Robust Tracking Designs for Both Holonomic and Nonholonomic Constrained Mechanical Systems: Adaptive Fuzzy Approach

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***Abstract*—Adaptive fuzzy-based tracking control designs will be proposed in this paper for both holonomic mechanical systems as well as a large class of nonholonomic mechanical systems with plant uncertainties and external disturbances. A unified and systematic procedure is employed to derive the controllers for both holonomic and nonholonomic mechanical control systems, respectively. First, a fuzzy logic system is introduced to learn the behavior of unknown (or uncertain) mechanical dynamics by using an adaptive algorithm. Next, the effect of approximation error on the tracking error must be efficiently eliminated by employing an additional robustifying algorithm. Consequently, hybrid adaptive-robust controllers can be constructed such that the resulting closed-loop mechanical systems guarantee a satisfactorily transient and asymptotic performance. Furthermore, a partitioned procedure with respect to the above developed adaptive fuzzy logic approximators is introduced such that the number of fuzzy IF–THEN rules is significantly reduced and the developed control schemes can be easily implemented from the viewpoint of practical applications. Finally, simulation examples are presented to illustrate the tracking performance of a two-link constrained robot manipulator and a vertical wheel rolling on a plane surface by the proposed adaptive fuzzy-based control algorithms.**

***Index Terms*—Adaptive fuzzy approach, constrained mechanical system, partitioned procedure, robust tracking design.**

I. Introduction

THE control of mechanical systems with holonomic or nonholonomic constraints as well as with plant uncertainties and external disturbances is of importance for numerous practical applications.

The constrained robot is the most remarkable holonomic control system. In many industrial tasks such as writing, scribing, and grinding, the robot's end-effector is required to keep contact with its environment. During the execution of such tasks, however, contact forces are induced between the end-effector and environmental constraint surfaces. Hence, a hybrid motion/force tracking control design for robot manipulators under this kind of constrained motion is necessary. As the complex dynamics of the holonomic constrained robot is exactly known and the external disturbance can be neglected, the modified computed-torque method has been employed to solve the tracking control problem [15], [18], [35]. Moreover, the strategies based on adaptive control [9], [19], [28] and robust control [11], [28] have been proposed, recently, to address the control design for constrained mechanical systems under plant uncertainties and external disturbances.

On the other hand, considerable attention has been paid toward studying the motion control of nonholonomic mechanical systems during last few years. It is well known that in rolling or cutting motions, the kinematic constraint equations are classical nonholonomic and the dynamics of such systems is also very well understood [3]–[5]. Several results have been published in the recent years wherein the motion control design of the nonholonomic mechanical system has been successfully treated [3]–[5], [22], [35]. In these designs, the dynamic models were assumed to be perfect, exactly known, and free of external disturbances. However, only a few studies have been carried out so far to address the control design for perturbative nonholonomic mechanical control systems [7], [27].

During the development of classical adaptive control algorithms (see for example, [9] [19], [27], [28]), the use of regressor matrix has become rather popular in adaptive control schemes. In such cases, nonlinear dynamics of a mechanical system with unknown (or uncertain) system parameters is assumed to be expressed as a product of a regressor matrix and an unknown parameter vector; that is, the property of linearity in the system parameters is used to obtain the adaptive control results. The parameter update law is then employed to estimate the unknown parameters, which are assumed to be constant or slowly varying. However, some potential difficulties are associated with this approach. For example, the unknown parameters may be quickly varying, the linear parametrizable property may not hold, computation of the regressor matrix happens to be a time-consuming task, and implementation of this approach also requires a precise knowledge of the structure of the entire dynamic model. Hence, development of an alternative approach to treat the adaptive control of mechanical systems with uncertainties is highly desirable.

Fuzzy control has been extensively applied to a wide variety of industrial systems and consumer products and has attracted the attention of several control researchers due to its computational simplicity and model free approach [8], [12], [14], [16], [25], [26], [29], [30], [33], [34], [36]. The fuzzy control algorithms attempt to make use of the information obtained from human experts, which is generally represented by fuzzy terms. In the early stage of investigation, however, only a few systematic procedures are available for analysis and design of

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fuzzy control. Most of the fuzzy control algorithms are proposed without analytical tools to guarantee the basic performance criteria and only simulations are performed by trial and error to show the validity of the proposed fuzzy approaches to the specific control problems [12], [14], [34], [36].

