Convex Optimization in Signal Processing and Communications

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Contents

List of contributors *page* iv

Part I 1

1 Convex Analysis for Non-negative Blind Source Separation with Application in Imaging 3

1.1 Introduction 3
1.2 Problem Statement 6
1.2.1 Assumptions 9
1.3 Review of Some Concepts in Convex Analysis 11
1.3.1 Affine Hull 11
1.3.2 Convex Hull 12
1.4 Non-negative Blind Source Separation Criterion via CAMNS 13
1.4.1 Convex Analysis of the Problem, and the CAMNS Criterion 13
1.4.2 An Alternative Form of the CAMNS Criterion 17
1.5 Systematic Linear Programming Method for CAMNS 20
1.6 Alternating Volume Maximization Heuristics for CAMNS 23
1.7 Numerical Results 27
1.7.1 Example of 3-Source Case: Cell Separation 28
1.7.2 Example of 4-Source Case: Ghosting Effect 29
1.7.3 Example of 5-Source Case: Human Face Separation 31
1.7.4 Monte Carlo Simulation: Noisy Environment 31
1.8 Summary and Discussion 34
1.9 Appendix 36
1.9.1 Proof of Lemma 1.2 36
1.9.2 Proof of Proposition 1.1 36
1.9.3 Proof of Lemma 1.5 37
1.9.4 Proof of Lemma 1.6 38
1.9.5 Proof of Lemma 1.7 38
1.9.6 Proof of Theorem 1.3 39

References 41
Index 44

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Part I

1 Convex Analysis for Non-negative Blind Source Separation with Application in Imaging

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In recent years, there has been a growing interest in blind separation of non-negative sources, as simply non-negative blind source separation (nBSS). Potential applications of nBSS include biomedical imaging, multi/hyper-spectral imaging, and analytical chemistry. In this chapter, we describe a rather new endeavor of nBSS, where convex geometry is utilized to analyze the nBSS problem. Called convex analysis of mixtures of non-negative sources (CAMNS), the framework described here makes use of a very special assumption called local dominance, which is a reasonable assumption for source signals exhibiting sparsity or high contrast. Under the local dominant and some usual nBSS assumptions, we show that the source signals can be perfectly identified by finding the extreme points of an observation-constructed polyhedral set. Two methods for practically locating the extreme points are also derived. One is analysis-based with some appealing theoretical guarantees, while the other is heuristic in comparison but is intuitively expected to provide better robustness against model mismatches. Both are based on linear programming and thus can be effectively implemented. Simulation results on several data sets are presented to demonstrate the efficacy of the CAMNS-based methods over several other reported nBSS methods.

1.1 Introduction

Blind source separation (BSS) is a signal processing technique the purpose of which is to separate source signals from observations, without information of how the source signals are mixed in the observations. BSS presents a technically very challenging topic to the signal processing community, but it has stimulated significant interest for many years due to its relevance to a wide variety of applications. BSS has been applied to wireless communications and speech processing, and recently there has been an increasing interest in imaging applications.

BSS methods are 'blind' in the sense that the mixing process is not known, at least not explicitly. But what is universally true for all BSS frameworks is that we make certain presumptions on the source characteristics (and sometimes on the mixing characteristics as well), and then exploit such characteristics during the blind separation process. For instance, independent component analysis (ICA) [1, 2], a major and very representative BSS framework on which many BSS methods are based, assumes that the sources are mutually uncorrelated/independent random processes possibly with non-Gaussian distributions. There are many other possibilities one can consider; for example, using quasi-stationarity [3, 4] (speech signals are quasi-stationary), and using boundness of the source magnitudes [5, 6, 7] (suitable for digital signals). In choosing a right BSS method for a particular application, it is important to examine whether the underlying assumptions of the BSS method are a good match to the application. For instance, statistical independence is a reasonable assumption in applications such as speech signal separation, but it may be violated in certain imaging scenarios such as hyperspectral imaging [8].

