H_∞ Decentralized Fuzzy Model Reference Tracking Control Design for Nonlinear Interconnected Systems

Chung-Shi Tseng, *Member, IEEE,* and Bor-Sen Chen, *Senior Member, IEEE*

Abstract—In general, due to the interactions among subsystems, it is difficult to design an H_∞ decentralized controller for nonlinear interconnected systems. In this study, the model reference tracking control problem of nonlinear interconnected systems is studied via H_∞ decentralized fuzzy control method. First, the nonlinear interconnected system is represented by an equivalent Takagi–Sugeno type fuzzy model. A state feedback decentralized fuzzy control scheme is developed to override the external disturbances such that the H_∞ model reference tracking performance is achieved. Furthermore, the stability of the nonlinear interconnected systems is also guaranteed. If states are not all available, a decentralized fuzzy observer is proposed to estimate the states of each subsystem for decentralized control. Consequently, a fuzzy observer-based state feedback decentralized fuzzy controller is proposed to solve the H_∞ tracking control design problem for nonlinear interconnected systems. The problem of H_∞ decentralized fuzzy tracking control design for nonlinear interconnected systems is characterized in terms of solving an eigenvalue problem (EVP). The EVP can be solved very efficiently using convex optimization techniques. Finally, simulation examples are given to illustrate the tracking performance of the proposed methods.

Index Terms—H_∞ decentralized fuzzy tracking control, linear matrix inequality problem (LMIP) and eigenvalue problem (EVP).

I. INTRODUCTION

THE past three decades have witnessed serious applications of large-scale interconnected system methodologies to urban planning, economic models, spacecraft dynamics, power systems, industrial processes, transportation networks and others. The properties of interconnected systems have been widely studied and many different approaches have been proposed to stabilize the interconnected linear systems [1]–[4]. On the other hand, there are few studies concerning with the stabilization control for the interconnected nonlinear systems [5], [6]. Since linearization technique and linear robust control are used, these results are always conservative and only applicable to some special nonlinear interconnected systems. Due to the physical configuration and high dimensionality of interconnected systems, a centralized control is neither economically feasible nor even necessary [7]. Therefore, decentralized scheme is preferred in control design of the large-scale interconnected systems [8]–[10]. In other words,

decentralized control scheme attempts to avoid difficulties in complexity of design, debugging, data gathering, and storage requirements. However, due to the effects of nonlinear interconnection among subsystems, there is still no efficient way to deal with the decentralized control problem of nonlinear interconnected systems, especially for the model reference tracking control case.

In the past few years, there has been rapidly growing interest in fuzzy control of nonlinear systems, and there have been many successful applications. The most important issue for fuzzy control systems is how to get a system design with the guarantee of stability and control performance, and recently there have been significant research efforts on these issues in fuzzy control systems [19]–[21], [23]–[27]. In these studies, a nonlinear plant was approximated by a Takagi–Sugeno fuzzy linear model [15], and then a model-based fuzzy control was developed to stabilize the Takagi–Sugeno fuzzy linear model. Similarly, there are very few studies concerning with the control problems for the nonlinear interconnected systems using Takagi–Sugeno fuzzy model.

The tasks of stabilization and tracking are two typical control problems. In general, tracking problems are more difficult than stabilization problems. Since fuzzy model is a suitable method to approximate a nonlinear system, a Takagi and Sugeno fuzzy model is employed in this study to approximate the nonlinear interconnected systems. In this paper, a state feedback decentralized fuzzy controller with constant control parameters is proposed to tackle the H_∞ model reference tracking control design problem for nonlinear interconnected systems. A robust technique is developed to efficiently override the effect of external disturbances and interconnections among subsystem to guarantee the stability of global system. The problem of state feedback decentralized H_∞ fuzzy tracking control design is characterized in terms of solving an eigenvalue problem (EVP).

All the results mentioned above are generated with state feedback. In practice, the states of the system are not all available. Several researchers have devoted to the observer design for large-scale systems; e.g., [11], [12]. If a full state observer constructed by the centralized method is used to estimate the state of a large-scale system, it will be difficult to implement and large estimation errors will also arise due to the number of computations involved [13], [14]. In this study, a decentralized fuzzy observer is proposed to estimate the states of each subsystem for decentralized control. Then, a fuzzy observer-based state feedback decentralized fuzzy controller is proposed to solve the H_∞ model reference tracking control design problem for nonlinear interconnected systems. The problem of output feedback H_∞ decentralized fuzzy tracking control design for nonlinear interconnected systems is also characterized in terms

of solving an eigenvalue problem (EVP). The EVP can be solved very efficiently using convex optimization techniques [31], [32].

The paper is organized as follows. The problem formulation is presented in Section II. In Section III, the design problems of H_∞ decentralized fuzzy tracking control for the nonlinear interconnected systems are introduced, while a fuzzy observer-based H_∞ decentralized tracking control design for the nonlinear interconnected systems is considered in Section IV. In Section V, simulation examples are provided to demonstrate the design procedures. Finally, concluding remarks are made in Section VI.

II. PROBLEM FORMULATION

Consider a class of nonlinear interconnected system S which is composed of N subsystems S_i $(i = 1, \ldots, N)$ as follows:

$$S_i : \dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(t)$$
$$+ \sum_{j=1, j \neq i}^{N} f_{ij}(x_j(t)) + w_i(t) \qquad (1)$$

where

$x_i(t) \in R^{n_i}$	state vector;
$u_i(t) \in R^{m_i}$	control signal;
$w_i(t)$	external disturbance of the ith subsystem;
$f_i(x_i(t)), g_i(x_i(t)), f_{ij}(x_j(t))$	smooth functions, where $f_{ij}(x_j(t))$ denotes the interconnection between the ith subsystem and the jth subsystem.

A fuzzy dynamic model has been proposed by Takagi and Sugeno [15] to represent locally linear input/output relations for nonlinear systems. This fuzzy dynamic model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem of a nonlinear interconnected system. The kth rule of this fuzzy model for the nonlinear interconnected subsystem S_i is proposed as the following form:

Plant Rule k:

If $z_{i_1}(t)$ is $F_{k_1} \ldots$ and $z_{i_{g_i}}(t)$ is $F_{k_{g_i}}$,
Then $\dot{x}_i(t) = A_{ik}x_i(t) + B_{ik}u_i(t)$
$$+ \sum_{j=1, j \neq i}^{N} A_{ijk}x_j(t) + w_i(t) \qquad (2)$$

for $k = 1, 2, \ldots, L$ where $x_i(t) \in R^{n_i}$ is the state and $u_i(t) \in R^{m_i}$ is the control signal of the ith subsystem, $F_{i_{g_i}}$ is the fuzzy set, L is the number of If-Then rules, the matrices A_{ik}, B_{ik} and A_{ijk} are of appropriate dimensions, and $z_{i_1}(t), z_{i_2}(t), \ldots, z_{i_{g_i}}(t)$ are the premise variables, for $i, j = 1, 2, \ldots, N$.

The overall fuzzy model of subsystem S_i can be rearranged as the following form [19], [20], [22], [23]:

$$\dot{x}_i(t) = \sum_{k=1}^{L} h_k(z_i(t)) \left[A_{ik}x_i(t) + B_{ik}u_i(t) \right.$$
$$\left. + \sum_{j=1, j \neq i}^{N} A_{ijk}x_j(t) \right] + w_i(t) \qquad (3)$$

where

$$\mu_k(z_i(t)) = \prod_{j=1}^{g_i} F_{kj}\left(z_{i_j}(t)\right)$$
$$h_k(z_i(t)) = \frac{\mu_k(z_i(t))}{\sum_{k=1}^{L} \mu_k(z_i(t))}$$
$$z_i(t) = [z_{i_1}(t), z_{i_2}(t), \ldots, z_{i_{g_i}}(t)] \qquad (4)$$

where $F_{i_{g_i}}(z_{i_{g_i}}(t))$ is the grade of membership of $z_{i_{g_i}}(t)$ in $F_{i_{g_i}}$ [16]–[18].

We assume

$$\mu_k(z_i(t)) \geq 0$$

and

$$\sum_{k=1}^{L} \mu_k(z_i(t)) > 0 \quad \text{for } k = 1, 2, \ldots, L$$

for all t.

Therefore, we get

$$h_k(z_i(t)) \geq 0 \quad \text{for } k = 1, 2, \ldots, L \qquad (5)$$

and

$$\sum_{k=1}^{L} h_k(z_i(t)) = 1. \qquad (6)$$

Consider a reference model for the ith subsystem as follows [29], [30]:

$$\dot{x}_{ri}(t) = A_{ri}x_{ri}(t) + r_i(t) \qquad (7)$$

where

$x_{ri}(t)$ denotes reference state;
A_{ri} denotes a specific asymptotically stable matrix;
$r_i(t)$ denotes bounded reference input.

It is assumed that $x_{ri}(t)$, for all $t \geq 0$, represents a desired trajectory for $x_i(t)$ to follow.

Suppose the following fuzzy decentralized controller is employed to deal with the stabilization problem of the above interconnected subsystem S_i.

Control Rule k:

If $z_{i_1}(t)$ is $F_{k_1} \ldots$ and $z_{i_{g_i}}(t)$ is $F_{k_{g_i}}$,
Then $u_i(t) = K_{ik}(x_i(t) - x_{ri}(t))$. $\qquad (8)$

Hence, the fuzzy decentralized controller is given by

$$u_i(t) = \sum_{k=1}^{L} h_k(z_i(t))[K_{ik}(x_i(t) - x_{ri}(t))]. \qquad (9)$$

Substituting (9) into (3) yields the closed-loop decentralized control of the subsystem S_i as the following:

$$\dot{x}_i(t) = \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t))$$
$$\times \left[(A_{ik} + B_{ik}K_{im})x_i(t) - B_{ik}K_{im}x_{ri}(t) \right.$$
$$\left. + \sum_{j=1, j \neq i}^{N} A_{ijk}x_j(t) \right] + w_i(t). \qquad (10)$$

Let us consider the H_∞ tracking performance related to the tracking error $x_i(t) - x_{ri}(t)$ as follows [28], [23]:

$$\frac{\int_0^{t_f} \{(x_i(t) - x_{ri}(t))^T Q_i(x_i(t) - x_{ri}(t))\}\, dt}{\int_0^{t_f} \bar{w}_i(t)^T \bar{w}_i(t)\, dt} \leq \rho^2 \quad (11)$$

or

$$\int_0^{t_f} \{(x_i(t) - x_{ri}(t))^T Q_i(x_i(t) - x_{ri}(t))\}\, dt$$
$$\leq \rho^2 \int_0^{t_f} \bar{w}_i(t)^T \bar{w}_i(t)\, dt \quad (12)$$

where $\bar{w}_i(t) = [w_i(t), r_i(t)]^T$ for all reference input $r_i(t)$ and external disturbance $w_i(t)$; t_f is terminal time of control, Q_i is symmetric positive definite weighting matrix, ρ is a prescribed attenuation level. The physical meaning of (11) or (12) is that the effect of any $\bar{w}_i(t)$ on tracking error $x_i(t) - x_{ri}(t)$ must be attenuated below a desired level ρ from the viewpoint of energy, no matter what $\bar{w}_i(t)$ is, i.e., the L_2 gain from $\bar{w}_i(t)$ to $x_i(t) - x_{ri}(t)$ must be equal to or less than a prescribed value ρ^2. The H_∞ tracking performance with a prescribed attenuation level is useful for a robust tracking design without knowledge of $w_i(t)$ and $r_i(t)$.

