CHAPTER 6

Noise

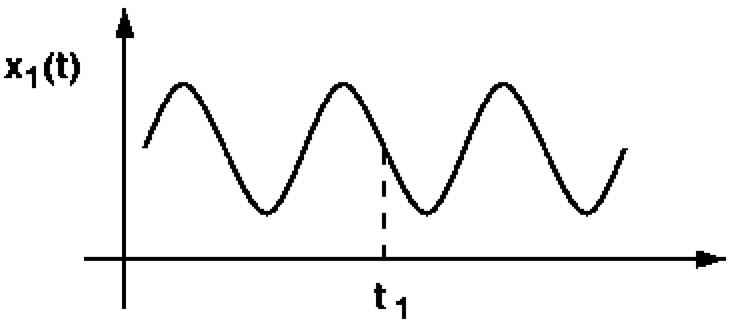
Syllabus

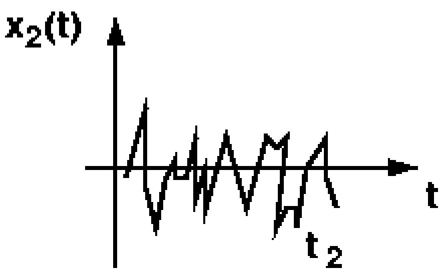
- Basic physics of MOS transistors and their equivalent circuit models (ch. 6)
- Elementary gain stages (ch. 7, ch. 9, and ch. 10)
- Operational amplifier basics (ch. 8)
- Frequency responses (ch. 11)
- Noise (ch.7, Design of analog CMOS ICs)
- Feedback, stability and compensation (ch. 12)
- Operational amplifiers (ch.9, Design of analog CMOS ICs)

Outline

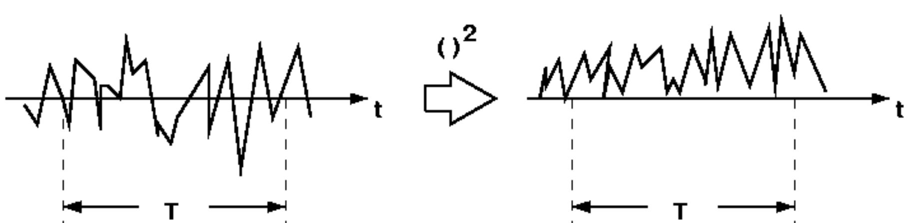
1. **Statistical Characteristics of Noise**
2. Types of Noise
3. Representation of Noise in Circuits
4. Noise in Single-Stage Amplifiers
5. Noise in Differential Pairs

Statistical Behavior of Noise





- Noise is a random process.
  - The value of noise can not be predicted at any time even if the past values are known.
  - Observe the noise for a long time and using the measured results to construct a "statistical model".
  - In many cases, the average power of noise is predictable.



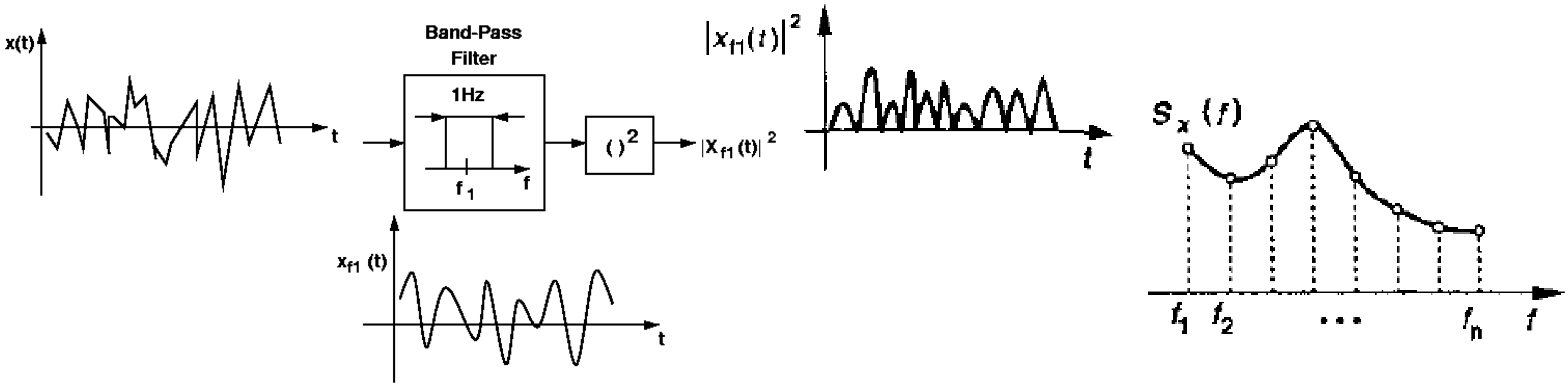
$$P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\frac{v^2(t)}{R_L}dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}x^2(t)dt$$

$$unit: (\mathrm{V}^2) \text{ in stead of } (\mathrm{W})$$

Normalize the area under waveform to T

Noise Spectrum

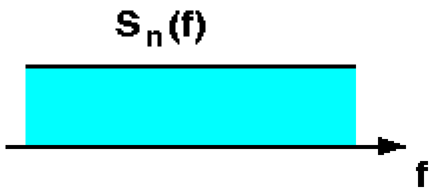
- The concept of noise power becomes more versatile if defined with regard to the *frequency content of noise – Power spectral density (PSD)*



- The PSD, $S_x(f)$, of a noise waveform *x(t)* is defined as the average power carried by *x(t)* in a one-Hertz bandwidth around *f*, expressed in V^2/Hz.
- The square root of $S_x(f)$ expressed in $V/\sqrt{Hz}$.
- Input noise = 3n $V/\sqrt{Hz}$ at 100Mhz means average power in a 1Hz bandwidth at 100Mhz = $(3e\text{-}9)^2V^2$.

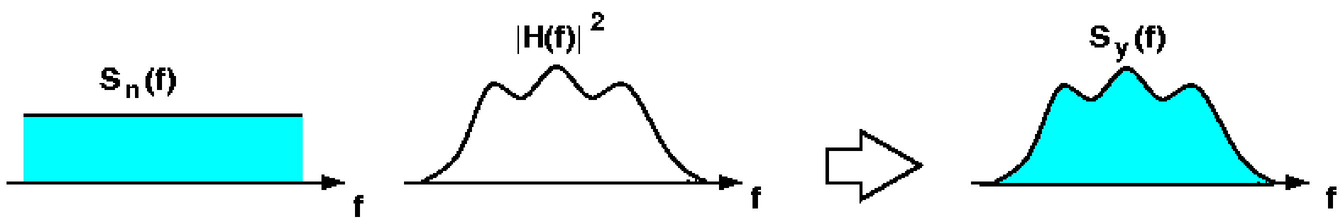
Spectral Shaping

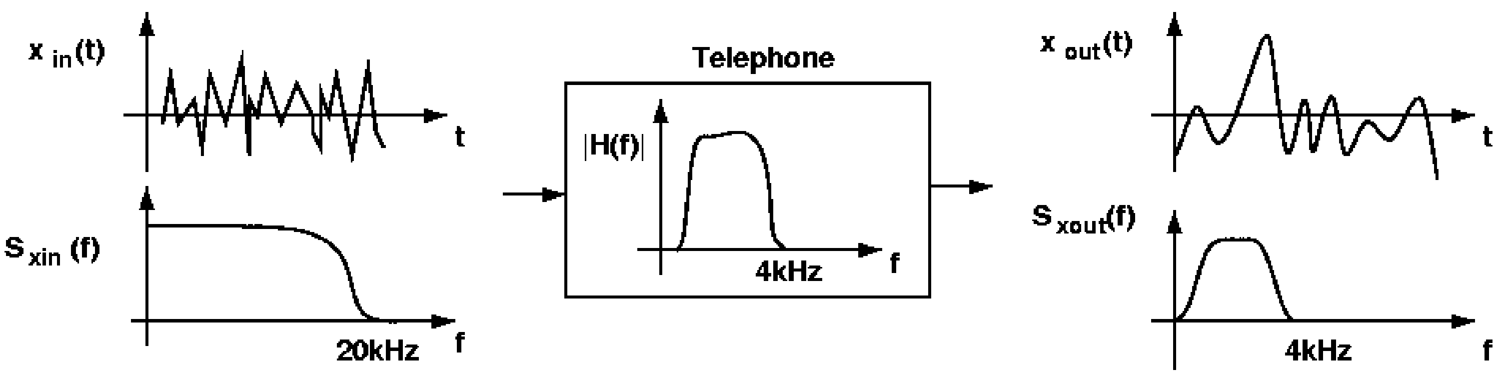
- White noise – the PSD displays the same value at all frequencies.