More recently, based on the universal approximation theorem [29], [30], [32] (fuzzy logic systems have been shown to be capable of uniformly approximating any well-defined nonlinear function to any degree of accuracy), many important adaptive fuzzy control schemes have been developed to directly incorporate the expert information systematically and various stable performance criteria are guaranteed by theoretical analyses [8], [25], [26], [29], [30], [33]. In [30], two identifiers of nonlinear dynamic systems are developed based on the fuzzy system models. The proposed fuzzy identifiers are shown to be capable of following the output of a very general nonlinear dynamic system to an arbitrary accuracy in any finite time interval and all signals in the fuzzy identifiers are shown to be uniformly bounded. Yin and Lee in [33] have proposed a fuzzy model-reference adaptive control for controlling a plant with unknown parameters, dependent on the known variables. The fuzzy basis function expansion has been used to represent the unknown parameters and the convergence and stability properties of the fuzzy model-reference adaptive control are preserved. A fuzzy logic controller equipped with a training (adaptive) algorithm is proposed in [8] to achieve H^∞ tracking performance for a class of uncertain (model free) nonlinear SISO systems with external disturbances. A bridge between H^∞ control design and fuzzy control design has been created, so as to supply H^∞ control design with more intelligence and fuzzy control design with better performance. In [25], both stable direct and indirect adaptive controllers are presented, based on Takagi–Sugeno fuzzy systems, conventional fuzzy systems, or a class of neural networks, to achieve asymptotic tracking of a reference signal for a class of continuous-time nonlinear plants with poorly understood dynamics.

Adaptive fuzzy-based tracking control designs are proposed in this paper for both holonomic and nonholonomic constrained mechanical systems with plant uncertainties and external disturbances. In the holonomic mechanical control design, the control of the most remarkable system, i.e., a constrained robot system, is considered in this paper. In the nonholonomic mechanical control design, this work focuses on a large class of nonholonomic mechanical systems such as nonholonomic Caplygin systems (which include a vertical wheel rolling without slipping on a plane surface, a mobile wheeled robot moving on a horizontal plane and a knife edge moving in point contact on a plane surface, etc). A unified and systematic procedure is employed to derive controllers for both holonomic and nonholonomic mechanical control systems. First, a fuzzy logic system is introduced to learn the behavior of unknown (or uncertain) mechanical dynamics by using an adaptive algorithm. Next, the effect of approximation error on the tracking error is efficiently eliminated by an additional robustifying algorithm. Consequently, hybrid adaptive-robust controllers, which consist of two parts: an adaptive fuzzy logic approximator and a robustifying controller, are constructed such that the resulting closed-loop mechanical systems guarantee a satisfactorily transient and asymptotic performance (e.g., L_∞ bounded, H^∞ performance, uniformly ultimately bounded). Moreover, there exists a tradeoff between the number of fuzzy IF–THEN rules and the PD control gain. Furthermore, a partitioned procedure with respect to the developed adaptive fuzzy logic approximators is introduced such that the basis functions can be easily determined, the fuzzy logic system can be tuned for a faster weighted update procedure, and the number of fuzzy IF–THEN rules is decreased significantly. Hence, the developed control scheme based on this approach can be easily implemented from the viewpoint of practical applications. Finally, simulation examples are presented to illustrate the tracking performances of a two-link constrained robot manipulator and a vertical wheel rolling on a plane surface by the proposed adaptive fuzzy-based control algorithms.

The organization of this paper is as follows. In Section II, the model descriptions of fuzzy logic systems and both holonomic and nonholonomic constrained mechanical systems are presented. Section III presents the adaptive fuzzy-based tracking control design for holonomic constrained robot control systems. Section IV presents the adaptive fuzzy-based tracking control design for a large class of nonholonomic constrained mechanical control systems. A simplified and partitioned procedure for both the proposed fuzzy logic approximators is introduced in Section V. In Section VI, simulations are made. Finally, important conclusions are summarized in Section VII.

In what follows, we shall use the following standard notations. The square norm of a vector $x \in R^n$ is denoted by $x^T x$ or $\|x\|^2$, and $\|x\|_Q^2 \triangleq x^T Q x$ with the weighting matrix Q. We say that x: $[0,T] \to R^n$ is in $L_2[0,T]$ if $\int_0^T \|x(t)\|^2\, dt < \infty$ and x: $[0,\infty) \to R^n$ is bounded (i.e., in $L_\infty[0,\infty)$) if $\|x(t)\| < \infty$ for all $t \in [0,\infty)$.