After manipulation, the augmented system can be expressed as the following form:

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}_{ri}(t) \end{bmatrix} = \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t))$$
$$\times \left\{ \begin{bmatrix} A_{ik} + B_{ik}K_{im} & -B_{ik}K_{im} \\ 0 & A_{ri} \end{bmatrix} \begin{bmatrix} x_i(t) \\ x_{ri}(t) \end{bmatrix} \right.$$
$$\left. + \begin{bmatrix} A_{ij_k} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_j(t) \\ x_{rj}(t) \end{bmatrix} + \begin{bmatrix} w_i(t) \\ r_i(t) \end{bmatrix} \right\}. \quad (13)$$

Let us denote

$$\bar{x}_i(t) = \begin{bmatrix} x_i(t) \\ x_{ri}(t) \end{bmatrix}$$
$$\bar{A}_{ikm} = \begin{bmatrix} A_{ik} + B_{ik}K_{im} & -B_{ik}K_{im} \\ 0 & A_{ri} \end{bmatrix}$$
$$\bar{B}_{ijk} = \begin{bmatrix} A_{ijk} & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{w}_i(t) = \begin{bmatrix} w_i(t) \\ r_i(t) \end{bmatrix}. \quad (14)$$

Therefore, the augmented system defined in (13) can be expressed as the following form:

$$\dot{\bar{x}}_i(t) = \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t))$$
$$\times (\bar{A}_{ikm}\bar{x}_i(t) + \bar{B}_{ijk}\bar{x}_j(t)) + \bar{w}_i(t). \quad (15)$$

Hence, the H_∞ tracking performance can be rewritten as follows

$$\int_0^{t_f} \{(x_i(t) - x_{ri}(t))^T Q_i(x_i(t) - x_{ri}(t))\}\, dt$$
$$= \int_0^{t_f} \bar{x}_i^T(t)\bar{Q}_i\bar{x}_i(t)\, dt$$
$$\leq \rho^2 \int_0^{t_f} \bar{w}_i(t)^T \bar{w}_i(t)\, dt \quad (16)$$

where

$$\bar{Q}_i = \begin{bmatrix} Q_i & -Q_i \\ -Q_i & Q_i \end{bmatrix}. \quad (17)$$

If the initial condition is also considered, the inequality (16) can be modified as

$$\int_0^{t_f} \bar{x}_i^T(t)\bar{Q}_i\bar{x}_i(t)\, dt$$
$$\leq \bar{x}_i^T(0)\bar{P}_i\bar{x}_i(0) + \rho^2 \int_0^{t_f} \bar{w}_i(t)^T \bar{w}_i(t)\, dt \quad (18)$$

where $\bar{P}_i$ is a symmetric positive definite weighting matrix.

The purpose of this study is to determine a decentralized fuzzy controller in (9) for the augmented system in (15) with the guaranteed H_∞ tracking performance in (18) for all $\bar{w}_i(t)$. Thereafter, the attenuation level ρ^2 can also be minimized so that the H_∞ tracking performance in (18) is reduced as small as possible. Furthermore, the closed-loop systems for whole interconnected systems

$$\dot{\bar{x}}_i(t) = \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t))$$
$$\times (\bar{A}_{ikm}\bar{x}_i(t) + \bar{B}_{ijk}\bar{x}_j(t)) \quad (19)$$

$i = 1, \ldots, N$ are stable.

III. H_∞ DECENTRALIZED TRACKING CONTROL OF INTERCONNECTED SYSTEMS

Stability is the most important issue in the control system. Obviously, it is appealing for control engineers to specify the decentralized control parameters K_{i_k} in the fuzzy controller (9) such that the stability for the whole interconnected nonlinear system S can be guaranteed. To prove the augmented closed-loop system in (19) to be stable, let us define a Lyapunov function for the system of (19) as

$$v_i(t) = \bar{x}_i^T(t)\bar{P}_i\bar{x}_i(t) \quad \text{for } i = 1, 2, \ldots, N \quad (20)$$

where the weighting matrix $\bar{P}_i = \bar{P}_i^T > 0$. Thus the Lyapunov function $V(t)$ for the overall interconnected system S is

$$V(t) = \sum_{i=1}^{N} v_i(t) = \sum_{i=1}^{N} \bar{x}_i^T(t)\bar{P}_i\bar{x}_i(t). \quad (21)$$

The time derivative of $V(t)$ is

$$\dot{V}(t) = \sum_{i=1}^{N} \left[\dot{\bar{x}}_i^T(t)\bar{P}_i\bar{x}_i(t) + \bar{x}_i^T(t)\bar{P}_i\dot{\bar{x}}_i(t) \right]. \quad (22)$$

By substituting (19) into (22), we get

$$\dot{V}(t) = \sum_{i=1}^{N} \left\{ \dot{\bar{x}}_i^T(t)\bar{P}_i\bar{x}_i(t) + \bar{x}_i^T(t)\bar{P}_i\dot{\bar{x}}_i(t) \right\}$$
$$= \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t))$$
$$\times \left\{ [\bar{A}_{ikm}\bar{x}_i(t) + \bar{B}_{ijk}\bar{x}_j(t)]^T \bar{P}_i\bar{x}_i(t) \right.$$
$$\left. + \bar{x}_i^T(t)\bar{P}_i[\bar{A}_{ikm}\bar{x}_i(t) + \bar{B}_{ijk}\bar{x}_j(t)] \right\}$$
$$= \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t))$$
$$\times \left\{ \begin{bmatrix} \bar{x}_i(t) \\ \bar{x}_j(t) \end{bmatrix}^T \begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i\bar{A}_{ikm} & \bar{P}_i\bar{B}_{ijk} \\ \bar{B}_{ijk}^T \bar{P}_i & 0 \end{bmatrix} \right.$$
$$\left. \times \begin{bmatrix} \bar{x}_i(t) \\ \bar{x}_j(t) \end{bmatrix} \right\}. \quad (23)$$

Then, we get the following result:

Theorem 1: In the augmented nonlinear interconnected system (19), if $\bar{P}_i = \bar{P}_i^T > 0$ is the common solution of the following matrix inequalities:

$$\begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i \bar{A}_{ikm} & \bar{P}_i \bar{B}_{ijk} \\ \bar{B}_{ijk}^T \bar{P}_i & 0 \end{bmatrix} \leq 0 \qquad (24)$$

for $i, j = 1, 2, \ldots, N$ $(j \neq i)$ and $k, m = 1, 2, \ldots, L$, then the whole interconnected nonlinear system S is stable in the sense of Lyapunov.

Proof: From (23), we get

$$\dot{V}(t) = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t)) h_m(z_i(t))$$
$$\times \left\{ \begin{bmatrix} \bar{x}_i(t) \\ \bar{x}_j(t) \end{bmatrix}^T \begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i \bar{A}_{ikm} & \bar{P}_i \bar{B}_{ijk} \\ \bar{B}_{ijk}^T \bar{P}_i & 0 \end{bmatrix} \right.$$
$$\left. \times \begin{bmatrix} \bar{x}_i(t) \\ \bar{x}_j(t) \end{bmatrix} \right\}. \qquad (25)$$

From (24), we obtain $\dot{V}(t) \leq 0$. Then, the whole interconnected nonlinear system S is stable in the sense of Lyapunov. This completes the proof. $\square$

Next, the design purpose in this study is to specify the decentralized fuzzy control in (9) to achieve H_∞ tracking control performance in (18).

Then, we obtain the following main result.

Theorem 2: In the nonlinear system (15), if $\bar{P}_i = \bar{P}_i^T > 0$ is the common solution of the following matrix inequalities:

$$\begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i \bar{A}_{ikm} + \bar{Q}_i & \bar{P}_i \bar{B}_{ijk} & \bar{P}_i \\ \bar{B}_{ijk}^T \bar{P}_i & 0 & 0 \\ \bar{P}_i & 0 & -\rho^2 I \end{bmatrix} \leq 0 \quad (26)$$

for $i, j = 1, 2, \ldots, N$ $(j \neq i)$ and $k, m = 1, 2, \ldots, L$. Then, the whole interconnected nonlinear system S is stable in the sense of Lyapunov if $\bar{w}_i(t) = 0$ and the H_∞ tracking control performance in (18) is guaranteed for a prescribed ρ^2.

Proof: Note that (26) implies

$$\begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i \bar{A}_{ikm} & \bar{P}_i \bar{B}_{ijk} \\ \bar{B}_{ijk}^T \bar{P}_i & 0 \end{bmatrix} \leq 0.$$

According to Theorem 1, the stability of the whole interconnected nonlinear system is immediately followed. From (16), we obtain

$$\int_0^{t_f} \left\{ (x_i(t) - x_{ri}(t))^T Q_i (x_i(t) - x_{ri}(t)) \right\} dt$$
$$= \int_0^{t_f} \bar{x}_i^T(t) \bar{Q}_i \bar{x}_i(t) \, dt$$
$$= \bar{x}_i^T(0) \bar{P}_i \bar{x}_i(0) - \bar{x}_i^T(t_f) \bar{P}_i \bar{x}_i(t_f)$$
$$+ \int_0^{t_f} \left\{ \bar{x}_i^T(t) \bar{Q}_i \bar{x}_i(t) + \frac{d}{dt} \left(\bar{x}_i^T(t) \bar{P}_i \bar{x}_i(t) \right) \right\} dt$$
$$\leq \bar{x}_i^T(0) \bar{P}_i \bar{x}_i(0) + \int_0^{t_f} \left\{ \bar{x}_i^T(t) \bar{Q}_i \bar{x}_i(t) \right.$$
$$\left. + \dot{\bar{x}}_i^T(t) \bar{P}_i \bar{x}_i(t) + \bar{x}_i^T(t) \bar{P}_i \dot{\bar{x}}_i(t) \right\} dt$$

$$= \bar{x}_i^T(0) \bar{P}_i \bar{x}_i(0) + \int_0^{t_f} \left\{ \bar{x}_i^T(t) \bar{Q}_i \bar{x}_i(t) \right.$$
$$+ \left(\sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t)) h_m(z_i(t)) \right.$$
$$\left. \times (\bar{A}_{ikm} \bar{x}_i(t) + \bar{B}_{ijk} \bar{x}_j(t) + \bar{w}_i(t))^T \bar{P}_i \bar{x}_i(t) \right.$$
$$+ \bar{x}_i^T(t) \bar{P}_i \left(\sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t)) h_m(z_i(t)) \right.$$
$$\left. \left. \times (\bar{A}_{ikm} \bar{x}_i(t) + \bar{B}_{ijk} \bar{x}_j(t) + \bar{w}_i(t)) \right) \right\} dt$$
$$= \bar{x}_i^T(0) \bar{P}_i \bar{x}_i(0)$$
$$+ \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t)) h_m(z_i(t))$$
$$\times \left\{ \int_0^{t_f} \bar{x}_i^T(t) \bar{Q}_i \bar{x}_i(t) + (\bar{A}_{ikm} \bar{x}_i(t) + \bar{B}_{ijk} \bar{x}_j(t) \right.$$
$$+ \bar{w}_i(t))^T \bar{P}_i \bar{x}_i(t) + \bar{x}_i^T(t) \bar{P}_i ((\bar{A}_{ikm} \bar{x}_i(t)$$
$$+ \bar{B}_{ijk} \bar{x}_j(t) + \bar{w}_i(t)) - \rho^2 \bar{w}_i(t)^T \bar{w}_i(t)$$
$$\left. + \rho^2 \bar{w}_i(t)^T \bar{w}_i(t) \right\} dt$$
$$= \bar{x}_i^T(0) \bar{P}_i \bar{x}_i(0)$$
$$+ \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t)) h_m(z_i(t))$$
$$\times \left\{ \int_0^{t_f} \begin{bmatrix} \bar{x}_i(t) \\ \bar{x}_j(t) \\ \bar{w}_i(t) \end{bmatrix}^T \right.$$
$$\times \begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i \bar{A}_{ikm} + \bar{Q}_i & \bar{P}_i \bar{B}_{ijk} & \bar{P}_i \\ \bar{B}_{ijk}^T \bar{P}_i & 0 & 0 \\ \bar{P}_i & 0 & -\rho^2 I \end{bmatrix}$$
$$\times \begin{bmatrix} \bar{x}_i(t) \\ \bar{x}_j(t) \\ \bar{w}_i(t) \end{bmatrix} + \rho^2 \bar{w}_i(t)^T \bar{w}_i(t) \right\} dt. \qquad (27)$$

By (26), we obtain

$$\int_0^{t_f} \bar{x}_i^T(t) \bar{Q}_i \bar{x}_i(t) \, dt$$
$$\leq \bar{x}_i^T(0) \bar{P}_i \bar{x}_i(0) + \rho^2 \int_0^{t_f} \bar{w}_i(t)^T \bar{w}_i(t) \, dt. \qquad (28)$$

Therefore, the H_∞ tracking control performance is achieved with a prescribed ρ^2. This completes the proof. $\square$

To obtain a better robust tracking performance, the robust tracking control problem can be formulated as the following minimization problem so that the H_∞ tracking performance in (18) is reduced as small as possible

$$\min_{\bar{P}_i} \rho^2$$
$$\text{subject to } \bar{P}_i = \bar{P}_i^T > 0 \quad \text{and} \quad (26). \qquad (29)$$

Note that the matrix inequalities in (26) can be transformed into certain forms of linear matrix inequalities (LMIs). There-

fore, the minimization problem in (29) can be formulated as a minimization problem subject to some LMIs, which is also called an eigenvalue problem (EVP).