- If a signal with spectrum $S_x(f)$ is applied to a linear time-invariant system with transfer function H(s), then the output spectrum is given by

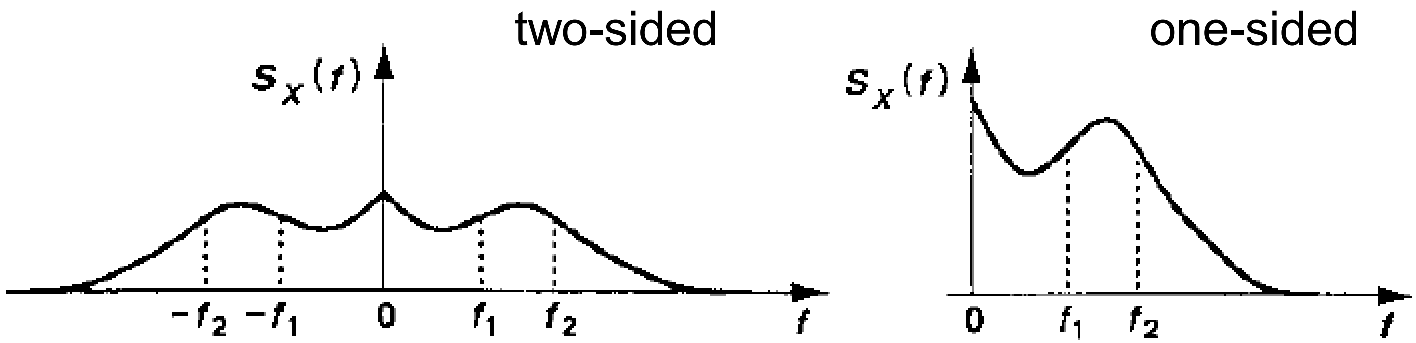
$$S_Y(f) = S_X(f)\,|H(f)|^2 \quad \text{where} \quad H(f) = H(s = j2\pi f)$$



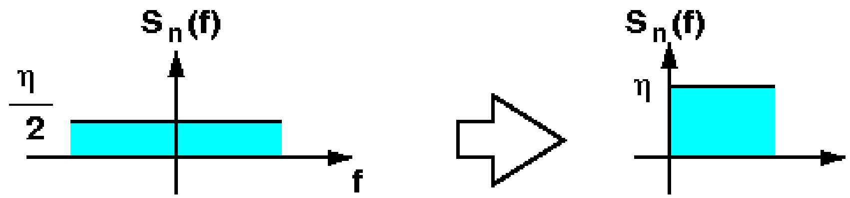


Noise Spectra

- Since $S_x(f)$ is an even function of f for real $x(t)$.



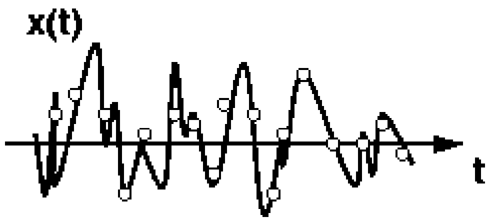
- The negative frequency part of the spectrum is folded around the vertical axis and added to the positive frequency part.

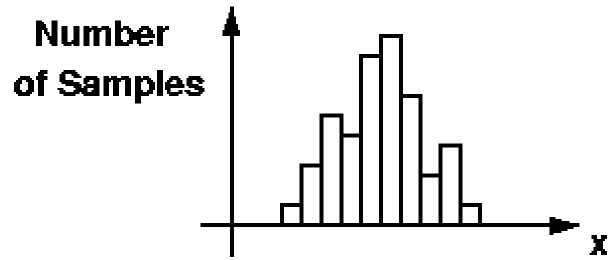


$$P_{f1,f2} = \int_{-f2}^{-f1} S_X(f)df + \int_{f1}^{f2} S_X(f)df = 2\int_{f1}^{f2} S_X(f)df$$

Amplitude Distribution

- Distribution of amplitude – *Probability density function (PDF).*
- The distribution of x(t) is defined as
  - $p_x(x)dx$ = *probability of* $x < X < x + dx$, X is the measured value of $x(t)$





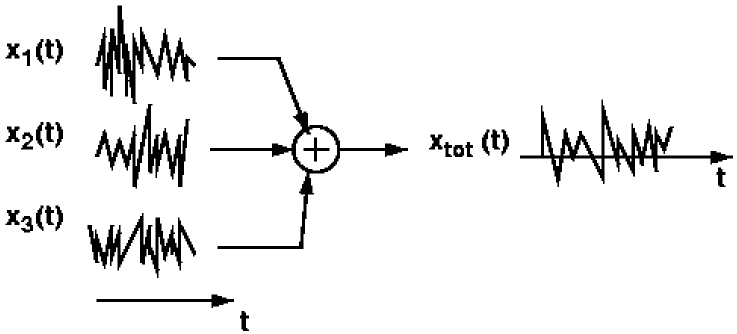
- *Gaussian (Normal) Distribution*
  - The *central limit theorem* states that if many independent random process with arbitrary PDFs are added, the PDF of the sum approaches a *Gaussian distribution.*
  - *The Gaussian PDF is defined as* $p_X(f) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-m)^2}{2\sigma^2}$

Correlated and Uncorrelated Sources

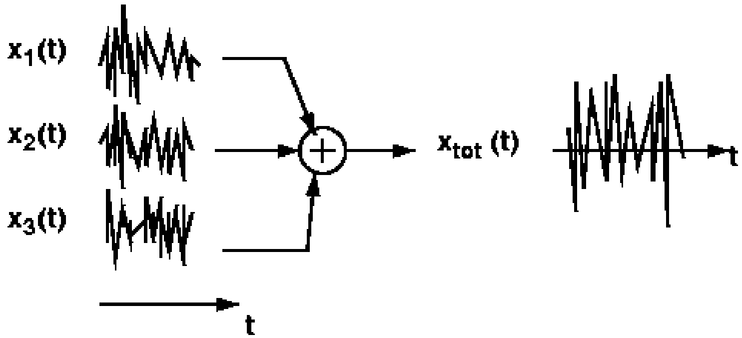
- For deterministic voltages and currents, use the superposition principle.
- In noise analysis, the average noise power is of interest.

$$P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left[x_1(t)+x_2(t)\right]^2 dt$$
$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}x_1^2(t)dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}x_2^2(t)dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}2x_1(t)x_2(t)dt$$
$$= P_{av1} + P_{av2} + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}2x_1(t)x_2(t)dt$$

  - P_{av1} and P_{av2} denote the average power of $x_1(t)$ and $x_2(t)$.
  - The third term is the "correlation" between $x_1(t)$ and $x_2(t)$.
  - If generated by independent devices, the correlation = 0.



uncorrelated noise



correlated noise

Outline

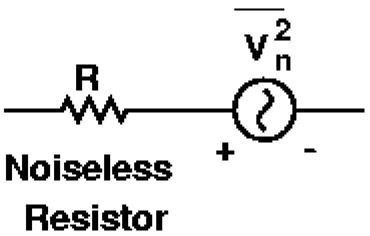
1. Statistical Characteristics of Noise
2. **Types of Noise**
3. Representation of Noise in Circuits
4. Noise in Single-Stage Amplifiers
5. Noise in Differential Pairs

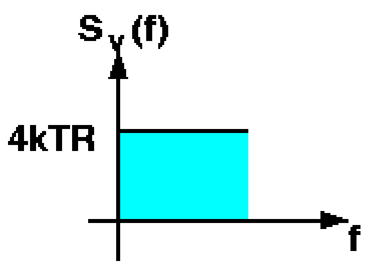
Types of Noise

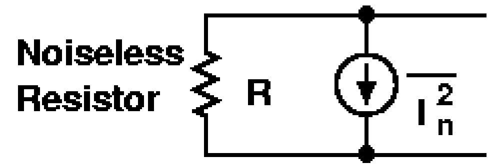
- Device electronics
  - Thermal noise
  - Flicker noise
- Environmental noise
  - Supply
  - Ground
  - Substrate

Thermal Noise

- Thermal noise (white noise)
  - Resistor thermal noise - the random motion of electrons in a conductor introduces fluctuations in the voltage measured across the conductor even if the average current is zero.
  - The spectrum of thermal noise is proportional to the absolute temperature.