II. Model Descriptions

A. Model Description of Fuzzy Logic Approximators

Fuzzy logic systems have been successfully employed to universally approximate the mathematical models of dynamical systems in the recent years [29], [30], [32]. Due to their approximation capabilities as well as inherent adaptive features, fuzzy logic systems offer an efficient alternative to classical methods of modelling of dynamical systems. In this section, we briefly describe the structure of fuzzy logic systems.

The basic configuration of the fuzzy logic system is depicted in Fig. 1 [29], [30], which is constructed from the fuzzy IF–THEN rules using some specific inference, fuzzification, and defuzzification strategies. The fuzzy logic system performs a mapping from $U \subset R^N$ to $V \subset R$. Let $U = U_1 \times \cdots \times U_N$ where $U_i \subset R$, $i = 1, 2, \cdots, N$. The fuzzifier maps a crisp point in U into a fuzzy set in U. The fuzzy rule base consists of a collection of fuzzy IF–THEN rules [29], [30], [32]

$$R^{(l)}: If\ x_1\ is\ F_1^l, \cdots and,\ x_N\ is\ F_N^l,\ Then\ y\ is\ G^l \quad (1)$$

in which $x = (x_1, \cdots, x_N)^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy logic system, respectively, F_i^l and G^l are fuzzy sets in U_i and V, respectively, and $l = 1, \cdots, M$ where M denotes the number of fuzzy IF–THEN rules. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy

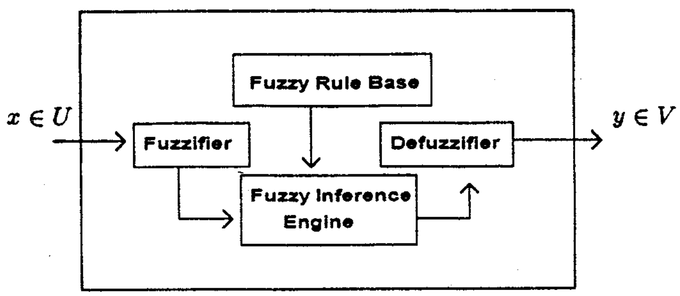


Fig. 1. The basic configuration of a fuzzy logic system.

sets in V, based upon the fuzzy IF–THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V. A more detailed description of these systems can be found in [29], [30], [32]. The fuzzy logic system depicted in Fig. 1 comprises a very rich class of static systems mapping from $U \subset R^N$ to $V \subset R$ since many different choices are available within each block and, in addition, many combinations of these choices can result in a useful subclass of fuzzy logic systems. One subclass of fuzzy logic systems is used here as a building block for the proposed adaptive fuzzy controller and is described by the following important result.

Lemma 2.1: [29], [30]: The fuzzy logic systems with center-average defuzzifier, product inference and singleton fuzzifier are of the following form:

$$y(x) = \frac{\sum_{l=1}^{M} \theta_l \left(\prod_{i=1}^{N} \mu_{F_i^l}(x_i) \right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{N} \mu_{F_i^l}(x_i) \right)} \tag{2}$$

where $\mu_{F_i^l}(\cdot)$ is the membership function of the fuzzy set F_i^l and θ_l is the point at which μ_{G^l} achieves its maximum value (it is assumed here that $\mu_{G^l}(\theta_l) = 1$). □

The above fuzzy logic system has been shown to be capable of uniformly approximating any well-defined nonlinear function over a compact set U to any degree of accuracy. More precisely, the universal approximation theorem is quoted as follows.

Theorem 2.1: [29], [30]: For any given real continuous function $f(x)$ on a compact subset $U \subset R^N$ and arbitrary $\epsilon > 0$ there exists a fuzzy logic system $y(x)$ in the form of (2) such that $\max_{x \in U} \ \|f(x) - y(x)\| < \epsilon$. □

This theorem provides a justification for applying the fuzzy logic systems to almost any nonlinear modeling problems. Although the fuzzy logic system described above is of single-output systems, it is straightforward to show that a multi-output system can always be approximated by a group of single-output approximation systems.

B. General Model of Constrained Mechanical Systems

Based on classical mechanics [1], a nonlinear dynamic model incorporating constraint effects has been derived and recognized as an excellent theoretical model for control of constrained mechanical systems.

