)/\partial \gamma_{i_1,i_2}$ is given by (5). Let us conclude this section with the following remarks:

(R1) The signal processing procedure for computing $e(m,n)$, $J(\gamma^{(i)})$ and $\mathbf{g}^{(i)}$ shown in Figure 2 is computationally efficient due to the simple parallel FIR filter bank structure.

(R2) The proposed algorithm provides a local optimum solution instead of the global optimum solution for $\widehat{\gamma}$. However, the obtained $\widehat{\gamma}$ guarantees the stability of both the optimum inverse filter $V(\omega_1,\omega_2)$ and $H(\omega_1,\omega_2) = 1/V(\omega_1,\omega_2)$.

3. APPLICATION TO TEXTURE SYNTHESIS

When $x(m,n)$ is a texture image that can be modeled as (1) under the previous assumptions, the proposed 2-D deconvolution algorithm shown in Figure 1 can be applied to obtain a synthetic image $\widehat{x}(m,n)$ that has the same statistical characteristics with $x(m,n)$ through the following procedure:

Texture Synthesis Procedure (TSP):

(1) Obtain the mean removed image $y(m,n) = x(m,n) - m_x$ where m_x is the mean of $x(m,n)$.

(2) Obtain the texture image model $H(\omega_1,\omega_2) = 1/V(\omega_1,\omega_2)$, modeled as an FSBM, and the optimum $e(m,n)$ by processing $y(m,n)$ with the proposed 2-D blind deconvolution algorithm.

(3) Generate an i.i.d. random field $u(m,n)$ that has the same histogram as $e(m,n)$. Then obtain the synthetic texture image $\widehat{x}(m,n)$ by

$$\widehat{x}(m,n) = u(m,n) * h(m,n) + m_x \quad (16)$$

which is then quantized into an integer valued image in terms of gray levels.

4. SIMULATION AND EXPERIMENTAL RESULTS

In the simulation and experiment, the proposed 2-D blind deconvolution algorithm was employed to estimate the FSBM parameters γ with $\lambda_{\min} = 10^{-10}$ and $\varepsilon = 10^{-8}$, $\gamma^{(0)} = \mathbf{0}$ and $\mathbf{R}^{(0)}$ equal to the identity matrix. Next, let us present some simulation results.

A zero-mean 256×256 exponentially distributed i.i.d. random field $u(m,n)$ and a 2-D ARMA model $h(m,n)$ with a non-symmetric support (taken from [4]) shown in Figure 3(a) were used to generate $x(m,n)$ for SNR = 20 dB and $w(m,n)$ being white Gaussian. The proposed 2-D deconvolution algorithm with $r = 2$ and $k = 3$ ($J_{2,3}$), $p_1 = p_2 = 5$ converged by spending 58 iterations and the estimated $\widehat{h}(m,n)$ is shown in Figure 3(b). One can see, from Figures 3(a) and 3(b), that $\widehat{h}(m,n)$ (an FSBM model) is a good approximation to $h(m,n)$ (an ARMA model).

Next, let us present some results for texture image synthesis that were obtained using the proposed 2-D deconvolution algorithm with $r = 2$ and $k = 4$ ($J_{2,4}$) through the TSP presented in Section 3. Figures 4(a) and 4(b) show a 128×128 sand texture image (taken from USC-SIPI Image Data Base 1.5.04) and the synthetic texture image, respectively. Figures 4(c) and 4(d) show a 128×128 wood grain texture image (taken from USC-SIPI Image Data Base 1.1.09) and the synthetic texture image, respectively. One can see from these figures that the synthetic texture images and the original texture images are quite similar. The synthetic texture images shown in Figures 4(b) and 4(d) were obtained by the proposed algorithm with $p_1 = p_2 = 5$ and 20 iterations, and with $p_1 = p_2 = 3$ and 68 iterations, respectively.

5. CONCLUSIONS

We have presented a 2-D blind deconvolution algorithm (shown in Figure 1) using Chi's 2-D nonseparable stable FSBM given by (4) and the inverse filter criteria $J_{r,k}(v(m,n))$ given by (3), that also shares the characteristics (B) of 1-D blind deconvolution algorithms using FSBM due to its simple signal processing procedure shown in Figure 2. Some simulation results and applications to texture image synthesis were presented to support the efficacy of the proposed algorithm.