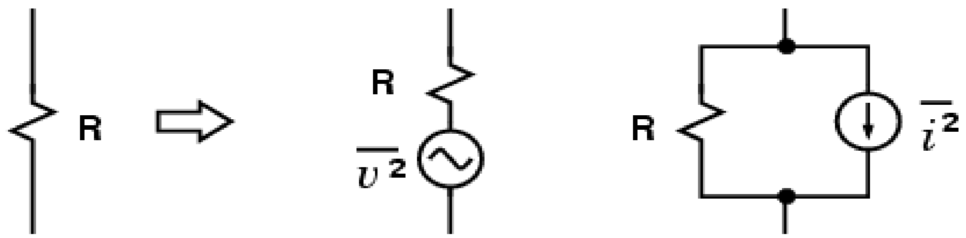
$S_v(f) = 4kTR\ (V^2/\mathrm{Hz})\quad f \geq 0,\quad \overline{V_n^2} = 4kTR \times 1Hz\ (V^2),\quad k = 1.38 \times 10^{-23}\ (J/K)$ Boltzmann constant

  - Example : a 50 Ω resistor at T = 300° K exhibits 8.28×10^{-19} V^2/Hz of thermal noise, or $0.91\,\mathrm{n}V/\sqrt{\mathrm{Hz}}$
  - The thermal noise of a resistor can be represented by a parallel current source as well. $\overline{I_n^2} = 4kT/R \quad (\mathrm{A}^2/\mathrm{Hz})$

Thermal Noise

- The thermal noise is white. In reality, $S_v(f)$ is flat for up to roughly 100 THz, dropping at higher frequencies.
- The polarity used for the voltage source is unimportant.

Thermal Noise



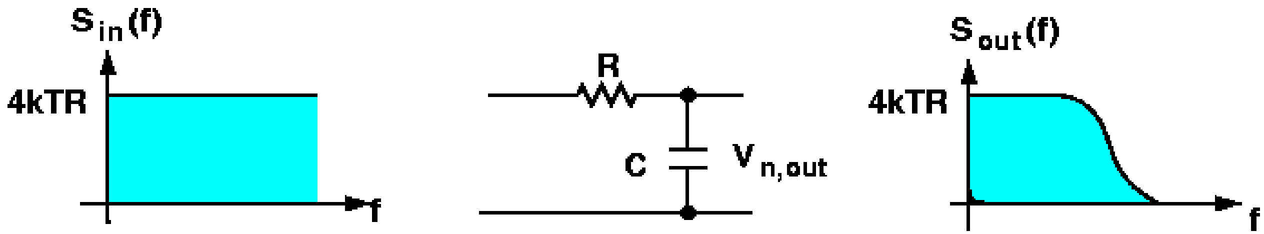
$$\frac{\overline{v^2}}{\Delta f} = 4kTR$$

$$\frac{\overline{i^2}}{\Delta f} = 4kT\frac{1}{R}$$

$$f = 0 - \infty$$

T = Absolute temperature in Kelvins
k = 1.38×10^{-23} watt/K-Hz (Boltzmann's constant)
Δf = Bandwidth per Hertz

Noise Spectrum Shaping : LP Filter



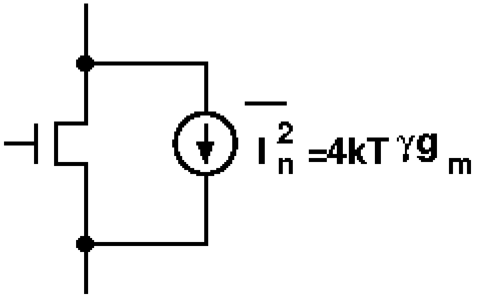
- Modeling the noise of *R* by a series voltage source V_R . The transfer function from V_R to V_{out}
$$\frac{V_{out}}{V_R}(s) = \frac{1}{1+sRC}$$
- We have
$$S_{out}(f) = S_R(f)\left|\frac{V_{out}}{V_R}(j\omega)\right|^2 = 4kTR\frac{1}{4\pi^2 R^2 C^2 f^2 + 1}$$
- The total noise power at the output
$$P_{n,out} = \int_0^\infty \frac{4kTR}{4\pi^2 R^2 C^2 f^2 + 1} df = \frac{2kT}{\pi C}\tan^{-1} u\Big|_{u=0}^{u=\infty} = \frac{kT}{C}\ (\mathrm{V}^2) \qquad \text{for} \qquad \int \frac{dx}{x^2+1} = \tan^{-1} x$$
- Example : for a 1 pF capacitor, the total noise voltage is equal to 64.3 μ$V_{rms.}$
- The total noise at the output of the circuit is independent of the value R.
- Low temperature operation can decrease noise in analog circuits. The mobility of charge carriers in MOS devices also increase at low temperatures.

Thermal Noise in MOSFET

- For long-channel MOS devices operating in saturation region, the channel noise can be modeled by

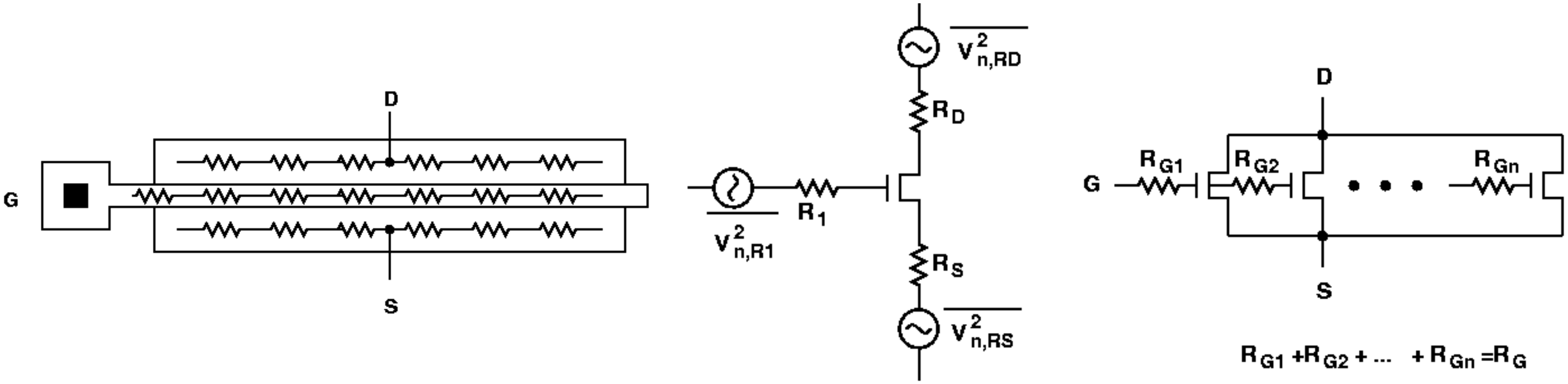
$$\overline{I_n^2} = 4kT\gamma g_m$$

- The coefficient γ is derived to be equal to 2/3 for long channel transistors and may need to be replaced by a larger value for submicron MOSFETs. (ex. 2.5 for 0.25-um MOS devices)

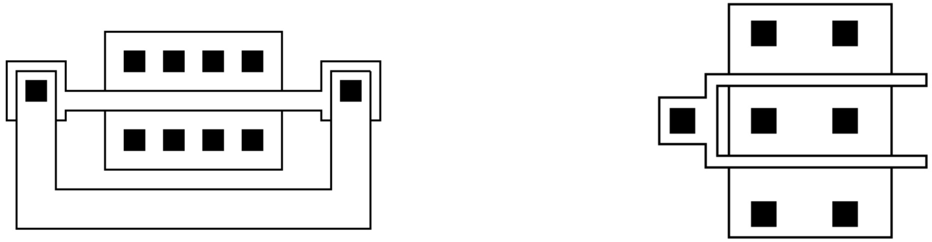


Thermal Noise in MOSFET

- The ohmic sections of a MOSFET also contribute thermal noise.
- For a relatively wide transistor, the gate distributed resistance may become noticeable.

