Bayesian Inference for Neighborhood Filters with Application in Denoising

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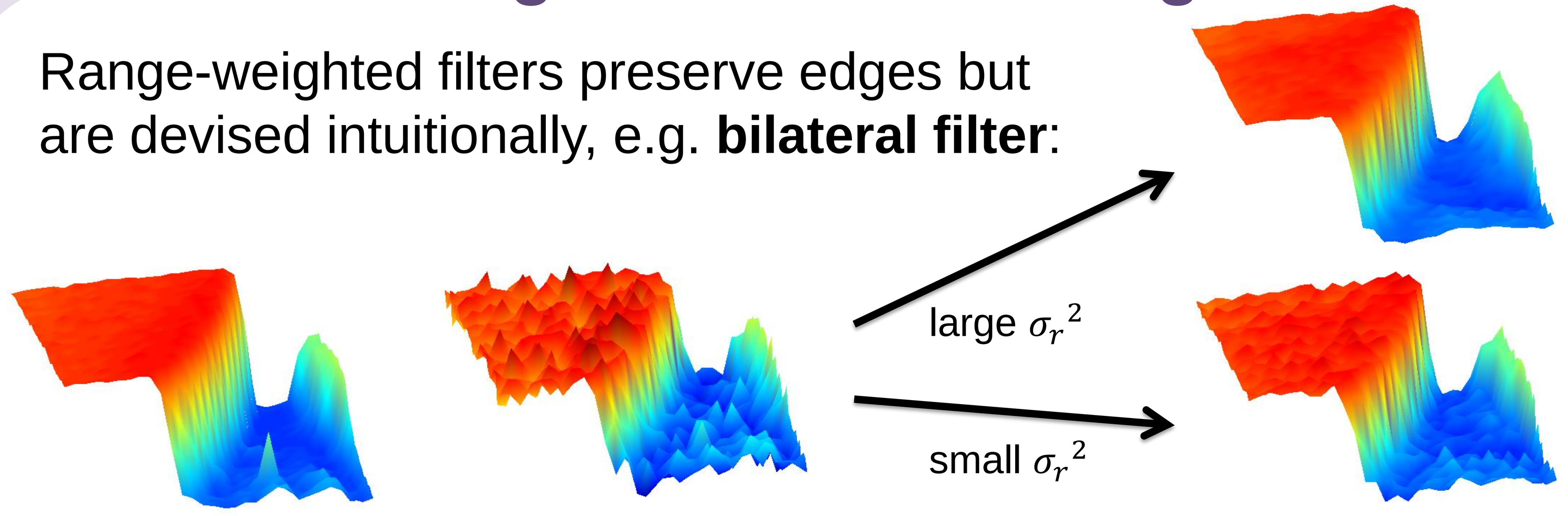
Neighborhood Filtering

Range-weighted filters preserve edges but are devised intuitionally, e.g. **bilateral filter**:



$$w_{l,i} = e^{-\frac{\|\boldsymbol{y}_l - \boldsymbol{y}_i\|^2}{2\sigma_r^2}} \qquad \hat{\boldsymbol{z}}_l = \frac{\sum_{i\in\Lambda_l} w_{l,i} d_{l,i} \boldsymbol{y}_i}{\sum_{i\in\Lambda_l} w_{l,i} d_{l,i}}$$

Range Weight — Weighted Average

Question:
Statistical reasoning?
Estimation for $\boldsymbol{\sigma_r}^2$?

Challenge and Contribution

Challenge

- Hard to build statistical model because the range weight w is linked to the noisy signal $\boldsymbol{y}$ adaptively

