Empirical Bayesian Light-Field Stereo Matching by Robust Pseudo Random Field Modeling

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Supplementary Material

Appendix A
Proof of Energy-Kernel Equation

In the following, we will show $E'(x) = K(x)$ based on their definitions from the prior function $G(w)$ and the extended formulation for the best guessed hidden $\hat{w}$ in (17). We can rewrite the energy function $E(x)$ in (7) by

$$E(x) = \left(\hat{w}x - \int G'(\hat{w}) d\hat{w} \right)_{|\hat{w}=K(x), G'(\hat{w})=x}, \quad \text{(A-1)}$$

$$= K(x) \cdot x - \int x \, dK(x), \quad \text{(A-2)}$$

$$= \int K(x) dx, \quad \text{(A-3)}$$

which gives $E'(x) = K(x)$. Note that the $E(x)$ and $G(w)$ follow a further constraint $E(0) = -G(1)$ when substituting the assumption $\hat{w}_{|x=0} = K(x=0) = 1$ into (7).

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Appendix B
EM Update for Smoothness Energy

The EM+ fitting contains two steps: EM and KLD updates. In the following, we present the details for the former, which can be easily extended to the latter. Define marginal energy $\mathcal{L}_{T_{\rm s}} \triangleq \mathbf{E}_{\tilde{t}_{\rm s}} \left[E_{T_{\rm s}}^{\rm soft}(\tilde{t}_{\rm s}; \boldsymbol{\theta}_{\rm s}) \right]$ for pixel difference $T_{\rm s}$ and $\mathcal{L}_H \triangleq \mathbf{E}_{\tilde{h}} \left[E_H^{\rm soft}(\tilde{h}; \boldsymbol{\theta}_{\rm s}) \right]$ for disparity difference H. Then we can update the parameters by minimizing the joint energy $\mathcal{L} = \mathcal{L}_{T_{\rm s}} + \eta \mathcal{L}_H$ in (14).

For the EM update $(\delta, \sigma_{\rm s}, \alpha_{\rm s}, \epsilon_{\rm s}) \Rightarrow (\hat{\delta}, \hat{\sigma}_{\rm s}, \hat{\alpha}_{\rm s}, \epsilon_{\rm s})$, we apply Jensen's inequality and set $\frac{\partial \mathcal{L}_H}{\partial \delta}$, $\frac{\partial \mathcal{L}_{T_{\rm s}}}{\partial \sigma_{\rm s}}$, and $\frac{\partial \mathcal{L}}{\partial \alpha_{\rm s}}$ to zero respectively to derive the update formulations:

$$\hat{\delta}^{\beta} = \beta \mathbf{E}_{\tilde{h}} \left[\tilde{h}^{\beta} \mathbf{E}_{U|H=\tilde{h}}[U] \right], \quad \text{(B-1)}$$

$$\hat{\sigma}_{\rm s}^2 = \frac{1}{k} \mathbf{E}_{\tilde{t}_{\rm s}} \left[\tilde{t}_{\rm s}^2 \mathbf{E}_{U|T_{\rm s}=\tilde{t}_{\rm s}}[U] \right], \quad \text{(B-2)}$$

$$H(\hat{\alpha}_{\rm s}, \epsilon_{\rm s}) = \frac{\mathbf{E}_{\tilde{t}_{\rm s}} \left[\mathbf{E}_{U|T_{\rm s}=\tilde{t}_{\rm s}}[G(U)] \right] + \eta \mathbf{E}_{\tilde{h}} \left[\mathbf{E}_{U|H=\tilde{h}}[G(U)] \right]}{1+\eta}, \quad \text{(B-3)}$$

where $H(\hat{\alpha}_{\rm s}, \epsilon_{\rm s}) = \mathbf{E}_{U;\hat{\alpha}_{\rm s},\epsilon_{\rm s}}[G(U)]$. The conditional distributions $f_{U|H}$ and $f_{U|T_{\rm s}}$ are all parameterized by the $(\delta, \sigma_{\rm s}, \alpha_{\rm s}, \epsilon_{\rm s})$ in the previous iteration.