By introducing a new matrix

$$
W_i = \begin{bmatrix} \bar{W}_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} \bar{P}_i^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}
$$

where $\bar{W}_i = \bar{P}_i^{-1}$ and multiplying it into (26), we obtain

$$
W_i \begin{bmatrix} \bar{A}_{ikm}^T \bar{P}_i + \bar{P}_i \bar{A}_{ikm} + \bar{Q}_i & \bar{P}_i \bar{B}_{ijk} & \bar{P}_i \\ \bar{B}_{ijk}^T \bar{P}_i & 0 & 0 \\ \bar{P}_i & 0 & -\rho^2 I \end{bmatrix} W_i
$$
$$
= \begin{bmatrix} \bar{W}_i \bar{A}_{ikm}^T + \bar{A}_{ikm} \bar{W}_i + \bar{W}_i \bar{Q}_i \bar{W}_i & \bar{B}_{ijk} & I \\ \bar{B}_{ijk}^T & 0 & 0 \\ I & 0 & -\rho^2 I \end{bmatrix}
$$
$$
\leq 0. \tag{30}
$$

By the Schur complements [31], (30) is equivalent to

$$
\begin{bmatrix} \bar{W}_i \bar{A}_{ikm}^T + \bar{A}_{ikm} \bar{W}_i & \bar{W}_i \left(\bar{Q}_i^{1/2} \right) & \bar{B}_{ijk} & I \\ \left(\bar{Q}_i^{1/2} \right)^T \bar{W}_i & -I & 0 & 0 \\ \bar{B}_{ijk}^T & 0 & 0 & 0 \\ I & 0 & 0 & -\rho^2 I \end{bmatrix} \leq 0. \tag{31}
$$

Therefore, the minimization problem in (29) is equivalent to

$$
\min_{\bar{W}_i} \rho^2
$$
$$
\text{subject to } \bar{W}_i = \bar{W}_i^T > 0 \quad \text{and} \quad (31). \tag{32}
$$

For the convenience of design, let

$$
\bar{P}_i = \begin{bmatrix} P_{i11} & 0 \\ 0 & P_{i11} \end{bmatrix} \tag{33}
$$

where $P_{i11} = P_{i11}^T > 0$ and thus

$$
\bar{W}_i = \begin{bmatrix} W_{i11} & 0 \\ 0 & W_{i11} \end{bmatrix} \tag{34}
$$

where $W_{i11} = P_{i11}^{-1}$.

With $Y_{im} = K_{im} W_{i11}$, (31) are equivalent to the following linear matrix inequalities (LMIs) (see (35) at the bottom of the page) where

$$
\begin{bmatrix} Q_{i11} & Q_{i12} \\ Q_{i21} & Q_{i22} \end{bmatrix} = \bar{Q}_i^{1/2}.
$$

Finally, the minimization problem in (32) is equivalent to the following eigenvalue problem (EVP).

$$
\min_{\{W_{i11}, Y_{im}\}} \rho^2
$$
$$
\text{subject to } W_{i11} = W_{i11}^T > 0 \quad \text{and} \quad (35) \tag{36}
$$

Based on the analysis above, the design procedures for the H_∞ fuzzy decentralized tracking control of interconnected systems are summarized as follows:

Design Procedure 1:

Step 1) Construct the fuzzy plant rules (2).

Step 2) Solve the EVP in (36) to obtain W_{i11} and Y_{im} (thus $K_{im} = Y_{im} W_{i11}^{-1}$) can also be obtained).

Step 3) Obtain fuzzy decentralized control rule in (9).

IV. DECENTRALIZED OBSERVER SYNTHESIS FOR NONLINEAR INTERCONNECTED SYSTEMS

In the previous sections, we assumed that all the state variables are available. In practice, this assumption often does not hold. In this situation, we need to estimate state vector $x_i(t)$ from output $y_i(t)$ of each subsystem for state feedback control. The kth rule of this fuzzy model for the nonlinear interconnected subsystem S_i is proposed as the following form:

Plant Rule k:

If $z_{i_1}(t)$ is $F_{k_1} \ldots$ and $z_{i_{g_i}}(t)$ is $F_{k_{g_i}}$
$$
\dot{x}_i(t) = A_{ik} x_i(t) + B_{ik} u_i(t),
$$
Then $+ \sum_{j=1, j\neq i}^{N} A_{ijk} x_j(t) + w_i(t)$
$$
y_i(t) = C_{ik} x_i(t) + v_i(t) \tag{37}
$$

where $v_i(t)$ is measurement noise of subsystem S_i.

Therefore, the overall output of subsystem S_i can be arranged as the following form:

$$
y_i(t) = \sum_{k=1}^{L} h_k(z_i(t)) C_{ik} x_i(t) + v_i(t). \tag{38}
$$

$$
\begin{bmatrix} W_{i11}A_{ik}^T + Y_{im}^T B_{ik}^T + A_{ik}W_{i11} + B_{ik}Y_{im} & -B_{ik}Y_{im} & W_{i11}Q_{i11} & W_{i11}Q_{i12} & A_{ijk} & 0 & I & 0 \\ -Y_{im}^T B_{ik}^T & W_{i11}A_{ri}^T + A_{ri}W_{i11} & W_{i11}Q_{i21} & W_{i11}Q_{i22} & 0 & 0 & 0 & I \\ Q_{i11}^T W_{i11} & Q_{i21}^T W_{i11} & -I & 0 & 0 & 0 & 0 & 0 \\ Q_{i12}^T W_{i11} & Q_{i22}^T W_{i11} & 0 & -I & 0 & 0 & 0 & 0 \\ A_{ijk}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & -\rho^2 I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & -\rho^2 I \end{bmatrix} \leq 0 \tag{35}
$$

Suppose the following fuzzy decentralized observer is proposed to deal with the state estimation of the nonlinear interconnected system

Observer Rule k:

If $z_1(t)$ is F_{k_1} ... and $z_{g_i}(t)$ is $F_{k_{g_i}}$

$$\dot{\hat{x}}_i(t) = A_{ik}\hat{x}_i(t) + B_{ik}u_i(t),$$

Then $+ \sum_{j=1, j \neq i}^{N} A_{ijk}\hat{x}_j(t)$

$$+ L_{ik}(y_i(t) - \hat{y}_i(t)) \tag{39}$$

where L_{ik} are the fuzzy observer gains for the kth observer rule for the ith interconnected subsystem S_i and $\hat{y}_i(t) = \sum_{k=1}^{L} h_k(z_i(t))C_{ik}\hat{x}_i(t)$.

The overall fuzzy decentralized observer for the ith subsystem is represented as follows:

$$\dot{\hat{x}}_i(t) = \sum_{k=1}^{L} h_k(z_i(t)) \left[A_{ik}\hat{x}_i(t) + B_{ik}u_i(t) \right.$$

$$\left. + \sum_{j=1, j \neq i}^{N} A_{ijk}\hat{x}_j(t) + L_{ik}(y_i(t) - \hat{y}_i(t)) \right]. \tag{40}$$

The fuzzy observer-based decentralized controller is modified as

$$u_i(t) = \sum_{m=1}^{L} h_m(z_i(t))[K_{im}(\hat{x}_i(t) - x_{ri}(t))] \tag{41}$$

for $i = 1, 2, \ldots, N$, where $\hat{x}_i(t)$ is obtained from the fuzzy observer in (40).

Remark 1: The premise variables $z_i(t)$ can be measurable state variables, outputs or combination of measurable state variables. For Takagi–Sugeno type fuzzy model, using state variables as premise variables are common, but not always [19], [21]–[23]. The limitation of this approach is that some state variables must be measurable to construct the fuzzy observer and fuzzy controller. This is a common limitation for control system design of Takagi–Sugeno fuzzy approach [22], [23]. If the premise variables of the fuzzy observer depend on the estimated state variables, i.e., $\hat{z}_i(t)$ instead of $z_i(t)$ in the fuzzy observer, the situation becomes more complicated. In this case, it is difficult to directly find decentralized control gains K_{im} and observer gains L_{ik}. □

Let us denote the estimation error as

$$e_i(t) = x_i(t) - \hat{x}_i(t). \tag{42}$$

By differentiating (42), we get error dynamic of fuzzy decentralized observer as follows:

$$\dot{e}_i(t) = \dot{x}_i(t) - \dot{\hat{x}}_i(t)$$

$$= \sum_{k=1}^{L} h_k(z_i(t)) \left[A_{ik}x_i(t) + B_{ik}u_i(t) \right.$$

$$\left. + \sum_{j=1, j \neq i}^{N} A_{ijk}x_j(t) + w_i(t) \right.$$

$$- \sum_{k=1}^{L} h_k(z_i(t)) \left[A_{ik}\hat{x}_i(t) + B_{ik}u_i(t) \right.$$

$$\left. + \sum_{j=1, j \neq i}^{N} A_{ijk}\hat{x}_j(t) + L_{ik}(y_i(t) - \hat{y}_i(t)) \right]$$

$$= \sum_{j=1, j \neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t))$$

$$\times [(A_{ik} - L_{ik}C_{im})e_i(t) + A_{ijk}e_j(t)$$

$$+ w_i(t) - L_{ik}v_i(t)]. \tag{43}$$

Then, the augmented system is equivalent to the following form:

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}_{ri}(t) \\ \dot{e}_i(t) \end{bmatrix}$$

$$= \begin{bmatrix} \left\{ \sum_{j=1, j \neq i}^{N} \sum_{k=1}^{L} \sum_{m=1}^{L} h_k(z_i(t))h_m(z_i(t)) \right. \\ \times [(A_{ik} + B_{ik}K_{im})x_i(t) - B_{ik}K_{im}x_{ri}(t) \\ \left. - B_{ik}K_{im}e_i(t) + A_{ijk}x_j(t) + w_i(t)] \right\} \\ \{A_{ri}x_{ri}(t) + r_i(t)\} \\ \left\{ \sum_{j=1, j \neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t)) \right. \\ [(A_{ik} - L_{ik}C_{im})e_i(t) + A_{ijk}e_j(t) \\ \left. + w_i(t) - L_{ik}v_i(t)] \right\} \end{bmatrix}. \tag{44}$$

After manipulation, (44) can be expressed as the following form:

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{x}_{ri}(t) \\ \dot{e}_i(t) \end{bmatrix}$$

$$= \sum_{j=1, j \neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t))$$

$$\times \left\{ \begin{bmatrix} A_{ik} + B_{ik}K_{im} & -B_{ik}K_{im} & -B_{ik}K_{im} \\ 0 & A_{ri} & 0 \\ 0 & 0 & A_{ik} - L_{ik}C_{im} \end{bmatrix} \right.$$

$$\times \begin{bmatrix} x_i(t) \\ x_{ri}(t) \\ e_i(t) \end{bmatrix} + \begin{bmatrix} A_{ijk} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{ijk} \end{bmatrix} \begin{bmatrix} x_j(t) \\ x_{rj}(t) \\ e_j(t) \end{bmatrix}$$

$$\left. + \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ I & 0 & -L_{ik} \end{bmatrix} \begin{bmatrix} w_i(t) \\ r_i(t) \\ v_i(t) \end{bmatrix} \right\}. \tag{45}$$

Let us denote

$$\tilde{A}_{ikm} = \begin{bmatrix} A_{ik} + B_{ik}K_{im} & -B_{ik}K_{im} & -B_{ik}K_{im} \\ 0 & A_{ri} & 0 \\ 0 & 0 & A_{ik} - L_{ik}C_{im} \end{bmatrix}$$

$$\tilde{x}_i(t) = \begin{bmatrix} x_i(t) \\ x_{ri}(t) \\ e_i(t) \end{bmatrix} \quad \tilde{B}_{ijk} = \begin{bmatrix} A_{ijk} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{ijk} \end{bmatrix}$$

$$\tilde{E}_{ik} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ I & 0 & -L_{ik} \end{bmatrix} \quad \tilde{w}_i(t) = \begin{bmatrix} w_i(t) \\ r_i(t) \\ v_i(t) \end{bmatrix}.$$

Therefore, the augmented system of the ith subsystem defined in (45) can be expressed as the following form:

$$
\dot{\tilde{x}}_i(t) = \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t))
$$
$$
\times [\tilde{A}_{ikm}\tilde{x}_i(t) + \tilde{B}_{ijk}\tilde{x}_j(t) + \tilde{E}_{ik}\tilde{w}_i(t)]. \tag{46}
$$

Hence, the H_∞ tracking performance in (18) can be modified as follows:

$$
\int_0^{t_f} \{(x_i(t) - x_{ri}(t))^T Q_i(x_i(t) - x_{ri}(t))\} \, dt
$$
$$
= \int_0^{t_f} \tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t) \, dt
$$
$$
\leq \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0) + \rho^2 \int_0^{t_f} \tilde{w}_i^T(t)\tilde{w}_i(t) \, dt \tag{47}
$$

where $\tilde{P}_i$ is a symmetric positive definite weighting matrix and

$$
\tilde{Q}_i = \begin{bmatrix} Q_i & -Q_i & 0 \\ -Q_i & Q_i & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{48}
$$

The purpose of this study is to determine a fuzzy controller in (41) for the augmented system in (46) with the guaranteed H_∞ tracking performance in (47) for all $\tilde{w}_i(t)$. Thereafter, the attenuation level ρ^2 can also be minimized so that the H_∞ tracking performance in (47) is reduced as small as possible. Furthermore, the closed-loop systems for whole interconnected systems

$$
\dot{\tilde{x}}_i(t) = \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t))
$$
$$
\times [\tilde{A}_{ikm}\tilde{x}_i(t) + \tilde{B}_{ijk}\tilde{x}_j(t)]. \tag{49}
$$

$i = 1, \ldots, N$ are stable.