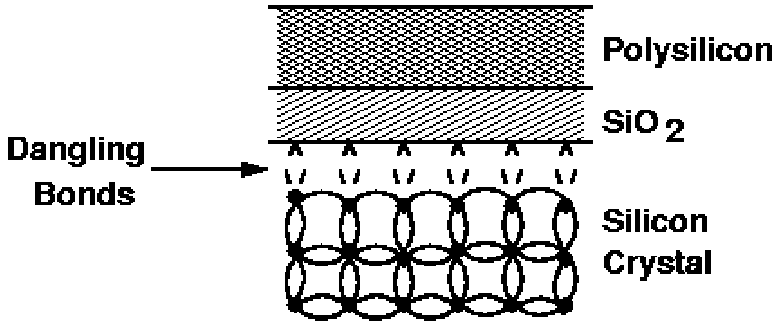


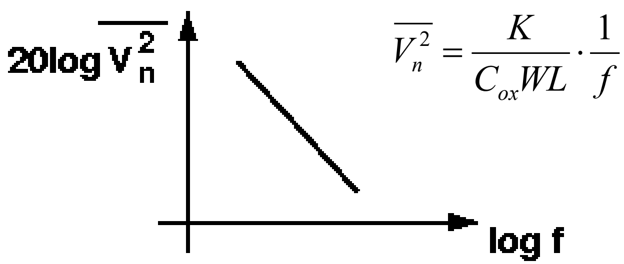
- A lumped resistor R_1 represents the distributed gate resistance. $R_1 = R_G / 3$
- Reduction of gate resistance by adding contacts to both sides or folding.



Flicker Noise

- Since the silicon crystal reaches an end at the interface, many dangling bonds appear, give rise to extra energy states.
- As carriers move to the interface, some are randomly trapped and later released by such energy states, introducing "flicker" noise in the drain current.





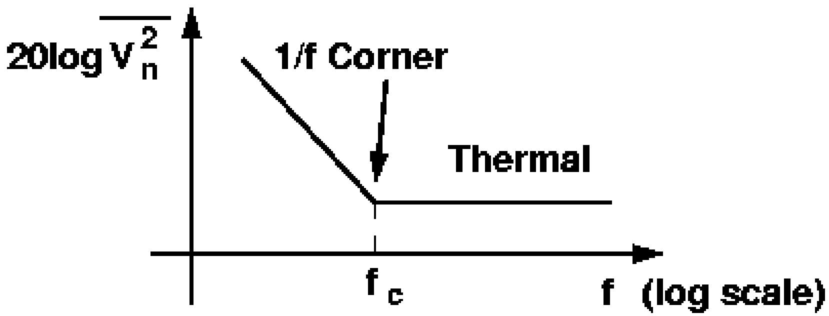
- The flicker noise is modeled as a voltage source series with the gate :

$$\overline{V_n^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}, \quad WL\uparrow, \quad f\uparrow, \quad \overline{V_n^2}\downarrow$$

  – Where K is a process-dependent constant on the order of 10^{-25} V^2 F.
  – Our notation assumes a bandwidth of *1 Hz*.
  – The trap-and-release phenomenon occurs at low frequencies more often. (*1/f noise*)
  – PMOS devices exhibits less 1/f noise than NMOS.

Flicker Noise Corner Frequency

- As *f* approaches DC, noise becomes indistinguishable from thermal drift or aging of devices.
- Corner frequency : the intersection point serves as a measure of what part of the band is mostly corrupted by flicker noise.

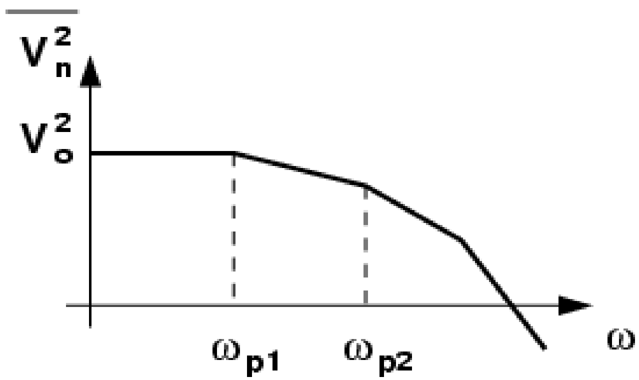


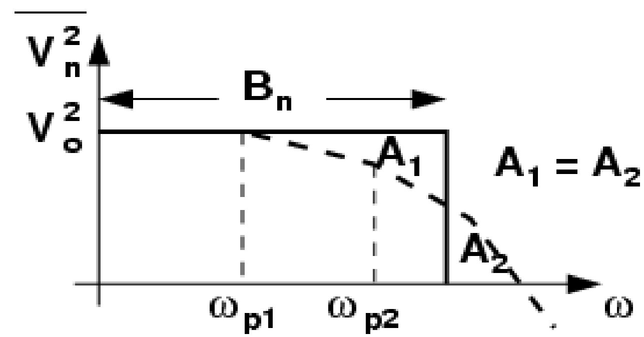
$$4kT\left(\frac{2}{3}g_m\right)=\frac{K}{C_{ox}WL}\cdot\frac{1}{f_C}\cdot g_m^2$$

$$f_C=\frac{K}{C_{ox}WL}g_m\frac{3}{8kT}$$

- f_C generally depends on device dimensions and bias current. However, for a given *L*, the dependence is relatively weak.
- The *1/f* noise corner is relatively constant, falling in the vicinity of *500 kHz* to *1 MHz* for submicron transistors.

Noise Bandwidth





- Noise bandwidth B_n : allows a fair comparison of circuits that exhibit the same low-frequency noise V_0^2 , but different high-frequency transfer functions.

$$\overline{V_{n,out,tot}^2} = \int_0^\infty \overline{V_{n,out}^2} df, \qquad V_0^2 \bullet B_n = \int_0^\infty \overline{V_{n,out}^2} df, B_n \equiv \text{ Noise Bandwidth}$$

- The total noise must be evaluated by calculating the total area under the spectral density, for a single-pole filter

$$H(s) = (1 + s/\omega_0)^{-1}, B_n = \int_0^\infty \left| H\left(j2\pi f \right) \right|^2 df = \int_0^\infty \left[1 + \left(\frac{f}{f_0} \right)^2 \right]^{-1} df = \frac{\pi}{2} f_0$$

