EE306001 Probability, Fall 2012

Homework Assignment #9

Reading assignment:

1. The supplementary lecturenotes on the Riemann-Stieltjes integral.

Basic Problems: The basic problems are for you to prepare the quiz on the Monday class. Please do not turn in their solutions.

1. Let $F(x) = 2.1\ u_0(x-3) + 4.6\ u_0(x-6) + 1.2\ u_0(x-8)$. Please evaluate the Riemann-Stieltjes integrals $\int_{[0,9]} x^2 dF(x)$ and $\int_{[3,8]} x^2 dF(x)$.
2. Let $F(x) = e^x$. Please evaluate the Riemann-Stieltjes integrals $\int_{[a,b]} dF(x)$.
3. Let $F(x) = x^3$. Please evaluate the Riemann-Stieltjes integrals $\int_{[-2,2]} x dF(x)$.
4. Let $F(x) = 1 - e^{-3x}$ on $[0, \infty)$. Does the improper Riemann-Stieltjes integrals $\int_{[0,\infty)} x dF(x)$ exist ? If exists, is it finite ? If finite, what is its value ?

Developed Problems: Please turn in your solutions on November 21st Wednesday in class. The TAs will check the similarity between homework solutions to detect improper conduct such as plagiarism.

1. Let F be monotone increasing on $[a,b]$ and continuous at a point $x_0 \in [a,b]$. Let $f(x_0) = 1$ and $f(x) = 0$ for all $x \neq x_0$. Please show that $f \in \mathfrak{R}_{[a,b]}(F)$ and $\int_{[a,b]} f dF = 0$.
2. Let f be nonnegative and continuous on $[a,b]$. Please show that if $\int_{[a,b]} f(x)dx = 0$, then $f(x) = 0$ for all $x \in [a,b]$. (Hint: Use the mean-value theorem in the supplementary lecturenotes.)
3. Let F and G be two distribution functions on $\mathbb{R}$, i.e., both F and G are monotone increasing and right-continuous on $\mathbb{R}$ with $F(-\infty) = G(-\infty) = 0$ and $F(+\infty) = G(+\infty) = 1$. The convolution $G * F$ of G with respect to F is defined as
$$(G * F)(x) \triangleq \int_{-\infty}^{\infty} G(x-y) dF(y),$$
by assuming here that the improper Riemann Stieltjes integral exists and is finite for each $x \in \mathbb{R}$.
   (a) Please show that the convolution $F * G$ of F with respect to G is well-defined on $\mathbb{R}$ and $F * G = G * F$. (Hint: Use the integration by part and the change of variable.)
   (b) Please show that $G * F$ is a distribution function on $\mathbb{R}$. (Hint: The order of the limiting process $\lim_{x \uparrow c}$ or $\lim_{x \downarrow c}$, $-\infty \leq c \leq +\infty$, and the integration process are exchangeable whenever the integrand is nonnegative.)
4. Theoretical Exercise 5.5 on page 227 of the textbook.