Let us choose a Lyapunov function for each subsystem (49) as follows:

$$
\tilde{v}_i(t) = \tilde{x}_i^T(t)\tilde{P}_i\tilde{x}_i(t), \quad \text{for } i = 1, 2, \ldots, N \tag{50}
$$

where the weighting matrix $\tilde{P}_i = \tilde{P}_i^T > 0$.

Thus, the Lyapunov function $\tilde{V}(t)$ for the overall interconnected system S is

$$
\tilde{V}(t) = \sum_{i=1}^{N} \tilde{v}_i(t) = \sum_{i=1}^{N} \tilde{x}_i^T(t)\tilde{P}_i\tilde{x}_i(t). \tag{51}
$$

By differentiating (51), we get

$$
\dot{\tilde{V}}(t) = \sum_{i=1}^{N} \left\{ \dot{\tilde{x}}_i^T(t)\tilde{P}_i\tilde{x}_i(t) + \tilde{x}_i^T(t)\tilde{P}_i\dot{\tilde{x}}_i(t) \right\}
$$
$$
= \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t))
$$
$$
\times [\tilde{A}_{ikm}\tilde{x}_i(t) + \tilde{B}_{ijk}\tilde{x}_j(t)]^T \tilde{P}_i\tilde{x}_i(t)
$$
$$
+ \tilde{x}_i^T(t)\tilde{P}_i[\tilde{A}_{ikm}\tilde{x}_i(t) + \tilde{B}_{ijk}\tilde{x}_j(t)]
$$
$$
= \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t))
$$

$$
\times \left\{ \begin{bmatrix} \tilde{x}_i(t) \\ \tilde{x}_j(t) \end{bmatrix}^T \begin{bmatrix} \tilde{A}_{ikm}^T\tilde{P}_i + \tilde{P}_i\tilde{A}_{ikm} & \tilde{P}_i\tilde{B}_{ijk} \\ \tilde{B}_{ijk}^T\tilde{P}_i & 0 \end{bmatrix} \right.
$$
$$
\left. \times \begin{bmatrix} \tilde{x}_i(t) \\ \tilde{x}_j(t) \end{bmatrix} \right\}. \tag{52}
$$

Then, we get the following results:

Theorem 3: In the augmented nonlinear interconnected system (49), if $\tilde{P}_i = \tilde{P}_i^T > 0$ is the common solution of the following matrix inequalities

$$
\begin{bmatrix} \tilde{A}_{ikm}^T\tilde{P}_i + \tilde{P}_i\tilde{A}_{ikm} & \tilde{P}_i\tilde{B}_{ijk} \\ \tilde{B}_{ijk}^T\tilde{P}_i & 0 \end{bmatrix} \leq 0 \tag{53}
$$

for $i, j = 1, 2, \ldots, N$ $(j \neq i)$ and $k, m = 1, \ldots, L$, then the whole interconnected nonlinear system S is stable in the sense of Lyapunov.

Proof: From (52) and (53), we obtain $\dot{\tilde{V}}(t) \leq 0$. Then, the whole interconnected nonlinear system S is stable in the sense of Lyapunov. This completes the proof. $\square$

Theorem 4: In the nonlinear augmented system (46), if $\tilde{P}_i = \tilde{P}_i^T > 0$ is the common solution of the following matrix inequalities

$$
\begin{bmatrix} \tilde{A}_{ikm}^T\tilde{P}_i + \tilde{P}_i\tilde{A}_{ikm} + \tilde{Q}_i & \tilde{P}_i\tilde{B}_{ijk} & \tilde{P}_i\tilde{E}_{ik} \\ \tilde{B}_{ijk}^T\tilde{P}_i & 0 & 0 \\ \tilde{E}_{ik}^T\tilde{P}_i & 0 & -\rho^2 I \end{bmatrix} \leq 0 \tag{54}
$$

for $i, j = 1, 2, \ldots, N$ $(j \neq i)$ and $k, m = 1, 2, \ldots, L$. Then, the whole interconnected nonlinear system S is stable in the sense of Lyapunov if $\tilde{w}_i(t) = 0$ and the H_∞ tracking control performance in (47) is guaranteed for a prescribed ρ^2.

Proof: According to Theorem 3 and by the same argument as that in Theorem 2, the stability of the whole interconnected nonlinear system is obtained. From (47), we obtain

$$
\int_0^{t_f} \{(x_i(t) - x_{ri}(t))^T Q_i(x_i(t) - x_{ri}(t))\} \, dt
$$
$$
= \int_0^{t_f} \tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t) \, dt
$$
$$
= \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0) - \tilde{x}_i^T(t_f)\tilde{P}_i\tilde{x}_i(t_f)
$$
$$
+ \int_0^{t_f} \left\{ \tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t) + \frac{d}{dt}\left(\tilde{x}_i^T(t)\tilde{P}_i\tilde{x}_i(t)\right) \right\} dt
$$
$$
\leq \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0) + \int_0^{t_f} \left\{ \tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t) \right.
$$
$$
+ \dot{\tilde{x}}_i^T(t)\tilde{P}_i\tilde{x}_i(t) + \tilde{x}_i^T(t)\tilde{P}_i\dot{\tilde{x}}_i(t) \Big\} dt
$$
$$
= \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0) + \int_0^{t_f} \left\{ \tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t) \right.
$$
$$
+ \left(\sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t)) \right.
$$
$$
\times (\tilde{A}_{ikm}\tilde{x}_i(t) + \tilde{B}_{ijk}\tilde{x}_j(t) + \tilde{E}_{ik}\tilde{w}_i(t))^T \tilde{P}_i\tilde{x}_i(t)
$$
$$
+ \tilde{x}_i^T(t)\tilde{P}_i \left(\sum_{j=1, j\neq i}^{N} \sum_{k=1}^{L} h_k(z_i(t)) \sum_{m=1}^{L} h_m(z_i(t)) \right.
$$
$$
\times (\tilde{A}_{ikm}\tilde{x}_i(t) + \tilde{B}_{ijk}\tilde{x}_j(t) + \tilde{E}_{ik}\tilde{w}_i(t)) \Big\} dt
$$

$$= \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0)$$
$$+ \sum_{j=1,j\neq i}^{N}\sum_{k=1}^{L}h_k(z_i(t))\sum_{m=1}^{L}h_m(z_i(t))$$
$$\times \int_0^{t_f}\Big\{\tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t)$$
$$+ (\tilde{A}_{ikm}\tilde{x}_i(t)+\tilde{B}_{ijk}\tilde{x}_j(t)+\tilde{E}_{ik}\tilde{w}_i(t))^T\tilde{P}_i\tilde{x}_i(t)$$
$$+ \tilde{x}_i^T(t)\tilde{P}_i(\tilde{A}_{ikm}\tilde{x}_i(t)+\tilde{B}_{ijk}\tilde{x}_j(t)+\tilde{E}_{ik}\tilde{w}_i(t))$$
$$- \rho^2\tilde{w}_i(t)^T\tilde{w}_i(t)+\rho^2\tilde{w}_i(t)^T\tilde{w}_i(t)\Big\}dt$$
$$= \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0)$$
$$+ \sum_{j=1,j\neq i}^{N}\sum_{k=1}^{L}h_k(z_i(t))\sum_{m=1}^{L}h_m(z_i(t))$$
$$\times \Bigg\{\int_0^{t_f}\begin{bmatrix}\tilde{x}_i(t)\\\tilde{x}_j(t)\\\tilde{w}_i(t)\end{bmatrix}^T$$
$$\times \begin{bmatrix}\tilde{A}_{ikm}^T\tilde{P}_i+\tilde{P}_i\tilde{A}_{ikm}+\tilde{Q}_i & \tilde{P}_i\tilde{B}_{ijk} & \tilde{P}_i\tilde{E}_{ik}\\ \tilde{B}_{ijk}^T\tilde{P}_i & 0 & 0\\ \tilde{E}_{ik}^T\tilde{P}_i & 0 & -\rho^2 I\end{bmatrix}$$
$$\times \begin{bmatrix}\tilde{x}_i(t)\\\tilde{x}_j(t)\\\tilde{w}_i(t)\end{bmatrix}+\rho^2\tilde{w}_i(t)^T\tilde{w}_i(t)\Bigg\}dt. \quad (55)$$

By (54), we obtain

$$\int_0^{t_f}\tilde{x}_i^T(t)\tilde{Q}_i\tilde{x}_i(t)\,dt$$
$$\leq \tilde{x}_i^T(0)\tilde{P}_i\tilde{x}_i(0)+\rho^2\int_0^{t_f}\tilde{w}_i(t)^T\tilde{w}_i(t)\,dt. \quad (56)$$

Therefore, the H_∞ tracking control performance is achieved with a prescribed ρ^2. This completes the proof. $\qquad\square$

To obtain a better robust racking performance, the robust tracking control problem can be formulated as the following minimization problem so that the H_∞ tracking performance in (47) is reduced as small as possible.

$$\min_{\tilde{P}_i}\rho^2$$
$$\text{subject to } \tilde{P}_i=\tilde{P}_i^T>0 \quad \text{and} \quad (54). \quad (57)$$

From the analysis above, the most important work of the H_∞ fuzzy decentralized observer-based tracking control problem is how to solve the common solution $\tilde{P}_i=\tilde{P}_i^T>0$ from the minimization problem (57). In general, it is not easy to analytically determine common solution $\tilde{P}_i=\tilde{P}_i^T>0$ for (57). Similarly, (57) can be transferred into a minimization problem subject to some linear matrix inequalities (LMIs) called eigenvalue problem (EVP) [31]. The EVP can be solved in a computationally efficient manner using a convex optimization technique such as the interior point method [31].

The following decoupling technique is employed to simplify the design problem. For the convenience of design, we assume

$$\tilde{P}_i=\begin{bmatrix}\tilde{P}_{i11} & 0 & 0\\ 0 & \tilde{P}_{i22} & 0\\ 0 & 0 & \tilde{P}_{i33}\end{bmatrix} \quad (58)$$

where $\tilde{P}_{i11}=\tilde{P}_{i11}^T>0$, $\tilde{P}_{i22}=\tilde{P}_{i22}^T>0$, $\tilde{P}_{i33}=\tilde{P}_{i33}^T>0$.