Novelty

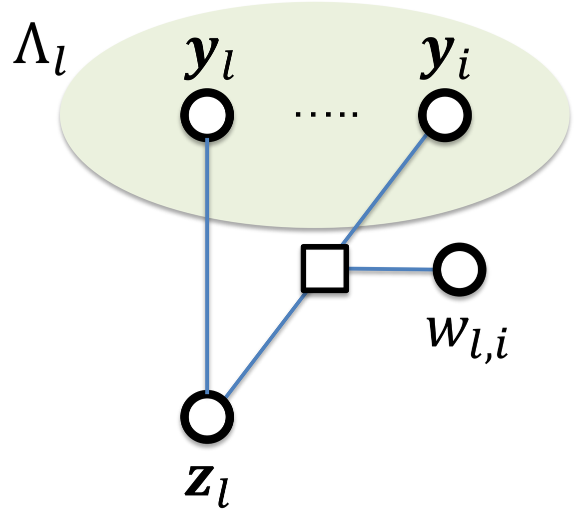
- Introduce a **soft-edge** random variable to infer the Gaussian range weight by MAP
- Formulate the l^2-norm of pixel difference in observable **chi scale mixtures** (**CSM**) to enable model fitting

Contribution

- Develop a **unified empirical Bayesian framework** to
  1. Infer neighborhood filters (**Property Reasoning**)
  2. Estimate σ_r^2 by model fitting (**Parameter Estimation**)
- Enable an iterative filtering scheme to improve performance

Noise Model and Inference

Neighborhood noise model



- Model neighbors in Gaussian scale mixture

$$\boldsymbol{y}_i = \boldsymbol{z}_l + \frac{\boldsymbol{n}_{l,i}}{\sqrt{w_{l,i}}} \qquad [\boldsymbol{n}_{l,i} \sim \mathcal{N}(0, \sigma^2 \boldsymbol{I}_k)]$$

- Define soft-edge prior distribution as

$$f_w(w; \varepsilon, \alpha) = \frac{1}{N(\varepsilon, \alpha)} w^{-\frac{k}{2}} w^{-\alpha w} e^{\alpha w}, w \in [\varepsilon, 1]$$

Bayesian inference

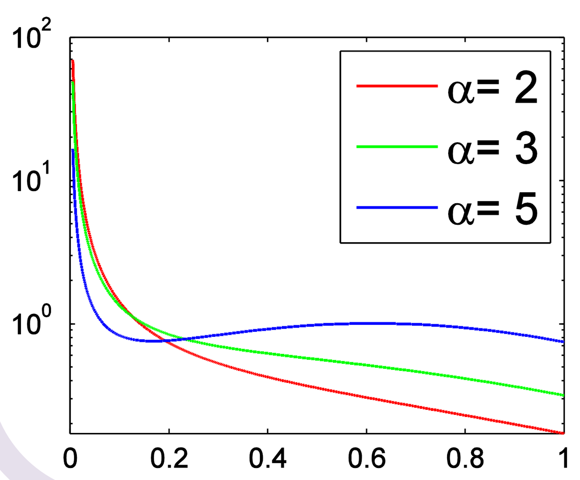
- First-iteration estimation for maximizing the posterior equals to
  1. MAP estimation for each $w_{l,i}$ ⇒ Range Weight ($\boldsymbol{\sigma_r}^2 = \boldsymbol{\alpha\sigma}^2$)
  2. ML estimation for $\boldsymbol{z}_l$ ⇒ Weighted Average
- Robust likelihood functions considering
  - Proximity ⇒ **Bilateral filter**
  - Patch similarity ⇒ Modified **non-local means** filter

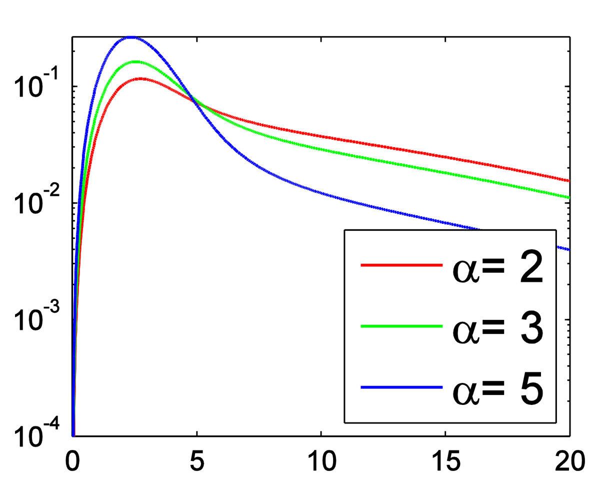
CSM fitting

- Formulate CSM ⇒ $s_{l,i} \triangleq \|\boldsymbol{y}_l - \boldsymbol{y}_i\|_2 \sim \sigma \sqrt{\frac{w_{l,i}+1}{w_{l,i}}} \chi_k$
- Fit empirical $P(s)$ to estimate $(\sigma, \varepsilon, \alpha)$