According to the Lagrange theory [1], the constrained dynamic equation with external disturbances can be formulated as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)\tau + J^T(q)\lambda + d \tag{3}$$

where $q \in R^n$ denotes the generalized coordinate, $\dot{q} \in R^n$ denotes the generalized velocity, $M(q) \in R^{n \times n}$ denotes the generalized moment of inertia, the term $C(q, \dot{q})\dot{q} \in R^n$ includes the centripetal forces and Coriolis forces (even the friction forces), $G(q) \in R^n$ denotes the gravitational forces, $\tau \in R^r$ is the control input, B: $R^n \to R^{n \times r}$ is the input matrix, $B(q)\tau \in R^n$ is a nonconservative generalized force along the direction of its corresponding generalized coordinate, d denotes the external disturbances, and $\lambda \in R^m$ denotes the constraint force due to the reaction of the following two cases.

Case 1: Holonomic constraint ($r = n, B(q)$ is nonsingular). Defining

$$J(q) = \frac{\partial \phi}{\partial q}\,(q)$$

we consider m independent frictionless constraints of the form [15], [18], [28]

$$\phi(q) = 0 \tag{4}$$

(where ϕ: $R^n \to R^m$) to be regarded as the rigid constraint surfaces, for example, in the constrained robot expressed by the joint-space coordinate frame. Then, the constraint force $\lambda \in R^m$ is known as task-space contact force between the end effector and this rigid environmental surfaces. Moreover, the holonomic constraint on the robot's end effector can be viewed as restricting only the dynamics on the constraint manifold, rather than to the space R^{2n}. Therefore, it is necessary to define the constraint manifold of holonomic systems as

$$\mathbf{M}_{hol} = \{(q, \dot{q}) \in R^n \times R^n \colon \phi(q) = 0, J(q)\dot{q} = 0\} \tag{5}$$

which is $(2n - 2m)$-dimensional. □

Case 2: Nonholonomic constraint ($n - m \le r < n, B(q)$ has full-row rank) The m nonintegrable and independent velocity constraints [3]–[5], [10], [35]

$$J(q)\dot{q} = 0 \tag{6}$$

(where J: $R^n \to R^{m \times n}$) are considered in this study and modelled for the expression of kinematic constraints, for example, in the vertical wheel rolling without slipping on a plane surface. Then, the constraint force $\lambda \in R^m$ is known as rolling friction force on the contact point between the rigid body and environmental surfaces. Moreover, each friction force presents slipping along the corresponding direction. Here, the constraint manifold of nonholonomic systems has the general form as

$$\mathbf{M}_{noh} = \{(q, \dot{q}) \in R^n \times R^n \colon J(q)\dot{q} = 0\} \tag{7}$$

which is $(2n-m)$-dimensional. It is important to note that since the nonholonomic constraint functions are nonintegrable, there is, in fact, no explicit restriction on the values of the configuration variables. □

Both the constraint manifolds $\mathbf{M}_{hol}$ and $\mathbf{M}_{noh}$, are assumed to be completely known in this study. Moreover, all the functions

$M(\cdot), C(\cdot), G(\cdot), B(\cdot)$, and $J(\cdot)$ are assumed to be continuous and defined on an appropriate open subset of the $(q, \dot{q})$ phase space. The matrices $M(\cdot), C(\cdot)$ and $G(\cdot)$ are, however, assumed to be unknown.

C. Reduced Form of Constrained Mechanical Systems

Obviously, the constrained dynamic equation (3) would not be suitable for analysing the dynamics and designing the controller. By assuming complete knowledge of the constrained manifolds and embedding the constraint equations (4) and (6) into the constrained dynamic equation (3), the reduced-form dynamic equations of both holonomic and nonholonomic mechanical systems are obtained here sequentially to realize the tracking control designs.

1) Reduced Form of Holonomic Mechanical Systems: In this study, we shall focus, without loss of generality, on the control of constrained robot systems, which is the most remarkable holonomic control system and $B(q) = I_{n\times n}$ in (3).