This choice is suitable for the separate design of the fuzzy controller and fuzzy observer. By substituting (58) into (54), we obtain (see (59) at the bottom of the page) where $M_{11}=(A_{ik}+B_{ik}K_{im})^T\tilde{P}_{i11}+\tilde{P}_{i11}(A_{ik}+B_{ik}K_{im})+Q_i$.

$$\left[\begin{array}{ccc}
M_{11} & -\tilde{P}_{i11}(B_{ik}K_{im})-Q_i & -\tilde{P}_{i11}(B_{ik}K_{im})\\
-(B_{ik}K_{im})^T\tilde{P}_{i11}-Q_i & A_{ri}^T\tilde{P}_{i22}+\tilde{P}_{i22}A_{ri}+Q_i & 0\\
-(B_{ik}K_{im})^T\tilde{P}_{i11} & 0 & (A_{ik}-L_{ik}C_{im})^T\tilde{P}_{i33}+\tilde{P}_{i33}(A_{ik}-L_{ik}C_{im})\\
A_{ijk}^T\tilde{P}_{i11} & 0 & 0\\
0 & 0 & 0\\
0 & 0 & A_{ijk}^T\tilde{P}_{i33}\\
\tilde{P}_{i11} & 0 & \tilde{P}_{i33}\\
0 & \tilde{P}_{i22} & 0\\
0 & 0 & -L_{ik}^T\tilde{P}_{i33}
\end{array}\right.$$

$$\left.\begin{array}{cccccc}
\tilde{P}_{i11}A_{ijk} & 0 & 0 & \tilde{P}_{11} & 0 & 0\\
0 & 0 & 0 & 0 & \tilde{P}_{i22} & 0\\
0 & 0 & \tilde{P}_{i33}A_{ijk} & \tilde{P}_{i33} & 0 & -\tilde{P}_{i33}L_{ik}\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\rho^2 I & 0 & 0\\
0 & 0 & 0 & 0 & -\rho^2 I & 0\\
0 & 0 & 0 & 0 & 0 & -\rho^2 I
\end{array}\right]\leq 0 \quad (59)$$

By introducing a new matrix

$$
\tilde{W}_i = \begin{bmatrix} \tilde{W}_{i11} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & I & 0 & \cdots & \cdots & \vdots \\ \vdots & 0 & \ddots & 0 & \cdots & \vdots \\ \vdots & \vdots & 0 & I & 0 & \vdots \\ \vdots & \vdots & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & I \end{bmatrix}
$$

$$
= \begin{bmatrix} \tilde{P}_{i11}^{-1} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & I & 0 & \cdots & \cdots & \vdots \\ \vdots & 0 & \ddots & 0 & \cdots & \vdots \\ \vdots & \vdots & 0 & I & 0 & \vdots \\ \vdots & \vdots & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & I \end{bmatrix}
$$

where $\tilde{W}_{i11} = \tilde{P}_{i11}^{-1}$ and multiplying it into (59), we obtain

$$
\tilde{W}_i \times \{\text{Left hand side of (59)}\} \times \tilde{W}_i \leq 0. \tag{60}
$$

With $Z_{ik} = P_{i33}L_{ik}$ and $Y_{im} = K_{im}\tilde{W}_{i11}$, the inequalities in (60) are equivalent to following matrix inequalities (see (61) at the bottom of the page) where $M_{22} = (A_{ik}\tilde{W}_{i11} + B_{ik}Y_{im})^T + (A_{ik}\tilde{W}_{i11} + B_{ik}Y_{im}) + \tilde{W}_{i11}Q_i\tilde{W}_{i11}$.

Therefore, the H_∞ fuzzy decentralized observer-based tracking control problem can be reformulated as the following optimization problem:

$$
\min_{\{\tilde{W}_{i11}, \tilde{P}_{i22}, \tilde{P}_{i33}\}} \rho^2
$$
$$
\text{subject to } W_{i11} = W_{i11}^T > 0,
$$
$$
P_{i22} = P_{i22}^T > 0,
$$
$$
P_{i33} = P_{i33}^T > 0 \quad \text{and} \quad \text{(61).} \tag{62}
$$

The analysis above shows that when dealing with the stabilization design problem of the fuzzy observer-based

decentralized control system, the most important task is to solve common solutions $W_{i11} = W_{i11}^T > 0$, $P_{i22} = P_{i22}^T > 0$ and $P_{i33} = P_{i33}^T > 0$ from (62). Since the variables K_{im} and Y_{im} $(=K_{im}\tilde{W}_{i11})$ are cross-coupled, there are no effective algorithms for solving these matrix inequalities till now. By the choice of (58), the control problem and observer problem can be decoupled and can be solved separately by the following two-stage procedures which solve the fuzzy control parameters first and then solve the fuzzy observer parameters. These will be discussed in detail in the following.

We first solve $\tilde{W}_{i11}$ and Y_{im} from the following matrix inequalities

$$
(A_{ik}\tilde{W}_{i11} + B_{ik}Y_{im})^T + (A_{ik}\tilde{W}_{i11} + B_{ik}Y_{im}) + \tilde{W}_{i11}Q_i\tilde{W}_{i11} \leq 0. \tag{63}
$$

Note, that by the Schur complements, (63) is equivalent to (see (64) at the bottom of the next page).

Note that solving $\tilde{W}_{i11}$ and Y_{im} from (64) is a convex linear matrix inequality problem (LMIP). After solving the LMIP in (64) to obtain $\tilde{W}_{i11}$ and Y_{im} (thus $K_{im} = Y_{im}\tilde{W}_{i11}^{-1}$) and by substituting $\tilde{W}_{i11}$ and Y_{im} and K_{im} into (61), (61) becomes a convex linear matrix inequalities (LMIs).

Second, we can solve the following eigenvalue problem (EVP)

$$
\min_{\{\tilde{P}_{i22}, \tilde{P}_{i33}, Z_{ik}\}} \rho^2
$$
$$
\text{subject to } \tilde{P}_{i22} = \tilde{P}_{i22}^T > 0,
$$
$$
\tilde{P}_{i33} = \tilde{P}_{i33}^T > 0 \quad \text{and} \quad \text{(61)} \tag{65}
$$

to obtain P_{i22}, P_{i33} and Z_{ik} (thus $L_{ik} = P_{i33}^{-1}Z_{ik}$).

Therefore, the H_∞ fuzzy decentralized observer-based tracking control problem in (62) is equivalent to solving

$$
\min_{\{\tilde{W}_{i11}, \tilde{P}_{i22}, \tilde{P}_{i33}\}} \rho^2
$$
$$
\text{subject to (64)} \quad \text{and} \quad \text{(65).} \tag{66}
$$

$$
\begin{bmatrix}
M_{22} & -B_{ik}K_{im} - \tilde{W}_{i11}Q_i & -B_{ik}K_{im} \\
-(B_{ik}K_{im})^T - Q_i\tilde{W}_{i11} & A_{ri}^T\tilde{P}_{i22} + \tilde{P}_{i22}A_{ri} + Q_i & 0 \\
-(B_{ik}K_{im})^T & 0 & (\tilde{P}_{i33}A_{ik} - Z_{ik}C_{im})^T + (\tilde{P}_{i33}A_{ik} - Z_{ik}C_{im}) \\
A_{ijk}^T & 0 & 0 \\
0 & 0 & A_{ijk}^T\tilde{P}_{i33} \\
I & 0 & \tilde{P}_{i33} \\
0 & \tilde{P}_{i22} & 0 \\
0 & 0 & -Z_{ik}^T
\end{bmatrix}
$$

$$
\begin{matrix}
A_{ijk} & 0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & 0 & \tilde{P}_{i22} & 0 \\
0 & 0 & \tilde{P}_{i33}A_{ijk} & \tilde{P}_{i33} & 0 & -Z_{ik} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\rho^2 I & 0 \\
0 & 0 & 0 & 0 & 0 & -\rho^2 I
\end{matrix} \leq 0 \tag{61}
$$

The design procedures for the H_∞ fuzzy observer-based decentralized tracking control of interconnected systems are summarized as follows:

Design Procedure 2:

Step 1) Select fuzzy plant rules (37).

Step 2) Solve the LMIP in (64) to obtain $\tilde{W}_{i11}$ and Y_{im} (thus $K_{im} = Y_{im}\tilde{W}_{i11}^{-1}$) can also be obtained).

Step 3) Substituting $\tilde{W}_{i11}, Y_{im}$, and K_{im} into (61) and then solve the EVP in (65) to obtain P_{i22}, P_{i33} and Z_{ik} (thus $L_{ik} = P_{i33}^{-1} Z_{ik}$ can also be obtained).

Step 4) Construct the fuzzy observer in (40).

Step 5) Obtain fuzzy decentralized control rule in (41).

V. SIMULATION EXAMPLES

We consider a two-machine interconnected system which is composed of two-machine subsystems S_i as follows [33]

$$
\begin{aligned}
S_i : \dot{x}_{i_1}(t) &= x_{i_2}(t) \\
\dot{x}_{i_2}(t) &= -\frac{D_i}{M_i}x_{i_2}(t) + \frac{1}{M_i}u_i(t) \\
&\quad + \sum_{j=1,j\neq i}^{2} \frac{E_i E_j Y_{ij}}{M_i}\left[\cos\left(\delta_{ij}^0 - \theta_{ij}\right)\right. \\
&\quad - \cos\left(x_{i_1}(t) - x_{j_1}(t) + \delta_{ij}^0 - \theta_{ij}\right)\bigr] \\
&\quad + w_{i_2}(t) \\
y_i(t) &= x_{i_1}(t) + v_i(t)
\end{aligned}
\tag{67}
$$

where

$x_{i_1}(t), x_{i_2}(t)$ absolute rotor angle and angular velocity of the ith machine, respectively;

M_i inertia coefficient;

D_i damping coefficient;

E_i internal voltage;

Y_{ij} modulus of the transfer admittance between the ith and jth machines;

θ_{ij} phase angle of the transfer admittance between the ith and jth machines;

$w_{i_2}(t)$ external disturbance which is assumed to be sinusoidal with amplitude 1 and period π;

$v_i(t)$ measurement noise which is assumed to be zero mean white noise with variance 0.1 for $i, j = 1, 2 \ (i \neq j)$.

We assume the two-machine interconnected systems' parameters as follows:

$$
\begin{aligned}
E_1 &= 1.017 & E_2 &= 1.005 \\
M_1 &= 1.03 & M_2 &= 1.25 \\
D_1 &= 0.8 & D_2 &= 1.2 \\
Y_{12} &= Y_{21} = 1.98 \\
\theta_{12} &= -\theta_{21} = 1.5 & \delta_{12}^0 &= -\delta_{21}^0 = 1.2
\end{aligned}
$$

Example 1: Suppose the state variables are available. Now, following the **Design Procedure 1** in the above section, a design procedure of fuzzy decentralized state feedback controller for model reference tracking of the interconnected system is given as follows:

Step 1) To use the fuzzy decentralized control approach, we represent the interconnected system (67) by a fuzzy model. The problem of constructing Takagi–Sugeno fuzzy model for nonlinear systems can be found in [35]–[37]. To minimize the design effort and complexity, we try to use as few rules as possible. Hence, we approximate the system by the following nine-rule fuzzy model.