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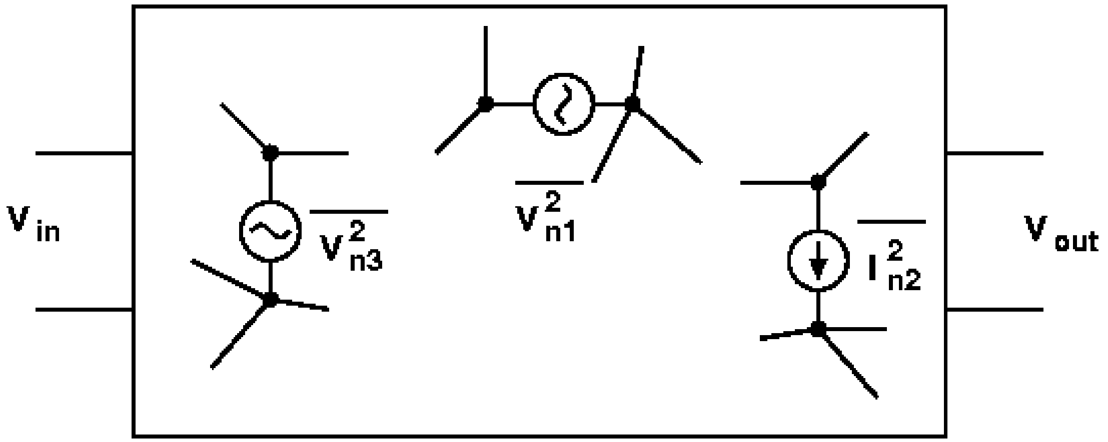
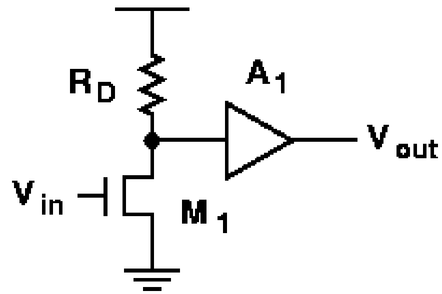
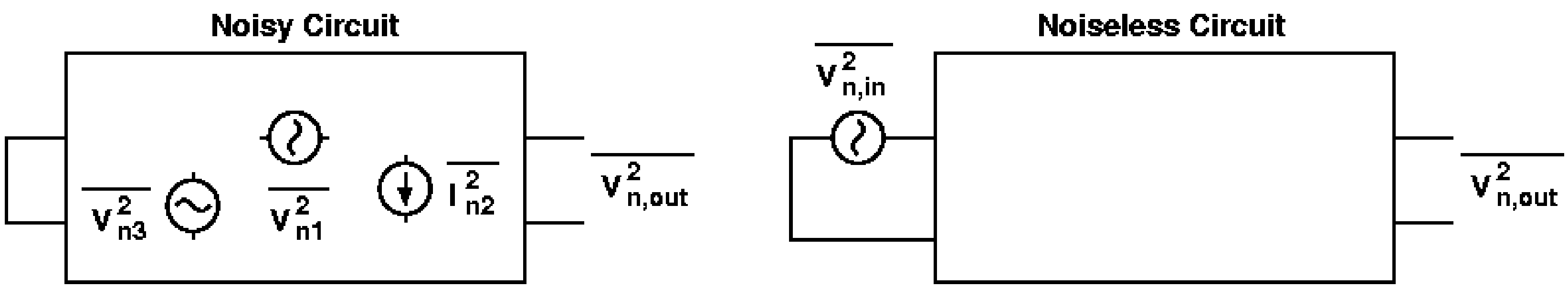
Representation of Noise in Circuits

- Set the input to zero and calculate the total noise at the output due to various sources of noise in the circuit.



Representation of Noise in Circuits

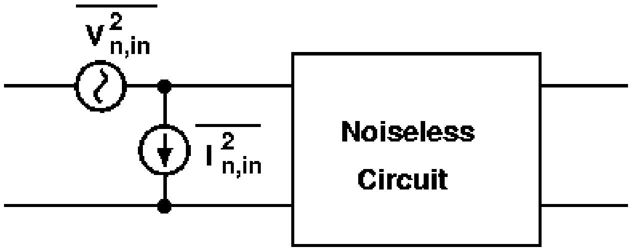




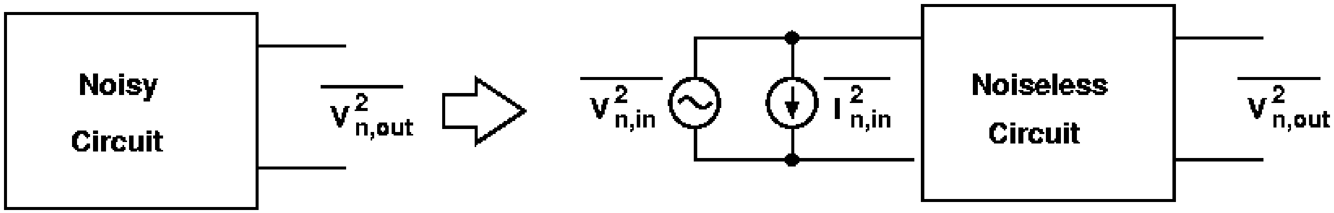
- A_1 amplifies the noise and signal also at the output.
- ***Input referred noise*** : to represent the effect of all noise sources in the circuit by a single source, $\overline{V_{n,in}^2}$.
  - If the voltage gain is A_v, then we must have $\overline{V_{n,out}^2} = A_v^2 \overline{V_{n,in}^2}$
  - Indicating how much the input signal is corrupted by the circuit's noise.

Representation of Noise by V/I Sources

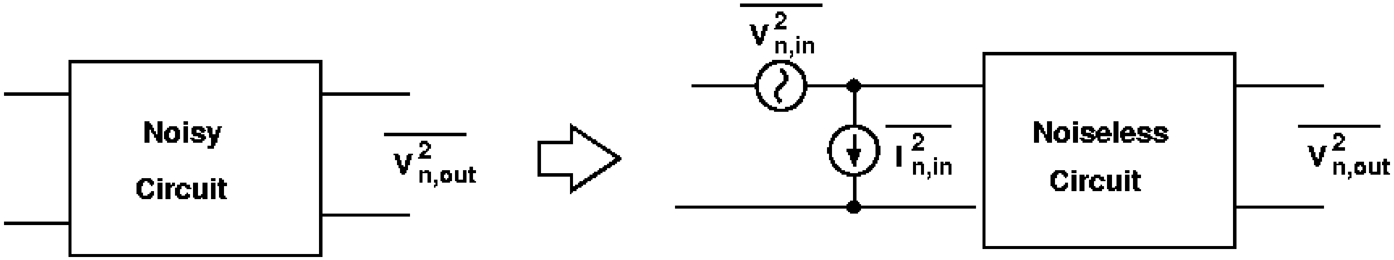
- Consider two extreme cases : zero and infinite source impedances.



- If the source impedance is zero, $\overline{I^2_{n,in}}$ flows through $\overline{V^2_{n,in}}$ and has no effect on the output.



- If the input is open, then $\overline{V^2_{n,in}}$ has no effect and the $\overline{V^2_{n,out}}$ is due to only $\overline{I^2_{n,in}}$



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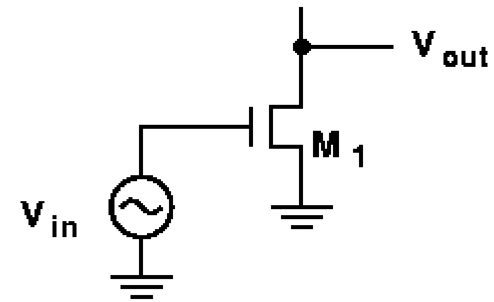
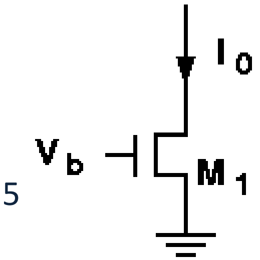
Common Source Stage

- The input-referred noise voltage per unit bandwidth of CS

$$\overline{V_{n,in}^2} = 4kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

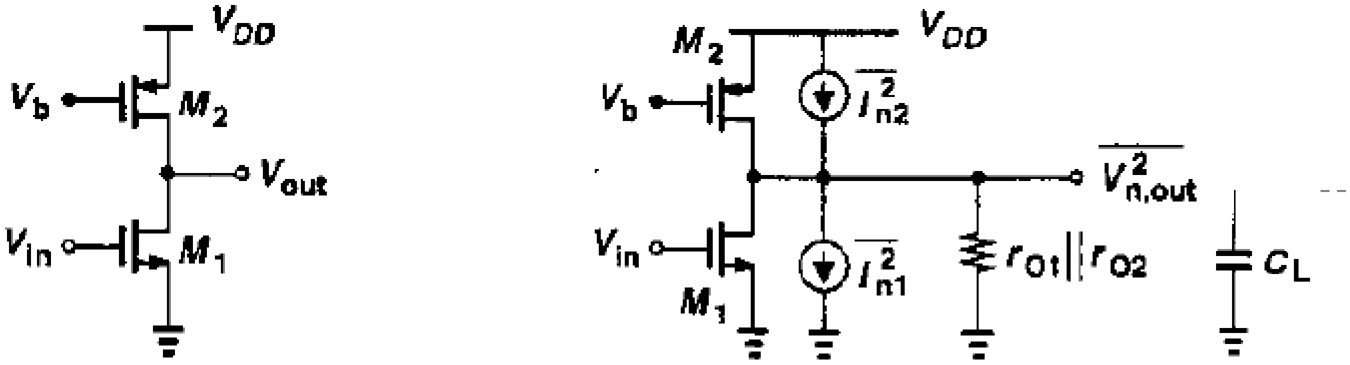


- To reduce the input-referred noise voltage
  - The g_m of M_1 must be maximized if the transistor is to amplify a voltage signal applied to its gate.
  - If $g_{m1} \uparrow \rightarrow I_D \uparrow \rightarrow$ greater power dissipation and limited voltage swings.
  - If $g_{m1} \uparrow \rightarrow W/L \uparrow \rightarrow$ larger input and output capacitance.
  - The transconductance of M_1 must be minimized if the transistor operates as a current source.