The presence of m constraints causes the manipulator to lose m degrees of freedom, hence, the manipulator is left with only $n-m$ degrees of freedom. According to the implicit function theorem, the vector $q \in R^n$ can be properly rearranged and partitioned into the form

$$q \triangleq \begin{bmatrix} q^1 \\ q^2 \end{bmatrix}$$

in which $q^1 = \left[q_1^1 \cdots q_{n-m}^1\right]^T$ describes the constrained motion of the manipulator and $q^2 = \left[q_1^2 \cdots q_m^2\right]^T$ denotes the remaining joint variables. Moreover, there is a unique function σ: $R^{n-m} \to R^m$ such that the constraint equation (4) can always be expressed explicitly as [18], [28]

$$q^2 = \sigma(q^1).$$

By defining

$$L(q^1) = \begin{bmatrix} I_{n-m} \\ \dfrac{\partial \sigma(q^1)}{\partial q^1} \end{bmatrix} \tag{8}$$

the dynamic model (3) of robots, when restricted to the constraint surface, can be expressed in the reduced form [28] as

$$M(q^1)L(q^1)\ddot{q}^1 + C_L(q^1, \dot{q}^1)\dot{q}^1 + G(q^1) = \tau + J^T(q)\lambda + d \tag{9}$$

where $C_L(q^1, \dot{q}^1) \triangleq M(q^1)\dot{L}(q^1) + C(q^1, \dot{q}^1)L(q^1)$. The matrices $M(q^1), C(q^1, \dot{q}^1)$, and $G(q^1)$ in (9) are obtained by substituting $q^2 = \sigma(q^1)$ and $\dot{q} = L\dot{q}^1$ into $M(q), C(q, \dot{q})$ and $G(q)$ in (3), respectively.

Several fundamental properties of the reduced-form dynamic equation (9) have been obtained as follows [28].

PH1: The matrix $A_L(q^1) \triangleq L^T(q^1)M(q^1)L(q^1)$ is symmetric and positive definite.

PH2: The matrix $\dot{A}_L(q^1) - 2L^T(q^1)C_L(q^1, \dot{q}^1)$ is skew symmetric.

PH3: $J(q^1)L(q^1) = L^T(q^1)J^T(q^1) = 0.$ □

2) Reduced Form of Nonholonomic Mechanical Systems: By following an embedding approach [5], [10], the nonholonomic dynamic equations (3), (6) can be reduced to the so-called reduced-form dynamic equation.

Let $r_1(q), \cdots, r_{n-m}(q)$ be a set of smooth and linearly independent vector fields in the null space of $J(q)$. Then, the following relations are satisfied in local coordinates:

$$J(q)R(q) = 0 \tag{10}$$

where $R(q) \triangleq [r_1(q) \cdots r_{n-m}(q)]$. The constraints (6) and (10) imply the existence of an $(n-m)$-vector $\dot{z}$ such that

$$\dot{q} = R(q)\dot{z} \tag{11}$$

and then the dynamics (3), which satisfies the nonholonomic constraint (6), can be rewritten in terms of the internal state variable $\dot{z}$ as [5], [10], [27]

$$\begin{aligned} M(q)R(q)\ddot{z} + C_R(q, \dot{q})\dot{z} + G(q) \\ = B(q)\tau + J^T(q)\lambda + d \end{aligned} \tag{12}$$

where $C_R(q, \dot{q}) \triangleq M(q)\dot{R}(q) + C(q, \dot{q})R(q)$. Several fundamental properties of the dynamic equation (12) have been proposed as follows [27].

PN1: The matrix $A_R(q) \triangleq R^T(q)M(q)R(q)$ is symmetric and positive definite.

PN2: The matrix $\dot{A}_R(q) - 2R^T(q)C_R(q, \dot{q})$ is skew symmetric.

PN3: $R^T(q)J^T(q) = 0.$ □

Since the equality constraint equations (4) and (6) are embedded into the dynamic equation (3) for both holonomic and nonholonomic systems, respectively, (9) and (12) are suitable for control purposes and form the basis for subsequent developments.

III. Adaptive Fuzzy-Based Tracking Control Design for Holonomic Mechanical Systems

An adaptive fuzzy-based tracking control problem for holonomic mechanical systems with plant uncertainties and external disturbances is considered and solved in this section. A fuzzy logic approximator with a projection update law is used to learn the behavior of uncertain dynamics, following which an adaptive fuzzy-based controller is constructed.