Rule 1

If $x_{1_1}(t)$ is about $-\frac{\pi}{2}$ and $x_{2_1}(t)$ is about $-\frac{\pi}{2}$,

Then $\dot{x}_i(t) = A_{i_1}x_i(t) + B_{i_1}u_i(t)$
$$
+ \sum_{j=1,j\neq i}^{2} A_{ij_1}x_j(t) + w_i(t).
$$

Rule 2

If $x_{1_1}(t)$ is about $-\frac{\pi}{2}$ and $x_{2_1}(t)$ is about 0,

Then $\dot{x}_i(t) = A_{i_2}x_i(t) + B_{i_2}u_i(t)$
$$
+ \sum_{j=1,j\neq i}^{2} A_{ij_2}x_j(t) + w_i(t)
$$

Rule 3

If $x_{1_1}(t)$ is about $-\frac{\pi}{2}$ and $x_{2_1}(t)$ is about $\frac{\pi}{2}$,

Then $\dot{x}_i(t) = A_{i_3}x_i(t) + B_{i_3}u_i(t)$
$$
+ \sum_{j=1,j\neq i}^{2} A_{ij_3}x_j(t) + w_i(t).
$$

Rule 4

If $x_{1_1}(t)$ is about 0   and $x_{2_1}(t)$ is about $-\frac{\pi}{2}$,

Then $\dot{x}_i(t) = A_{i_4}x_i(t) + B_{i_4}u_i(t)$
$$
+ \sum_{j=1,j\neq i}^{2} A_{ij_4}x_j(t) + w_i(t).
$$

Rule 5

If $x_{1_1}(t)$ is about 0 and $x_{2_1}(t)$ is about 0,

Then $\dot{x}_i(t) = A_{i_5}x_i(t) + B_{i_5}u_i(t)$
$$
+ \sum_{j=1,j\neq i}^{2} A_{ij_5}x_j(t) + w_i(t).
$$

$$
\begin{bmatrix}
(A_{ik}\tilde{W}_{i11} + B_{ik}Y_{im})^T + (A_{ik}\tilde{W}_{i11} + B_{ik}Y_{im}) & \tilde{W}_{i11} \\
\tilde{W}_{i11} & -Q_i^{-1}
\end{bmatrix} \leq 0.
\tag{64}
$$

Rule 6

If $x_{1_1}(t)$ is about 0 and $x_{2_1}(t)$ is about $\dfrac{\pi}{2}$,

Then $\dot{x}_i(t) = A_{i_6} x_i(t) + B_{i_6} u_i(t)$
$$+ \sum_{j=1, j \neq i}^{2} A_{ij_6} x_j(t) + w_i(t).$$

Rule 7

If $x_{1_1}(t)$ is about $\dfrac{\pi}{2}$ and $x_{2_1}(t)$ is about $-\dfrac{\pi}{2}$,

Then $\dot{x}_i(t) = A_{i_7} x_i(t) + B_{i_7} u_i(t)$
$$+ \sum_{j=1, j \neq i}^{2} A_{ij_7} x_j(t) + w_i(t).$$

Rule 8

If $x_{1_1}(t)$ is about $\dfrac{\pi}{2}$ and $x_{2_1}(t)$ is about 0,

Then $\dot{x}_i(t) = A_{i_8} x_i(t) + B_{i_8} u_i(t)$
$$+ \sum_{j=1, j \neq i}^{2} A_{ij_8} x_j(t) + w_i(t).$$

Rule 9

If $x_{1_1}(t)$ is about $\dfrac{\pi}{2}$ and $x_{2_1}(t)$ is about $\dfrac{\pi}{2}$,

Then $\dot{x}_i(t) = A_{i_9} x_i(t) + B_{i_9} u_i(t)$
$$+ \sum_{j=1, j \neq i}^{2} A_{ij_9} x_j(t) + w_i(t)$$

where A_{ik}, B_{ik}, A_{ijk} are listed in Appendix A, $w_i(t) = [0, \ w_{i_2}(t)]^T$ and $C_{i_k} = [1 \ 0]$ for $i = 1, 2$ and $k = 1, \ldots, 9$.

The reference models are given as

$$A_{ri} = \begin{bmatrix} 0 & 1 \\ -100 & -101 \end{bmatrix}$$

and

$$r_i(t) = \begin{bmatrix} 0 \\ 100 \cos(0.5t) \\ 0 \\ 100 \sin(0.5t) \end{bmatrix} \quad \text{for } i = 1, 2.$$

For the convenience of desin, triangle type membership functions are chosen for Rule 1–Rule 9.

Step 2) Solve EVP using the LMI optimization toolbox in Matlab [34]. In this case

$$W_{111} = \begin{bmatrix} 0.0092 & -0.0376 \\ -0.0376 & 2.6558 \end{bmatrix} \times 10^5$$

and

$$W_{211} = \begin{bmatrix} 0.0098 & -0.0471 \\ -0.0471 & 3.4882 \end{bmatrix} \times 10^5.$$

Step 3) The control parameters can be found in Appendix B.

Figs. 1–4 present the simulation results. Initial condition is assumed to be $(x_{1_1}(0), x_{1_2}(0), x_{2_1}(0), x_{2_2}(0))^T = (1, 0, 1, 0)^T$ in the simulations. The trajectories of the state variable $x_{1_1}(t)$ including reference state variable $x_{r1_1}(t)$ are shown in Fig. 1.
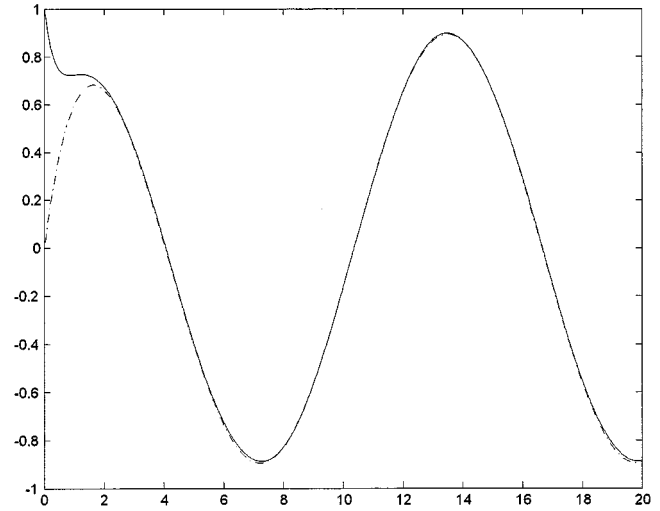

Fig. 1. The trajectories of the state variable $x_{1_1}(t)$ (solid line) and reference state variable $x_{r1_1}(t)$ (dashdot line).
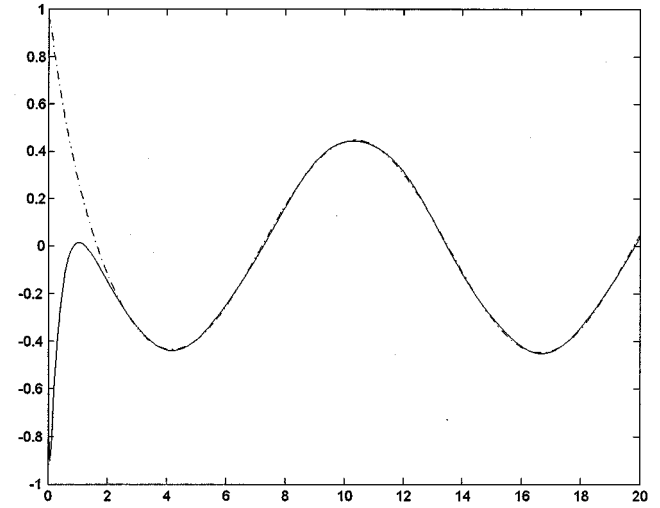

Fig. 2. The trajectories of the state variable $x_{1_2}(t)$ (solid line) and reference state variable $x_{r1_2}(t)$ (dashdot line).
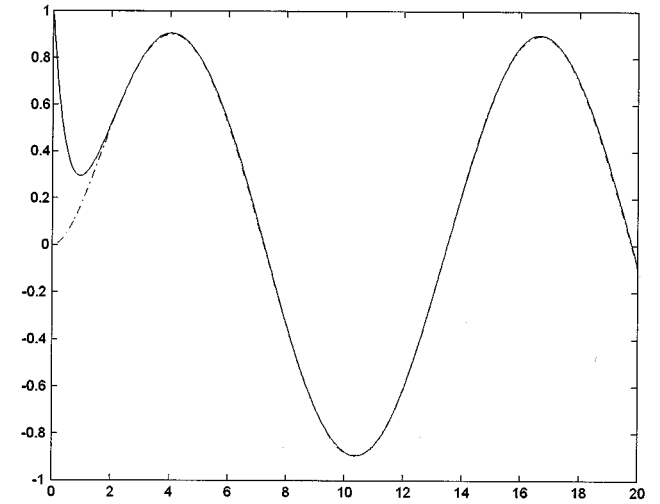

Fig. 3. The trajectories of the state variable $x_{2_1}(t)$ (solid line) and reference state variable $x_{r2_1}(t)$ (dashdot line).

The trajectories of the state variable $x_{1_2}(t)$ including reference state variable $x_{r1_2}(t)$ are shown in Fig. 2. The trajectories of the
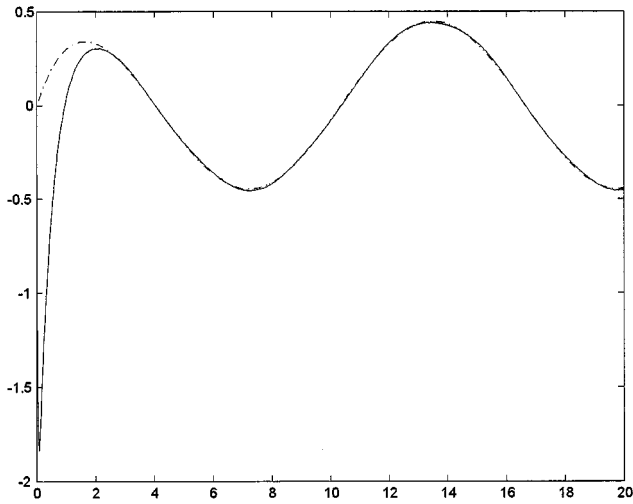
Fig. 4. The trajectories of the state variable $x_{2_2}(t)$ (solid line) and reference state variable $x_{r2_2}(t)$ (dashdot line).
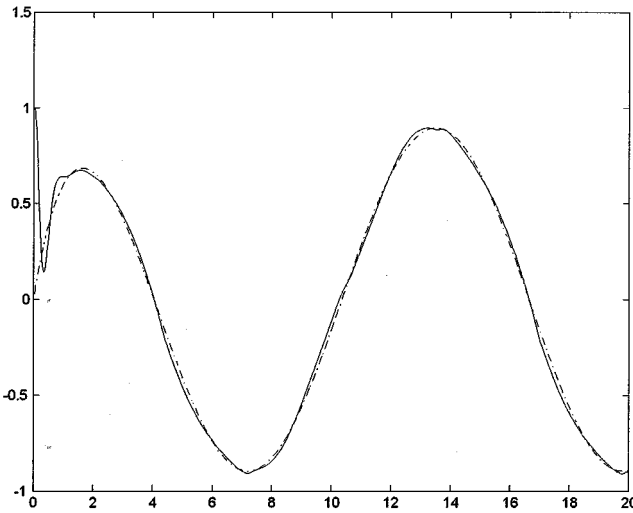

Fig. 6. The trajectories of the state variable $x_{1_2}(t)$ (solid line) and reference state variable $x_{r1_2}(t)$ (dashdot line).

Fig. 5. The trajectories of the state variable $x_{1_1}(t)$ (solid line) and reference state variable $x_{r1_1}(t)$ (dashdot line).

Fig. 7. The trajectories of the state variable $x_{1_2}(t)$ (solid line) and estimated state variable $\hat{x}_{1_2}(t)$ (dashdot line).

state variable $x_{2_1}(t)$ including reference state variable $x_{r2_1}(t)$ are shown in Fig. 3. The trajectories of the state variable $x_{2_2}(t)$ including reference state variable $x_{r2_2}(t)$ are shown in Fig. 4.

Example 2: Suppose the state variables are not all available. Then, following the **Design Procedure 2** in the above section, a design procedure of fuzzy observer-based decentralized controller for the model reference tracking of the interconnected system is given as follows. We assume the state variables $x_{1_1}(t) \in [-\pi/2, \pi/2]$ and $x_{2_1}(t) \in [-\pi/2, \pi/2]$ can be measured to construct the fuzzy model. In other words, the state variables $x_{1_2}(t)$ and $x_{2_2}(t)$ should be estimated.