  - Noise contributed by R_D decreases as R_D increases.
    - Limiting the voltage headroom and lowering the speed.
  - The noise voltage due to R_D at the output proportional to $(R_D)^{0.5}$
  - The voltage gain of the circuit is proportional to R_D.
  - Trade-off between noise, power dissipation, voltage headroom, and speed.

SNR of CS Stage



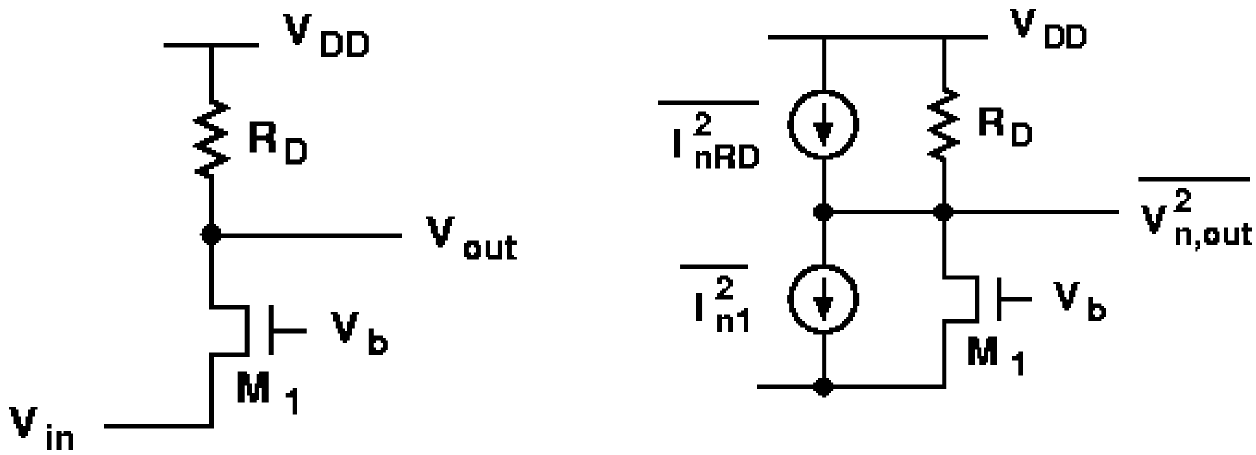
$$\overline{V_{n,out}^2} = 4kT\frac{2}{3}(g_{m1}+g_{m2})(r_{O1}\|r_{O2})^2 = \overline{V_{n,in}^2}(g_{m1})^2(r_{O1}\|r_{O2})^2,\ \overline{V_{n,in}^2} = 4kT\frac{2}{3}\frac{(g_{m1}+g_{m2})}{(g_{m1})^2}$$

$$\overline{V_{n,out,tot}^2} = \int_0^\infty 4kT\frac{2}{3}(g_{m1}+g_{m2})(r_{O1}\|r_{O2})^2\frac{df}{1+(r_{O1}\|r_{O2})^2C_L^2(2\pi f)^2} = \frac{2}{3}(g_{m1}+g_{m2})(r_{O1}\|r_{O2})\frac{kT}{C_L}$$

$$SNR_{out} = \left[\frac{g_{m1}(r_{O1}\|r_{O2})V_m}{\sqrt{2}}\right]^2\frac{1}{(2/3)(g_{m1}+g_{m2})(r_{O1}\|r_{O2})(kT/C_L)} = \frac{3C_L}{4kT}\frac{g_{m1}^2(r_{O1}\|r_{O2})}{g_{m1}+g_{m2}}V_m^2$$

- $C_L\uparrow$ bandwidth$\downarrow$ $SNR\uparrow$
- $g_{m1}\uparrow$ $g_{m2}\downarrow$ $V_m\uparrow$ $SNR\uparrow$ (V_m: amplitude of sinusoid input)

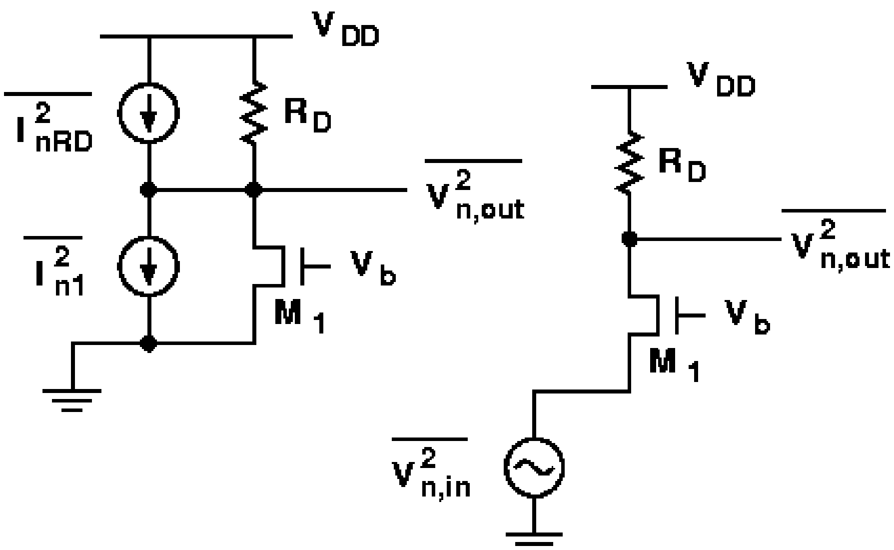
Common-Gate Stage



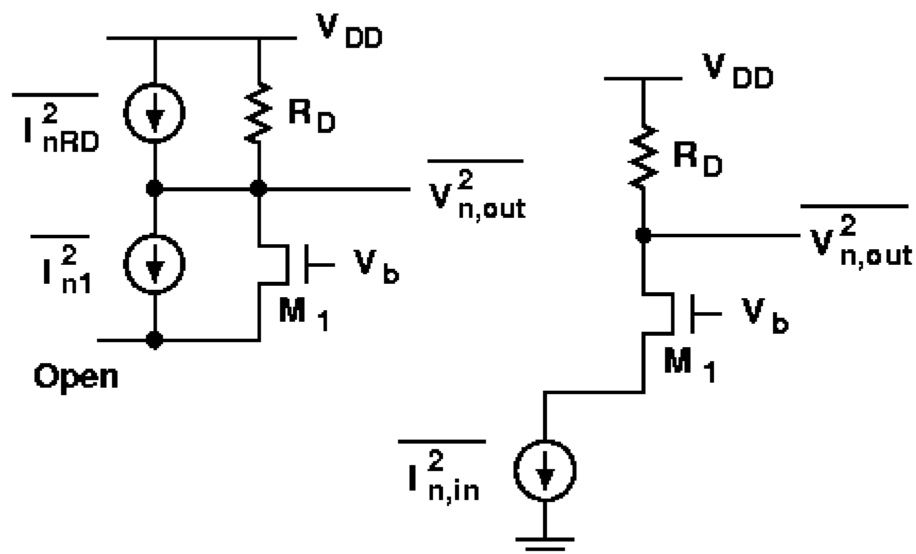
- Neglect channel length modulation effect. Represent the thermal noise of M_1 and R_D by two current sources.
- Due to low input impedance of CG, the input-referred noise current is not negligible at low frequencies
- They directly refer the noise current produced by the load to the input (current-gain = 1).