A. Problem Formulation for Holonomic Control Systems

Consider the constrained dynamic equation (3) together with m independent holonomic constraints (4). The motion $q(t)$ and force $\lambda(t)$ will be tracked, simultaneously, in this study, to the desired trajectories. Given a desired joint trajectory $q_d(t)$ and a desired constraint force $f_d(t)$, or, equivalently, a desired multiplier $\lambda_d(t)$, that satisfy the imposed constraints, i.e., $\phi(q_d(t)) = 0$ and $f_d(t) = J^T(q_d(t))\lambda_d(t)$, the following bounded assumption is required throughout this section.

AH1: The desired reference trajectory $q_d(t)$ is assumed to be bounded, has bounded derivatives up to the second order, and belongs to a compact set Ω_d. The desired constraint force $\lambda_d(t)$ is also assumed to be bounded. □

It should be noted that since $q^2 = \sigma(q^1)$, it is only required to evolve a control law such that $q^1(t) \to q_d^1(t)$ as $t \to \infty$. An adaptive tracking control problem for perturbative holonomic constrained systems based on fuzzy approaches can be formulated as follows.

Problem 1: Consider the holonomic mechanical system (3), (4) with plant uncertainties and external disturbances. Develop an adaptive fuzzy-based controller of the form

$$\dot{\theta} = \alpha_1(t, \theta, q^1, \dot{q}^1) \tag{13}$$

$$\tau = \alpha_2(t, \theta, q^1, \dot{q}^1) \tag{14}$$

where θ denotes the tunable fuzzy logic parameters such that for all $(q(0), \dot{q}(0)) \in \mathbf{M}_{hol}$, all the variables of the closed-loop system (9), (13), (14) are bounded for all $t \geq 0$ and the trajectory errors $q^1 - q_d^1, \dot{q}^1 - \dot{q}_d^1$ and $\lambda - \lambda_d$ should be as small as possible. □

B. Controller Design and Stability Analysis for Holonomic Control Systems

Based on the reduced form (9), the reduced state–space error dynamics will be explicitly obtained and employed to derive the adaptive fuzzy-based controller for the holonomic control system.

Premultiplying both sides of (9) by $L^T(q^1)$, we get

$$\begin{aligned} A_L(q^1)\ddot{q}^1 + L^T(q^1)C_L(q^1, \dot{q}^1)\dot{q}^1 + L^T(q^1)G(q^1) \\ = L^T(q^1)(\tau + d). \end{aligned} \tag{15}$$

Define the position tracking error $\overline{x}_1(t)$ and the filtered link-tracking error $\overline{x}_2(t)$ as [23], [28]

$$\begin{aligned} \overline{x}(t) &\triangleq \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix} \\ &\triangleq \begin{bmatrix} q^1(t) - q_d^1(t) \\ \dot{q}^1(t) - \dot{q}_d^1(t) + p(q^1(t) - q_d^1(t)) \end{bmatrix} \end{aligned} \tag{16}$$

for some constant $p > 0$. Then, the error dynamic equations with respect to $\overline{x}_1$ and $\overline{x}_2$ are obtained as

$$\dot{\overline{x}}_1 = -p\overline{x}_1 + \overline{x}_2 \tag{17}$$

$$\begin{aligned} A_L(q^1)\dot{\overline{x}}_2 = &-L^T(q^1)F(x_e) - L^T(q^1)C_L(q^1, \dot{q}^1)\overline{x}_2 \\ &+ L^T(q^1)(\tau + d) \end{aligned} \tag{18}$$

where $x_e \triangleq \left[q^{1^T}\ \dot{q}^{1^T}\ q_d^{1^T}\ \dot{q}_d^{1^T}\ \ddot{q}_d^{1^T}\right]^T$ and $F(x_e) \triangleq M(q^1)L(q^1)(\ddot{q}_d^1 - p\dot{\overline{x}}_1) + C_L(q^1, \dot{q}^1)(\dot{q}_d^1 - p\overline{x}_1) + G(q^1)$. Since $M(q), C(q, \dot{q})$ and $G(q)$ are assumed to be unknown, the term $F(x_e)$ in (18) is uncertain. A fuzzy logic system, described in Section II-A, with an adaptive update law is constructed in this section to learn the behavior of the uncertain term $F(x_e)$. The uncertain term $F(x_e)$ is an n-dimensional vector, hence, a fuzzy logic system must be constructed whose output is an n-vector. A universal approximation system $\hat{F}(x_e, \hat{\Theta}_1)$ with input vector $x_e \in U_{x_e}$ for some compact set $U_{x_e} \subset R^{5(n-m)}$ is proposed here to approximate the uncertain term $F(x_e)$ where $\hat{\Theta}_1$ is a vector containing the tunable approximation parameters. Let

$$F(x_e) = \begin{bmatrix} F_1(x_e) \\ \vdots \\ F_n(x_e) \end{bmatrix}$$

$$\hat{F}(x_e, \hat{\Theta}_1) = \begin{bmatrix} \hat{F}_1(x_e, \hat{\theta}_1) \\ \vdots \\ \hat{F}_n(x_e, \hat{\theta}_n) \end{bmatrix}. \tag{19}$$