  Step 1) The same as **Example 1**.
  Step 2) Solve LMIP and EVP using the LMI optimization toolbox in Matlab. In this case

$$\tilde{W}_{111} = \begin{bmatrix} 0.0077 & -0.0493 \\ -0.0493 & 1.7958 \end{bmatrix} \times 10^7$$

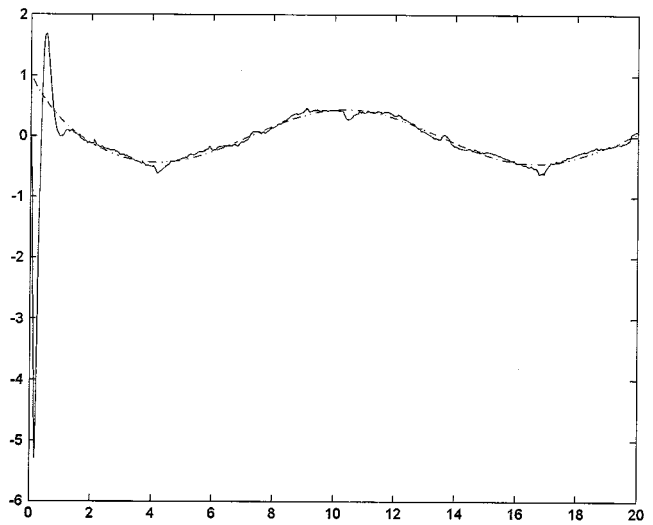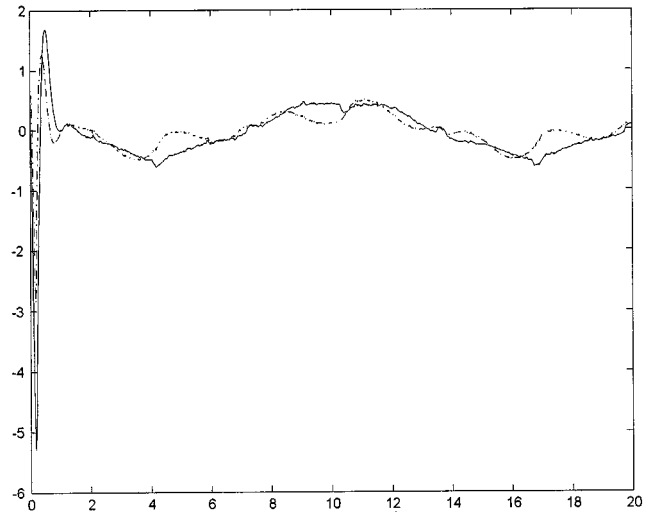$$\tilde{W}_{211} = \begin{bmatrix} 0.0079 & -0.0488 \\ -0.0488 & 1.7743 \end{bmatrix} \times 10^7$$

$$\tilde{P}_{122} = \begin{bmatrix} 0.5596 & 0.0070 \\ 0.0070 & 0.0025 \end{bmatrix}$$

$$\tilde{P}_{222} = \begin{bmatrix} 33.8545 & 33.5434 \\ 33.5434 & 33.8967 \end{bmatrix}$$

$$\tilde{P}_{133} = \begin{bmatrix} 0.0686 & -0.0067 \\ -0.0067 & 0.0017 \end{bmatrix}$$

$$\tilde{P}_{233} = \begin{bmatrix} 0.0924 & -0.0088 \\ -0.0088 & 0.0020 \end{bmatrix}.$$

  Step 3) The observer parameters can be found in Appendix C.
  Step 4) The control parameters can be found in Appendix D.

Figs. 5–10 present the simulation results. The trajectories of the state variable $x_{1_1}(t)$ including reference state variable $x_{r1_1}(t)$ is shown in Fig. 5. The trajectories of the state variable $x_{1_2}(t)$ including reference state variable $x_{r1_2}(t)$ is shown in Fig. 6. The trajectories of the state variable $x_{1_2}(t)$ including
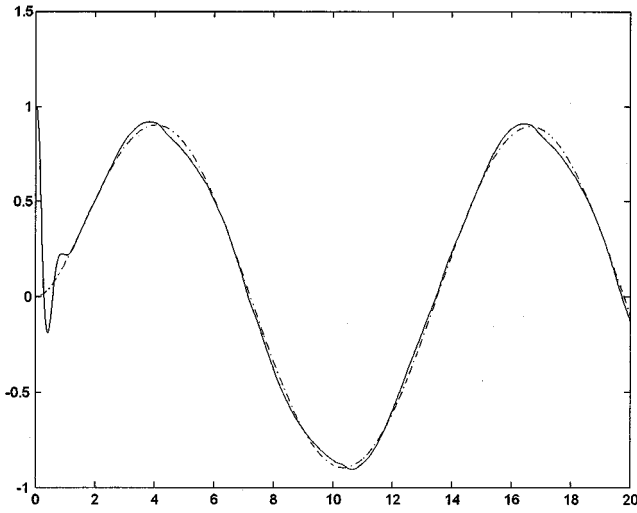
Fig. 8. The trajectories of the state variable $x_{2_1}(t)$ (solid line) and reference state variable $x_{r2_1}(t)$ (dashdot line).

Fig. 10. The trajectories of the state variable $x_{2_2}(t)$ (solid line) and estimated state variable $\hat{x}_{r2_2}(t)$ (dashdot line).
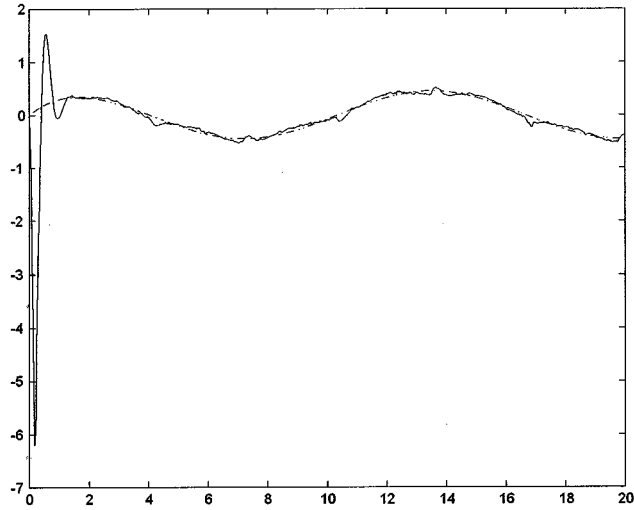

Fig. 9. The trajectories of the state variable $x_{2_2}(t)$ (solid line) and reference state variable $x_{r2_2}(t)$ (dashdot line).

estimated state variable $\hat{x}_{1_2}(t)$ is shown in Fig. 7. The trajectories of the state variable $x_{2_1}(t)$ including reference state variable $x_{r2_1}(t)$ is shown in Fig. 8. The trajectories of the state variable $x_{2_2}(t)$ including reference state variable $x_{r2_2}(t)$ is shown in Fig. 9. The trajectories of the state variable $x_{2_2}(t)$ including estimated state variable $\hat{x}_{2_2}(t)$ is shown in Fig. 10. From the simulation results, the proposed decentralized H_∞ control scheme can solve the tracking problem for nonlinear interconnected systems effectively and systematically with the aid of LMI toolbox in Matlab.

VI. CONCLUSION

In this paper, a Takagi and Sugeno fuzzy model is proposed to study the model reference tracking control problems for nonlinear interconnected systems using fuzzy decentralized control. A fuzzy decentralized controller is proposed to override the effect of external disturbances such that the H_∞ model reference tracking performance is achieved. Furthermore, the stability of the nonlinear interconnected systems is also guaranteed. If states
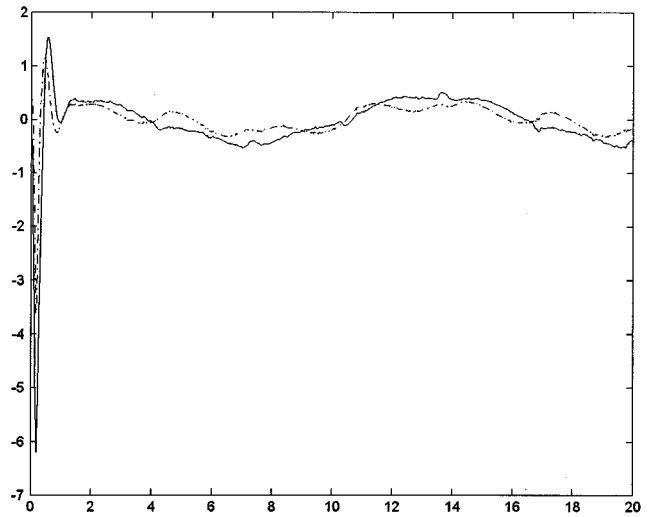
are not all available, a decentralized fuzzy observer is also proposed to estimate the states of each subsystem for decentralized control. Consequently, a fuzzy observer-based state feedback decentralized fuzzy controller is proposed to solve the H_∞ tracking control design problem for nonlinear interconnected systems. The problem of H_∞ decentralized fuzzy tracking control design for nonlinear interconnected systems is characterized in terms of solving an eigenvalue problem (EVP). The EVP can be efficiently solved using convex optimization techniques. The proposed fuzzy observer-based decentralized control scheme is simple without complex control algorithms. Therefore, it is suitable for practical applications. Simulation example is given to illustrate the design procedure and tracking performance of the proposed method.

APPENDIX A

$$A_{1_1} = \begin{bmatrix} 0 & 1 \\ -0.7046 & -0.7767 \end{bmatrix}$$

$$B_{1_1} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_1} = \begin{bmatrix} 0 & 0 \\ 0.5086 & 0 \end{bmatrix}$$

$$A_{1_2} = \begin{bmatrix} 0 & 1 \\ -1.4809 & -0.7767 \end{bmatrix}$$

$$B_{1_2} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_2} = \begin{bmatrix} 0 & 0 \\ 1.5483 & 0 \end{bmatrix}$$

$$A_{1_3} = \begin{bmatrix} 0 & 1 \\ -1.4536 & -0.7767 \end{bmatrix}$$

$$B_{1_3} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_3} = \begin{bmatrix} 0 & 0 \\ 1.4556 & 0 \end{bmatrix}$$

$$A_{1_4} = \begin{bmatrix} 0 & 1 \\ 1.0472 & -0.7767 \end{bmatrix}$$

$$B_{1_4} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_4} = \begin{bmatrix} 0 & 0 \\ -0.7669 & 0 \end{bmatrix}$$

$$A_{1_5} = \begin{bmatrix} 0 & 1 \\ -0.5139 & -0.7767 \end{bmatrix}$$

$$B_{1_5} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_5} = \begin{bmatrix} 0 & 0 \\ 0.5249 & 0 \end{bmatrix}$$

$$A_{1_6} = \begin{bmatrix} 0 & 1 \\ -1.5480 & -0.7767 \end{bmatrix}$$

$$B_{1_6} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_6} = \begin{bmatrix} 0 & 0 \\ 1.4812 & 0 \end{bmatrix}$$

$$A_{1_7} = \begin{bmatrix} 0 & 1 \\ 1.1261 & -0.7767 \end{bmatrix}$$

$$B_{1_7} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_7} = \begin{bmatrix} 0 & 0 \\ -1.1295 & 0 \end{bmatrix}$$

$$A_{1_8} = \begin{bmatrix} 0 & 1 \\ 0.7686 & -0.7767 \end{bmatrix}$$

$$B_{1_8} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_8} = \begin{bmatrix} 0 & 0 \\ -1.0486 & 0 \end{bmatrix}$$

$$A_{1_9} = \begin{bmatrix} 0 & 1 \\ -0.5066 & -0.7767 \end{bmatrix}$$

$$B_{1_9} = \begin{bmatrix} 0 \\ 0.9709 \end{bmatrix} \quad A_{12_9} = \begin{bmatrix} 0 & 0 \\ 0.7017 & 0 \end{bmatrix}$$

$$A_{2_1} = \begin{bmatrix} 0 & 1 \\ 0.2776 & -0.96 \end{bmatrix}$$

$$B_{2_1} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_1} = \begin{bmatrix} 0 & 0 \\ -0.4410 & 0 \end{bmatrix}$$

$$A_{2_2} = \begin{bmatrix} 0 & 1 \\ -0.6478 & -0.96 \end{bmatrix}$$

$$B_{2_2} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_2} = \begin{bmatrix} 0 & 0 \\ 0.9436 & 0 \end{bmatrix}$$

$$A_{2_3} = \begin{bmatrix} 0 & 1 \\ -0.9528 & -0.96 \end{bmatrix}$$

$$B_{2_3} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_3} = \begin{bmatrix} 0 & 0 \\ 0.9284 & 0 \end{bmatrix}$$

$$A_{2_4} = \begin{bmatrix} 0 & 1 \\ 1.2532 & -0.96 \end{bmatrix}$$

$$B_{2_4} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_4} = \begin{bmatrix} 0 & 0 \\ -1.2188 & 0 \end{bmatrix}$$

$$A_{2_5} = \begin{bmatrix} 0 & 1 \\ 0.3200 & -0.96 \end{bmatrix}$$

$$B_{2_1} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_5} = \begin{bmatrix} 0 & 0 \\ -0.2912 & 0 \end{bmatrix}$$

$$A_{2_6} = \begin{bmatrix} 0 & 1 \\ -0.9129 & -0.96 \end{bmatrix}$$

$$B_{2_6} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_6} = \begin{bmatrix} 0 & 0 \\ 0.6472 & 0 \end{bmatrix}$$

$$A_{2_7} = \begin{bmatrix} 0 & 1 \\ 1.2135 & -0.96 \end{bmatrix}$$

$$B_{2_7} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_7} = \begin{bmatrix} 0 & 0 \\ -1.2104 & 0 \end{bmatrix}$$

$$A_{2_8} = \begin{bmatrix} 0 & 1 \\ 1.2206 & -0.96 \end{bmatrix}$$

$$B_{2_8} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_8} = \begin{bmatrix} 0 & 0 \\ -1.2670 & 0 \end{bmatrix}$$

$$A_{2_9} = \begin{bmatrix} 0 & 1 \\ -0.5133 & -0.96 \end{bmatrix}$$

$$B_{2_9} = \begin{bmatrix} 0 \\ 0.800 \end{bmatrix} \quad A_{21_9} = \begin{bmatrix} 0 & 0 \\ -0.3509 & 0 \end{bmatrix}.$$