This book chapter focuses on non-negative blind source separation (nBSS), in which the source signals are assumed to take on non-negative values. Naturally, images are non-negative signals. Potential applications of nBSS include biomedical imaging [9], hyperspectral imaging [10], and analytical chemistry [11]. In biomedical imaging, for instance, there are realistic, meaningful problems where nBSS may serve as a powerful image analysis tool for practitioners. Such examples will be briefly described in this book chapter.

In nBSS, how to cleverly utilize source non-negativity to achieve clean separation has been an intriguing subject that has received much attention recently. Presently available nBSS methods may be classified into two groups. One group is similar to ICA: Assume that the sources are mutually uncorrelated or independent, but with non-negative source distributions. Methods falling in this class include non-negative ICA (nICA) [12], stochastic non-negative ICA (SNICA) [13], and Bayesian positive source separation (BPSS) [14]. In particular, in nICA the blind separation criterion can theoretically guarantee perfect separation of sources [15], under an additional assumption where the source distributions are non-vanishing around zero (this is called the well-grounded condition).

Another group of nBSS methods does not rely on statistical assumptions. Roughly speaking, these methods explicitly exploit source non-negativity or even mixing matrix non-negativity, with an attempt to achieve some kind of least square fitting criterion. Methods falling in this group are generally known as (or may be vaguely recognized as) non-negative matrix factorization (NMF) [16, 17]. An advantage with NMF is that it does not operate on the premise of mutual uncorrelatedness/independence as in the first group of nBSS methods. NMF is a nonconvex constrained optimization problem. A popular way of handling NMF is to apply gradient descent [17], but it is known to be suboptimal and slowly convergent. A projected quasi-Newton method has been incorporated in NMF to speed up its convergence [18]. Alternatively, alternating least squares (ALS)

[19, 20, 21, 22] can also be applied. Fundamentally, the original NMF [16, 17] does not always yield unique factorization, and this means that NMF may fail to provide perfect separation. Possible circumstances under which NMF draws a unique decomposition can be found in [23]. Simply speaking, unique NMF would be possible if both the source signals and mixing process exhibit some form of sparsity. Some recent works have focused on incorporating additional penalty functions or constraints, such as sparse constraints, to strengthen the NMF uniqueness [24, 25].

In this chapter we introduce an nBSS framework that is different from the two groups of nBSS approaches mentioned above. Called convex analysis of mixtures of non-negative sources (CAMNS) [26], this framework is deterministic using convex geometry to analyze the relationships of the observations and sources in a vector space. Apart from source non-negativity, CAMNS adopts a special deterministic assumption called *local dominance*. We initially introduced this assumption to capture the sparse characteristics of biomedical images [27, 28], but we also found that local dominance can be perfectly or approximately satisfied for high-contrast images such as human portraits. (We however should stress that the local dominance assumption is different from the sparsity assumption in compressive sensing.) Under the local dominance assumption and some standard nBSS assumptions, we can show using convex analysis that the true source vectors serve as the extreme points of some observation-constructed polyhedral set. This geometrical discovery is surprising, with a profound implication that perfect blind separation can be achieved by solving an extreme point finding problem that is not seen in the other BSS approaches to our best knowledge. Then we will describe two methods for practical realizations of CAMNS. The first method is analysis-based, using LPs to locate all the extreme points systematically. Its analysis-based construction endows it with several theoretical appealing properties, as we will elaborate upon later. The second method is heuristic in comparison, but intuitively it is expected to have better robustness against mismatch of model assumptions. In our simulation results with real images, the second method was found to exhibit further improved separation performance over the first.

In Figure 1.1 we use diagrams to give readers some impressions on how CAMNS works.

The chapter is organized as follows. In Section 1.2, the problem statement is given. In Section 1.3, we review some key concepts of convex analysis, which would be useful for understanding of the mathematical derivations that follow. CAMNS and its resultant implications on nBSS criteria are developed in Section 1.4. The systematic, analysis-based LP method for implementing CAMNS is described in Section 1.5. We then introduce an alternating volume maximization heuristics for implementing CAMNS in Section 1.6. Finally, in Section 1.7, we use simulations to evaluate the performance of the proposed CAMNS-based nBSS methods and some other existing nBSS methods.