Input-referred Noise of CG

- To calculate input referred noise V_n :



- To calculate input referred noise I_n :

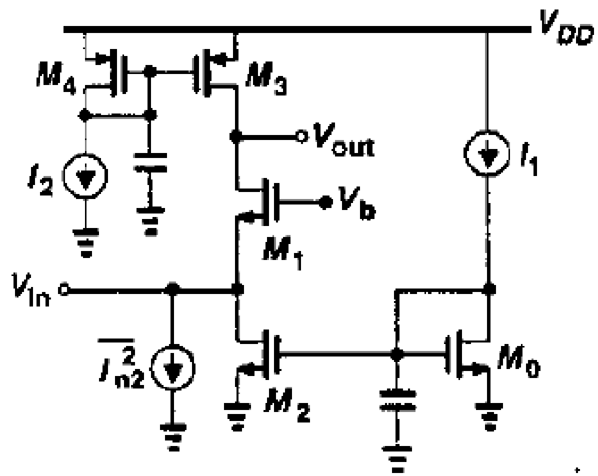


$$4kT\left(\frac{2}{3}g_m+\frac{1}{R_D}\right)R_D^2=\overline{V_{n,in}^2}\left(g_m+g_{mb}\right)^2 R_D^2,\quad \overline{V_{n,in}^2}=\frac{4kT\left(\frac{2}{3}g_m+\frac{1}{R_D}\right)}{\left(g_m+g_{mb}\right)^2}$$

$$I_{n1}+I_{D1}=0,\ \overline{I_{n,in}^2 R_D^2}=4kTR_D,\quad \overline{I_{n,in}^2}=\frac{4kT}{R_D}$$

- They directly refer the noise current produced by the load to the input (I-gain = 1).

Noise Contributed by Current Source



- The drain noise current of M_2 directly adds to the input referred noise current of $\overline{I^2_{n2}}$.
- For low noise performance, $g_{m2}\downarrow$, $g_{m2} = \dfrac{2I_{D2}}{(V_{GS2} - V_{TH2})}$
- For a given bias current, this requires a high voltage of V_b and limiting the voltage swing at the output node.
- For input-referred noise voltage, short the input to ground

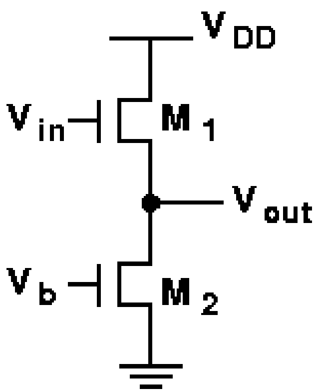
$$\overline{V^2_{n,out}} = 4kT\frac{2}{3}(g_{m1} + g_{m3})(r_{O1} \| r_{O3})^2 = \overline{V^2_{n,in}}(g_{m1} + g_{mb1})^2(r_{O1} \| r_{O3})^2$$

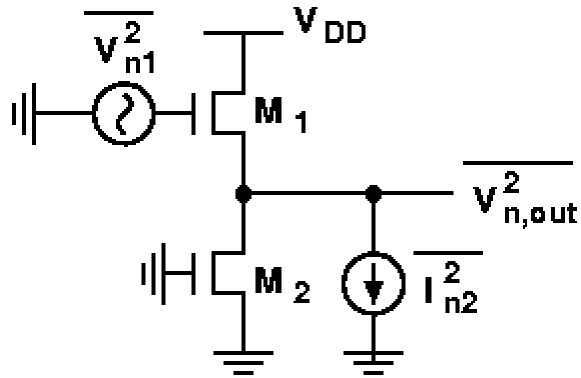
$$\overline{V^2_{n,in}} = 4kT\frac{2}{3}\frac{(g_{m1} + g_{m3})}{(g_{m1} + g_{mb1})^2} \quad \propto \quad g_{m3}$$

- For input-referred noise current, open the input

$$\overline{V^2_{n,out}} = (\overline{I^2_{n2}} + \overline{I^2_{n3}})R^2_{out} = \overline{I^2_{n,in}}R^2_{out}, \qquad \overline{I^2_{n,in}} = (\overline{I^2_{n2}} + \overline{I^2_{n3}}) = 4kT\frac{2}{3}(g_{m2} + g_{m3}) \quad \propto \quad g_{m2},\ g_{m3}$$

Source Followers





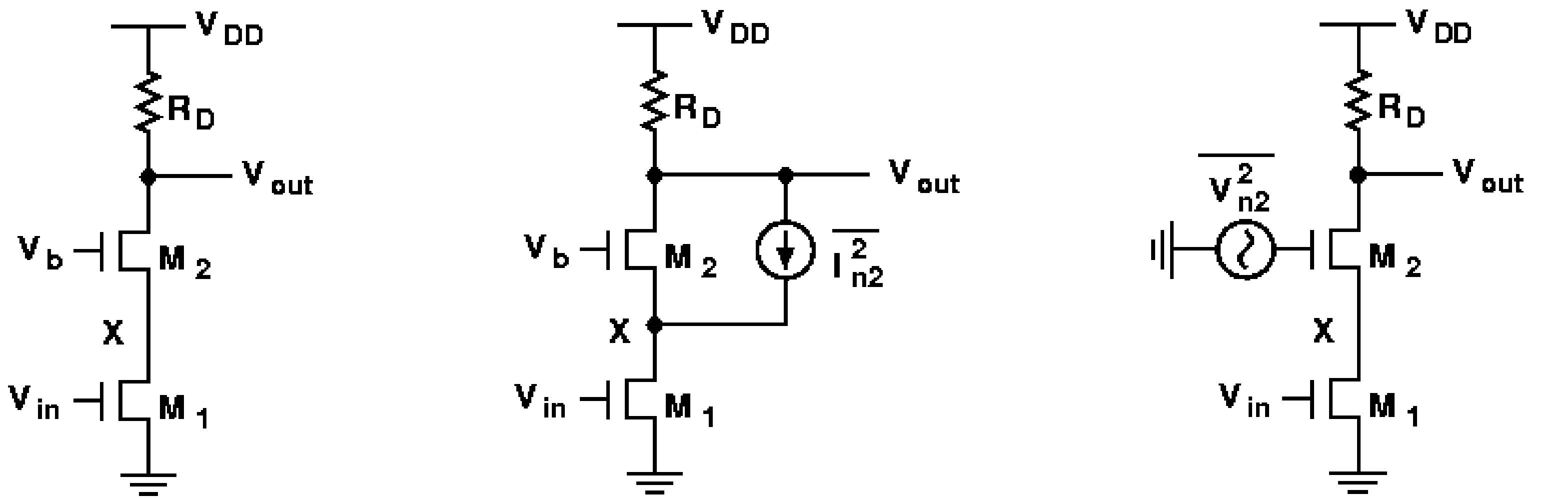
$$\overline{V_{n,in}^2} = \overline{V_{n1}^2} + \frac{\overline{V_{n,out}^2}|_{M2}}{A_v^2} = 4kT\frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)$$

$$\text{or}\quad \overline{V_{n,in}^2} = \frac{\left(\overline{I_{n1}^2} + \overline{I_{n2}^2}\right)R_{out}^2}{A_v^2},\quad \frac{R_{out}}{A_v} = \frac{1}{g_{m1}}$$

$$R_{out} = \left(\frac{1}{g_{m1}} \parallel R_o\right),\quad A_v = \frac{R_o}{R_o + 1/g_{m1}}$$

- M_2 serves as the bias current source .
- Since the input impedance of the circuit is quite high, even at relatively high frequencies, the input-referred noise current can usually be neglected for moderate driving source impedances.
- Since source followers add noise to the input signal while providing a voltage gain less than unity, they are usually avoided in low-noise amplification.

