# EE306001, Probability, Fall 2012

## Quiz #3, Problems and Solutions

**Prob. 1:**What is the probability that at least one of a pair of fair dice lands on 6, given that

(1) the sum of the dice is 2?

(2) the sum of the dice is 7?

**Solution.** Define the sample space as the set of all outcomes rolling tow dices. that is ,

$$\begin{aligned} S = \{&(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ &(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ &(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ &(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ &(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ &(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}. \end{aligned}$$

Define the $\sigma$-field

$$\mathcal{F} = 2^S.$$

Define the probability function $P : S \to \mathbb{R}$ as

$$P = \frac{|E|}{|S|}, \ \forall E \in \mathcal{F}.$$

Define the event

$$\begin{aligned} E =&\{\text{at least one of a pair of dice lands on 6}\}\\ =&\{(1,6),(2,6),(3,6),(4,6),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5)\} \end{aligned}$$

(1) The given event is

$$\begin{aligned} F =&\{\text{the sum of the dice is 2}\}\\ =&\{(1,1)\}. \end{aligned}$$

The intersection

$$E \cap F = \emptyset.$$

Hence the probability

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = 0.$$

(2) The given event is

$$\begin{aligned} F =&\{\text{the sum of the dice is 7}\}\\ =&\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}. \end{aligned}$$

The intersection

$$E \cap F = \{(1,6),(6,1)\}.$$

Hence the probability

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}.$$

**<u>Prob. 2:</u>** Show that if $\mathcal{P}(A) > 0$, then $\mathcal{P}(A \cap B|A) \geq \mathcal{P}(A \cap B|A \cup B)$.

**Solution.** For any event $B \in \mathcal{F}$, $A \subset (A \cup B)$ implies $\mathcal{P}(A) \leq \mathcal{P}(A \cup B)$. Hence

$$\mathcal{P}(A \cap B|A) = \frac{\mathcal{P}\big((A \cap B) \cap A\big)}{\mathcal{P}(A)} = \frac{\mathcal{P}(A \cap B)}{\mathcal{P}(A)}$$
$$\geq \frac{\mathcal{P}(A \cap B)}{\mathcal{P}(A \cup B)} = \frac{\mathcal{P}\big((A \cap B) \cap (A \cup B)\big)}{\mathcal{P}(A \cup B)} = \mathcal{P}(A \cap B|A \cup B).$$

**<u>Prob. 3:</u>** Urn A conatins 2 white balls and 1 black ball, and urn B contains 2 white balls ans 5 black balls. Now a ball is randomly drawn from urn A and is put in urn B. A ball is then drawn from urn B and it happens to be white. What's the probability that the ball transferred from A to B is white? ( Hint. You can let

$$E = \{\text{The ball transferred from A to B is white.}\},$$
$$F = \{\text{The ball we finally select from B is white.}\}.$$

then use the technique of Baye's formula to compute $\mathcal{P}(E|F)$. )

**Solution.** We have

$$\mathcal{P}(E|F) = \frac{\mathcal{P}(E)\mathcal{P}(F|E)}{\mathcal{P}(E)\mathcal{P}(F|E) + \mathcal{P}(E^c)\mathcal{P}(F|E^c)},$$

where

$$\mathcal{P}(E) = \frac{2}{3},\ \mathcal{P}(F|E) = \frac{3}{8},\ \mathcal{P}(E^c) = \frac{1}{3},\ \mathcal{P}(F|E^c) = \frac{2}{8}.$$

Therefore $\mathcal{P}(E|F) = \frac{3}{4}$.