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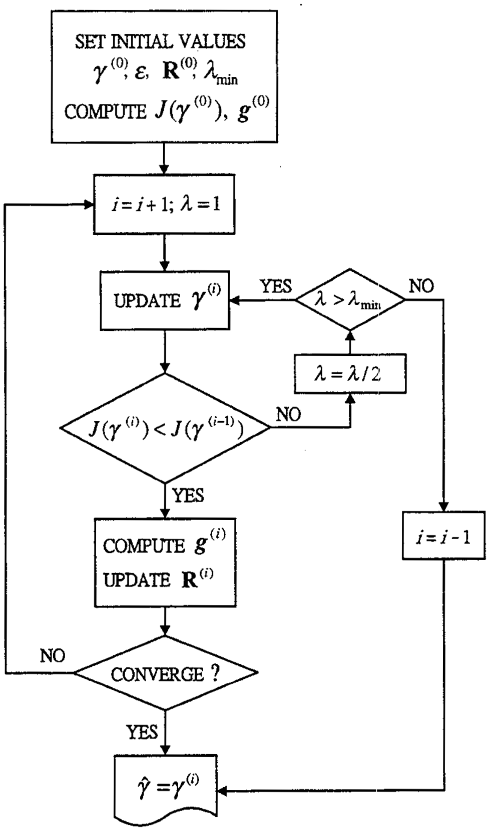

Fig. 1. The proposed 2-D deconvolution algorithm.

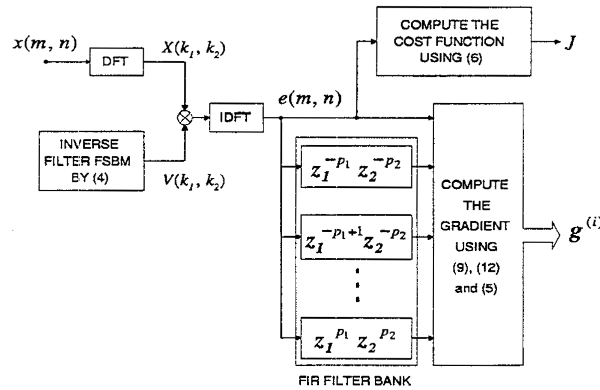

Fig. 2. The signal processing procedure of the proposed 2-D deconvolution algorithm.

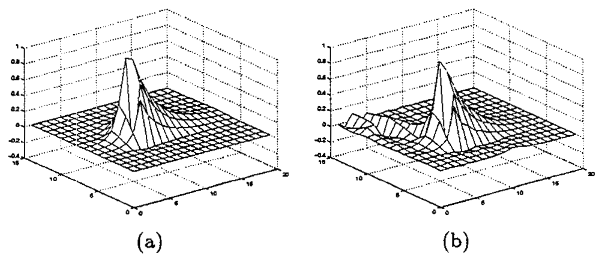

Fig. 3. (a) The impulse response of the true 2-D system $h(m,n)$ (an ARMA system) and that of the estimated 2-D system $\widehat{h}(m,n)$ (an FSBM) with $p_1 = p_2 = 5$.

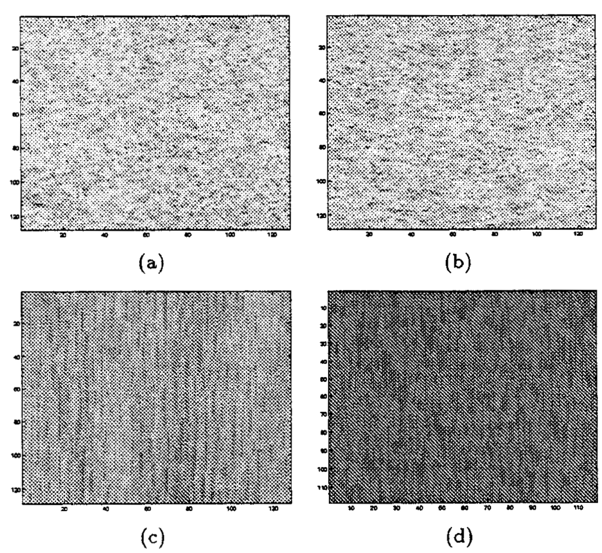

Fig. 4. (a) The original 128×128 sand texture image and (b) the synthetic 128×128 sand texture image; and (c) the original 128×128 wood grain texture image and (d) the synthetic 128×128 wood grain texture image.

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