Cascode Stage



- Consider the noise current of M_1 and R_D . At low frequencies, the noise currents of M_1 and R_D flow through R_D

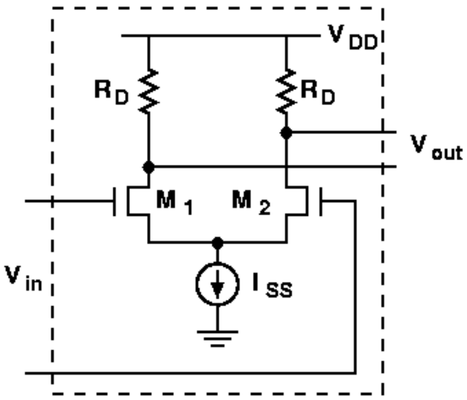
$$\left.\overline{V_{n,in}^2}\right|_{M1,RD} = 4kT\left(\frac{2}{3g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)$$

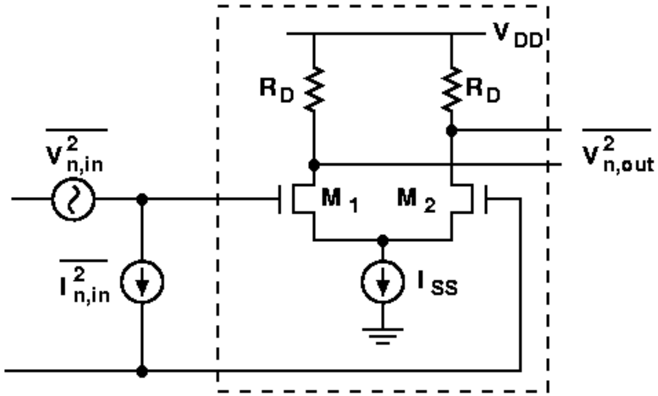
- Consider the noise current of M_2 *,(CS +Source degeneration)*

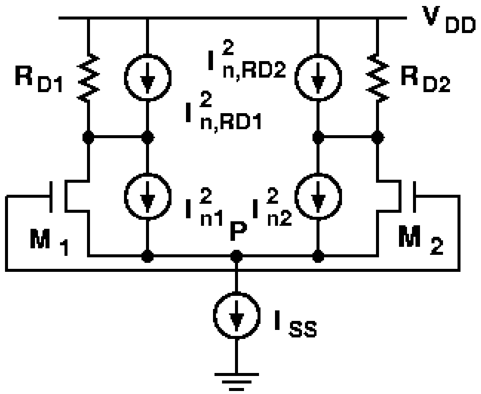
$$\frac{V_{n,out}}{V_{n2}} \approx \frac{-R_D}{1/sC_X + 1/g_{m2}}$$

- The input referred noise of a cascode stage may rise considerably at high freq.

Noise in Differential Pairs

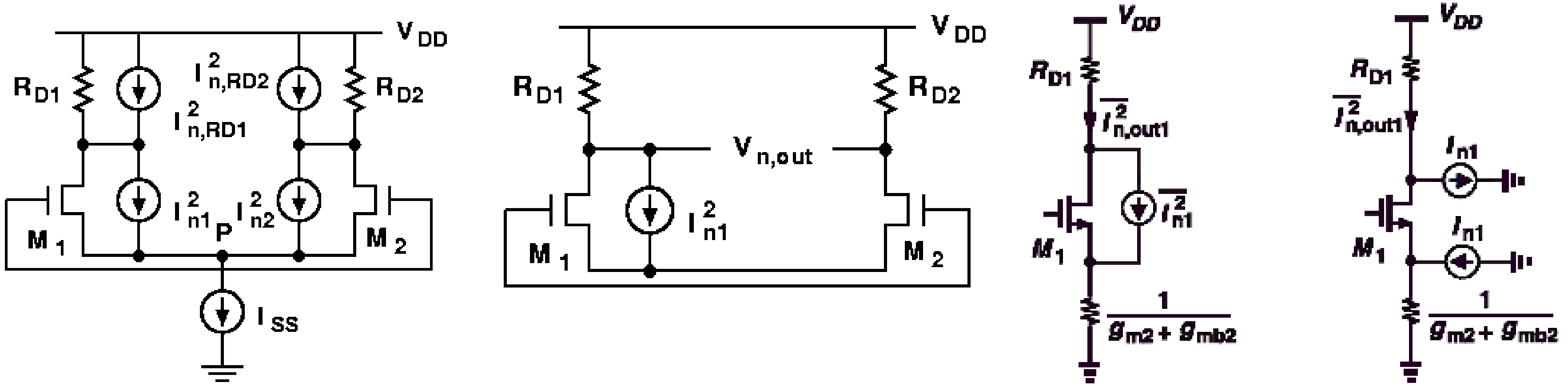






- For low frequency operation, the magnitude of $\overline{I_{n,in}^2}$ is typically negligible.
- To calculate the thermal component of $\overline{V_{n,in}^2}$, short input
  - Since $\overline{I_{n1}^2}$ and $\overline{I_{n2}^2}$ are uncorrelated, node *P* can not be considered a virtual ground.

Input Referred Noise of DP



- Decomposing I_{n1} into two (correlated) current sources and calculating their effect at the output.

$$V_{n,out}\Big|_{\mathrm{M1}} = \frac{I_{n1}}{2}R_{D1} + \frac{I_{n1}}{2}R_{D2}$$

- If $R_{D1} = R_{D2} = R_D$

$$\overline{V^2_{n,out}}\Big|_{M1} = \overline{I^2_{n1}}R_D^2, \qquad \overline{V^2_{n,out}}\Big|_{M2} = \overline{I^2_{n2}}R_D^2, \qquad \overline{V^2_{n,out}}\Big|_{M1,M2} = \left(\overline{I^2_{n1}} + \overline{I^2_{n2}}\right)R_D^2$$

- Taking into account the noise of $R_{D1} = R_{D2}$

$$\boxed{\overline{V^2_{n,in}}(CS) = 4kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right)}$$

$$\overline{V^2_{n,out}} = \left(\overline{I^2_{n1}}R_D^2 + \overline{I^2_{n2}}R_D^2\right) + 2(4kTR_D) = 8kT\left(\frac{2}{3}g_m R_D^2 + R_D\right)$$

- Dividing the result by the square of the differential gain:

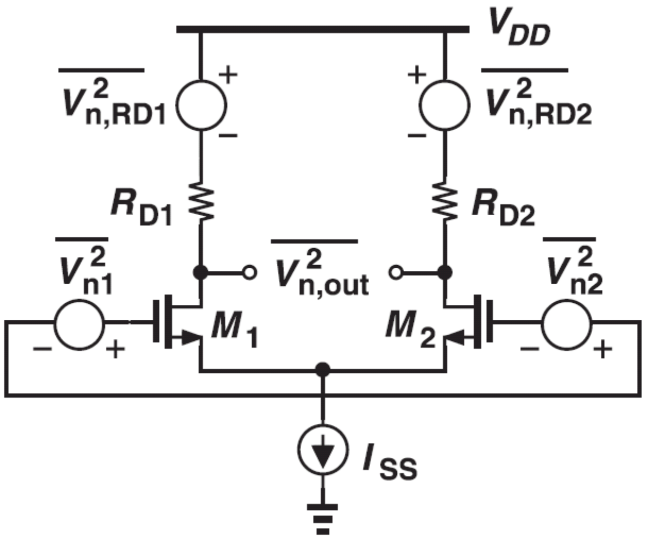
$$\overline{V^2_{n,in}} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right)$$

Input-Referred Noise of DP

- Placing the voltage sources given by $K/(C_{ox}WL)$ in series with each gate

$$\overline{V^2_{n,in}} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{2K}{C_{ox}WL}\frac{1}{f}$$

$$\boxed{\overline{V^2_{n,in}}(CS) = 4kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}}$$



- The noise of tail current modulates the transconductance of each device

$$\Delta I_{D1} - \Delta I_{D2} = g_m \Delta V_{in} = \sqrt{2\mu_n C_{ox}\frac{W}{L}\left(\frac{I_{SS}+I_n}{2}\right)}\Delta V_{in}$$

- In essence, the noise modulates the transconductance of each device

$$\Delta I_{D1} - \Delta I_{D2} \approx \sqrt{2\mu_n C_{ox}\frac{W}{L}\cdot\frac{I_{SS}}{2}}\left(1+\frac{I_n}{2I_{SS}}\right)\Delta V_{in} = g_{m0}\left(1+\frac{I_n}{2I_{SS}}\right)\Delta V_{in}$$

g_{m0} : g_m of noiseless circuit

- The effect is nonetheless usually negligible.