CSM Parameter
σ: noise intensity
ε, α: edge distribution

Examples of Model Distributions





$\alpha \searrow$ (or $\varepsilon \searrow$)
⇓
$f_w(w)$ leans left (more edges)
$f_s(s)$ has thicker tail
⇓
CSM fitting:
ε, α for the shape
σ for the scale

Experiments on Denoising

Datasets: Twelve standard test images (RGB color) + AWGN (σ_n)
Evaluation: Compare PSNR and σ_r^2 accuracy to
1) Best result by scanning σ_r^2
2) MAD+SURE: State-of-the-art (multi-pass) estimator

Bilateral filter (9x9)

- **Individual fitting results**

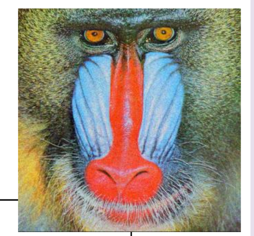
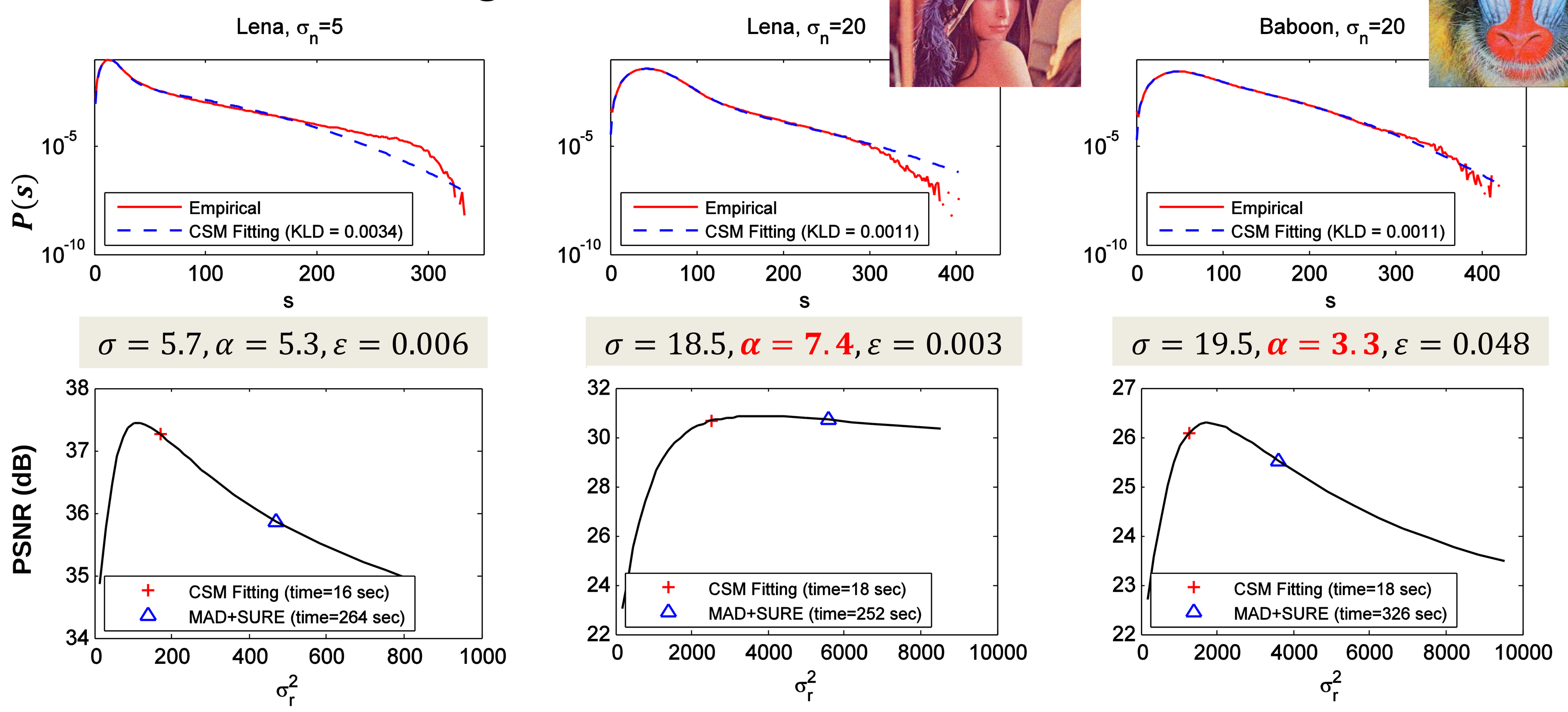
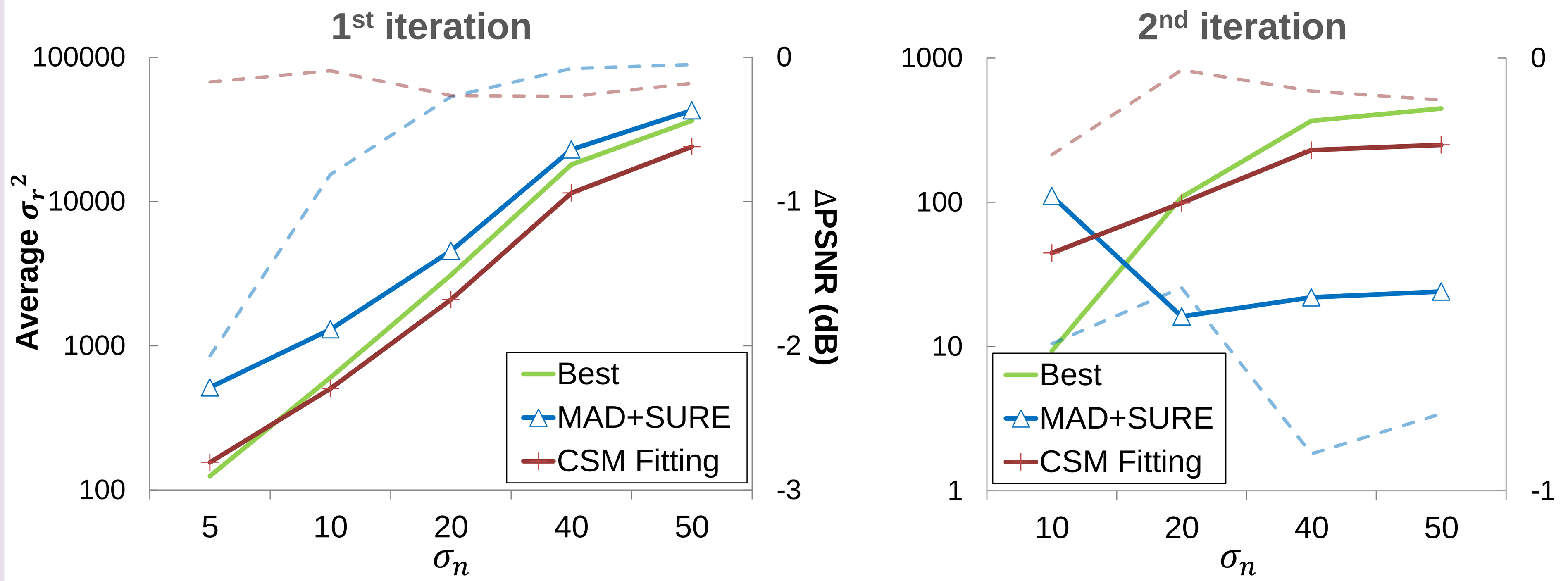


Image-dependent σ_r^2 tracked!



- **Average fitting results (and iterative filtering)**



CSM fitting even good for non-Gaussian noise, which enables the iterative filtering and improves PSNR by up to **1 dB**.

More results and further extensions

(incl. different filter/kernel/noise, multispectral image, image gradient) ⇒