EE306001 Probability, Fall 2012

Homework Assignment #4

Please turn in your solutions in class on October 19th Friday.

1. (Prove a special case of Theorem 1.2.8.) Let X and Y be two discrete r.v.'s. Let $Z = g(Y)$ for some real-valued function $g(y)$ on the real line. Prove that if both X and XZ have finite expectations, then

$$E[X \cdot Z|Y] = Z \cdot E[X|Y] \text{ w.p.1.}$$

2. (Prove a special case of Theorem 1.2.9.) Let X and Y be two discrete r.v.'s. Prove that if X has a finite expectation and X and Y are statistically independent, then

$$E[X|Y] = E[X] \text{ w.p.1.}$$

3. Let X be a r.v. with a finite expectation, Z a bounded r.v., i.e., $|Z(\omega)| \leq c$ for all ω in the sample space S for some positive constant c, and $Y_1, \ldots, Y_n$ arbitrary r.v.'s. Please show that

$$E[E[X|Y_1, \ldots, Y_n] \cdot Z] = E[X \cdot E[Z|Y_1, \ldots, Y_n]] \text{ w.p.1.}$$

(Hint: Use Theorem 1.2.7 in the lecture notes.)

4. Let X and Y be jointly absolutely continuous with a jpdf $f(x, y)$ such that $E[X]$ exists and is finite. Please show that one version of the conditional expectation $E[X|X + Y]$ of X given $X + Y$ is $h(X + Y)$ with

$$h(z) = \frac{\int_{-\infty}^{\infty} x f(x, z - x) dx}{\int_{-\infty}^{\infty} f(x, z - x) dx}.$$