APPENDIX B

$$K_{11} = [-140.6451 \quad -67.8928]$$
$$K_{12} = [-140.3757 \quad -67.7941]$$
$$K_{13} = [-140.3836 \quad -67.7969]$$
$$K_{14} = [-141.3106 \quad -68.1321]$$
$$K_{15} = [-140.7160 \quad -67.9187]$$
$$K_{16} = [-140.3494 \quad -67.7843]$$
$$K_{17} = [-141.3433 \quad -68.1446]$$
$$K_{18} = [-141.2039 \quad -68.0953]$$
$$K_{19} = [-140.7202 \quad -67.9204]$$
$$K_{21} = [-184.0177 \quad -85.4650]$$
$$K_{22} = [-183.7923 \quad -85.4344]$$
$$K_{23} = [-183.7179 \quad -85.4238]$$
$$K_{24} = [-184.2922 \quad -85.5030]$$
$$K_{25} = [-184.0277 \quad -85.4662]$$
$$K_{26} = [-183.7261 \quad -85.4246]$$
$$K_{27} = [-184.2813 \quad -85.5016]$$
$$K_{28} = [-184.2854 \quad -85.5023]$$
$$K_{29} = [-184.0760 \quad -85.4728].$$

APPENDIX C

$$L_{11} = \begin{bmatrix} 15.2449 \\ 58.9788 \end{bmatrix} \quad L_{12} = \begin{bmatrix} 15.2449 \\ 58.2905 \end{bmatrix}$$

$$L_{13} = \begin{bmatrix} 15.2449 \\ 58.8063 \end{bmatrix} \quad L_{14} = \begin{bmatrix} 15.2449 \\ 58.1325 \end{bmatrix}$$

$$L_{15} = \begin{bmatrix} 15.2449 \\ 58.9632 \end{bmatrix} \quad L_{16} = \begin{bmatrix} 15.2449 \\ 58.9628 \end{bmatrix}$$

$$L_{17} = \begin{bmatrix} 15.2449 \\ 58.9342 \end{bmatrix} \quad L_{18} = \begin{bmatrix} 15.2449 \\ 58.4381 \end{bmatrix}$$

$$L_{19} = \begin{bmatrix} 15.2449 \\ 58.9351 \end{bmatrix} \quad L_{21} = \begin{bmatrix} 17.2505 \\ 83.9651 \end{bmatrix}$$

$$L_{22} = \begin{bmatrix} 17.2505 \\ 85.2093 \end{bmatrix} \quad L_{23} = \begin{bmatrix} 17.2505 \\ 88.9561 \end{bmatrix}$$

$$L_{24} = \begin{bmatrix} 17.2505 \\ 86.4055 \end{bmatrix} \quad L_{25} = \begin{bmatrix} 17.2505 \\ 83.9011 \end{bmatrix}$$

$$L_{26} = \begin{bmatrix} 17.2505 \\ 85.9321 \end{bmatrix} \quad L_{27} = \begin{bmatrix} 17.2505 \\ 89.3109 \end{bmatrix}$$

$$L_{28} = \begin{bmatrix} 17.2505 \\ 85.6217 \end{bmatrix} \quad L_{29} = \begin{bmatrix} 17.2505 \\ 83.8479 \end{bmatrix}.$$

APPENDIX D

$$K_{11} = [-225.3436 \quad -23.8345]$$
$$K_{12} = [-224.4896 \quad -23.8259]$$
$$K_{13} = [-224.5195 \quad -23.8262]$$
$$K_{14} = [-227.2713 \quad -23.8540]$$
$$K_{15} = [-225.5534 \quad -23.8366]$$
$$K_{16} = [-224.4156 \quad -23.8251]$$

$$K_{17} = \begin{bmatrix} -227.3581 & -23.8548 \end{bmatrix}$$
$$K_{18} = \begin{bmatrix} -226.9647 & -23.8509 \end{bmatrix}$$
$$K_{19} = \begin{bmatrix} -225.5614 & -23.8367 \end{bmatrix}$$
$$K_{21} = \begin{bmatrix} -239.0267 & -24.5687 \end{bmatrix}$$
$$K_{22} = \begin{bmatrix} -237.8031 & -24.5578 \end{bmatrix}$$
$$K_{23} = \begin{bmatrix} -237.3998 & -24.5542 \end{bmatrix}$$
$$K_{24} = \begin{bmatrix} -240.3168 & -24.5803 \end{bmatrix}$$
$$K_{25} = \begin{bmatrix} -239.0827 & -24.5692 \end{bmatrix}$$
$$K_{26} = \begin{bmatrix} -237.4526 & -24.5547 \end{bmatrix}$$
$$K_{27} = \begin{bmatrix} -240.2643 & -24.5798 \end{bmatrix}$$
$$K_{28} = \begin{bmatrix} -240.2737 & -24.5799 \end{bmatrix}$$
$$K_{29} = \begin{bmatrix} -239.3384 & -24.5715 \end{bmatrix}.$$

REFERENCES

[1] W. E. Thompson, "Exponential stability of interconnected systems," *IEEE Trans. Automatic Contr.*, vol. AC-15, pp. 504–506, 1970.

[2] L. T. Grujić and D. D. Siljak, "Asymptotic stability and instability of large scale systems," *IEEE Trans. Automatic Contr.*, vol. AC-18, pp. 636–645, 1973.

[3] W. S. Chan and C. A. Desoer, "Eigenvalue assignment and stabilization of interconnected systems using local feedback," *IEEE Trans. Automatic Contr.*, vol. AC-25, pp. 106–107, 1980.

[4] D. J. N. Limebeer and Y. S. Hung, "Robust stability of interconnected system," *IEEE Trans. Circuit Syst.*, vol. CAS-30, pp. 397–403, 1983.

[5] E. J. Davison, "The decentralized stabilization and control of unknow nonlinear time varying systems," *Automatica*, vol. 10, pp. 309–316, 1974.

[6] B. S. Chen and W. S. You, "Robust stabilization in observer-based feedback control systems under nonlinear time-varying perturbations or unmodeled dynamics," *IEEE Trans. Automatic Contr.*, vol. AC-32, no. 12, pp. 1131–1135, 1987.

[7] M. Jamshidi, *Large-Scale Systems: Modeling and Control*. New York: Elsevier, 1983.

[8] Y. H. Chen, "Decentralized robust control system design for large-scale uncertain systems," *Int. J. Contr.*, vol. 111, pp. 1195–1205, 1988.

[9] W. J. Wang and C. F. Cheng, "Robustness of perturbed large-scale systems with local constant state feedback," *Int. J. Contr.*, vol. 50, pp. 373–384, 1989.

[10] A. K. Mahalanabis and R. Singh, "On decentralized feedback stabilization of large-scale interconnected systems," *Int. J. Contr.*, vol. 32, pp. 115–126, 1980.

[11] B. Shahian, "Decentralized control using observers," *Int. J. Contr.*, vol. 44, pp. 1125–1135, 1986.

[12] D. D. Siljak and M. Vukcevic, "Decentralization, stabilization, and estimation of large-scale systems," *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 363–366, 1976.

[13] D. G. Luenberger, "Observers for multivariable system," *IEEE Trans. Automatic Contr.*, vol. 11, pp. 190–197, 1966.

[14] R. T. Stefani, "Observer steady-state errors induced by errors in relization," *IEEE Trans. Automatic Contr.*, vol. 21, pp. 280–282, 1976.

[15] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. 15, pp. 116–132, 1985.

[16] G. C. Hwang and S. C. Lin, "A stability approach to fuzzy control design for nonlinear systems," *Fuzzy Sets Syst.*, vol. 48, pp. 279–287, 1992.

[17] J. J. Buckley, "Theory of fuzzy controller: An introduction," *Fuzzy Sets Syst.*, vol. 51, pp. 249–258, 1992.

[18] C. C. Lee, "Fuzzy logic in control systems: Fuzzy logic controller—Part I and part II," *IEEE Trans. Syst., Man, Cybern.*, vol. 20, pp. 404–435, 1990.

[19] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issues," *IEEE Trans. Fuzzy Syst.*, vol. 4, pp. 14–23, 1996.

[20] M. Johansson, A. Rantzer, and K.-E. Arzen, "Piecewise quadratic stability of fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 7, pp. 713–722, 1999.

[21] B. S. Chen, C. S. Tseng, and H. J. Uang, "Mixed H_2/H_∞ fuzzy output feedback control design for nonlinear dynamic systems: An LMI approach," *IEEE Trans. Fuzzy Syst.*, vol. 8, pp. 249–265, 2000.

[22] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs," *IEEE Trans. Fuzzy Syst.*, vol. 6, pp. 250–265, 1998.

[23] B. S. Chen, C. S. Tseng, and H. J. Uang, "Robustness design of nonlinear dynamic systems via fuzzy linear control," *IEEE Trans. Fuzzy Syst.*, vol. 7, pp. 571–585, 1999.

[24] C. S. Tseng, B. S. Chen, and H. J. Uang, Fuzzy tracking control design for nonlinear dynamic systems via TS fuzzy model, in IEEE Trans. Fuzzy Systems, vol. 9, no. 3, pp. 381–392, 2001.

[25] E. Kim and H. Lee, "New approaches to relaxed quadratic stability condition of fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 8, pp. 523–534, 2000.

[26] K. Tanaka, T. Ikeda, and H. O. Wang, "A unified approach to controlling chaos via LMI-based fuzzy control system design," *IEEE Trans. Circuit Syst. I*, vol. 45, pp. 1021–1040, 1998.

[27] Z. M. Yeh, "A systematic method for design of multivariable fuzzy logic control systems," *IEEE Trans. Fuzzy Syst.*, vol. 7, pp. 741–752, 1999.

[28] A. Isidori and A. Asolfi, "Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 1283–1293, 1992.

[29] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[30] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[31] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM, 1994.

[32] C. Scherer and P. Gahinet, "Multiobjective output-feedback control via LMI optimization," *IEEE Trans. Automat. Contr.*, vol. 42, pp. 896–911, 1997.

[33] D. D. Siljak, *Large-Scale Dynamic Systems: Stability and Structure*: North-Holland, 1978.

[34] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*. Natick, MA: The MathWorks, 1995.

[35] M. Sugeno and K. Tanaka, "Successive identification of fuzzy model," *Fuzzy Sets Syst.*, vol. 28, pp. 15–33, 1988.

[36] E. Kim, M. Park, S. Ji, and M. Park, "A new approach to fuzzy modeling," *IEEE Trans. Fuzzy Syst.*, vol. 5, pp. 328–337, 1997.

[37] M. Sugeno and T. Yasukawa, "A fuzzy-logic-based approach to qualitative modeling," *IEEE Trans. Fuzzy Syst.*, vol. 1, pp. 7–31, 1993.

Chung-Shi Tseng (M'01) received the B.S. degree from National Cheng Kung University, Tainan, Taiwan, the M.S. degree from the University of New Mexico, Albuquerque, and the Ph.D. degree, from the National Tsing-Hua University, Hsin-Chu, Taiwan.

He is currently an Associate Professor with the Ming Hsin Institute of Technology and Commerce, Hsin-Chu, Taiwan. His research interests include nonlinear robust control, adaptive control, fuzzy control, and robotics.

Bor-Sen Chen (M'82–SM'89) received the B.S. degree from Tatung Institute of Technology, Taiwan, the M.S. degree from National Central University, Taiwan, and the Ph.D degree from the University of Southern California, Los Angeles, in 1970, 1973, and 1982, respectively.

From 1973 to 1987, he was a Lecturer, Associate Professor, and Professor at Tatung Institute of Technology. Currently, he is a Professor with National Tsing Hua University, Hsin-Chu, Taiwan, R.O.C. His research interests include control and signal processing.

Dr. Chen has received the Distinguished Research Award from National Science Council of Taiwan four times. He is a Research Fellow of the National Science Council and the Chair of the Outstanding Scholarship Foundation.