The fuzzy logic system $\hat{F}(x_e, \hat{\Theta}_1)$ is assumed to be expressed as

$$\begin{aligned} \hat{F}(x_e, \hat{\Theta}_1) &= \begin{bmatrix} \hat{F}_1(x_e, \hat{\theta}_1) \\ \vdots \\ \hat{F}_n(x_e, \hat{\theta}_n) \end{bmatrix} = \begin{bmatrix} y_1^T\hat{\theta}_1 \\ \vdots \\ y_n^T\hat{\theta}_n \end{bmatrix} \\ &= \begin{bmatrix} y_1^T & 0 & \cdots & 0 \\ 0 & y_2^T & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & y_n^T \end{bmatrix} \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \vdots \\ \hat{\theta}_n \end{bmatrix} \\ &\triangleq Y_1(x_e)\hat{\Theta}_1 \end{aligned} \tag{20}$$

where $Y_1(x_e)$ denotes a basis matrix.

Consider the constraint region of parameter $\hat{\Theta}_1$ to be defined as

$$\Omega_{\theta_1} \triangleq \left\{\hat{\Theta}_1 | \hat{\Theta}_1^T\hat{\Theta}_1 \leq M_\theta, M_\theta > 0\right\}$$

where M_θ is a positive constant specified by the designer. The following assumptions are used in this section:

AH2: Assume that there is a finite parameter value $\Theta_1^* \in \Omega_{\theta_1}$ known as the optimal approximation parameter, i.e., [8], [26], [29], [30]

$$\Theta_1^* \triangleq \underset{\hat{\Theta}_1 \in \Omega_{\theta_1}}{\operatorname{argmin}} \left(\max_{x_e \in U_{x_e}} \|\hat{F}(x_e, \hat{\Theta}_1) - F(x_e)\| \right)$$

such that $\hat{F}(x_e, \Theta_1^*)$ can approximate the uncertain dynamics $F(x_e)$ as best as possible.

Let $w \triangleq Y_1(x_e)\Theta_1^* - F(x_e) + d$ denote the approximation error plus the external disturbance. In general, the approximation error is assumed to be bounded by a fixed constant in the analysis of the previous arguments. However, in practice, the numbers of fuzzy IF–THEN rules are finite and it is possible that during the early stages of learning, the initial fuzzy system approximations may be quite poor. Hence, a weaker assumption than a fixed constant bound due to the approximation error is necessary.

AH3: There exist three positive constants w_0, w_1 and w_2 such that $\|w\| \leq w_0 + w_1\|\overline{x}_1\| + w_2\|\overline{x}_2\|$. □

Throughout this study, the linearly parametrized fuzzy model [8], [26], [29], [30] is employed in the approximation procedure of the uncertain dynamics. The fuzzy basis function $Y_1(x_e)$ is specified beforehand and explicit expressions for the computation of Θ_1^* are not required since this value can be learned by using the adaptive algorithm.

In order to derive the adaptive controller, we choose the Lyapunov function candidate as

$$V(t,\overline{x},\tilde{\Theta}_1) = \frac{\alpha_1}{2}\,\overline{x}_1^T\overline{x}_1 + \frac{1}{2}\,\overline{x}_2^T A_L \overline{x}_2 + \frac{1}{2\gamma_1}\,\tilde{\Theta}_1^T\tilde{\Theta}_1 \quad (21)$$

for some $\alpha_1 > 0$ and adaptive gain $\gamma_1 > 0$ where $\tilde{\Theta}_1 \triangleq \hat{\Theta}_1 - \Theta_1^*$ and select any positive constants c_1 and c_2, which can be arbitrarily large. Let

$$\Omega_0 \triangleq \{\overline{x} | \|\overline{x}_1\| \le c_1, \|\overline{x}_2\| \le pc_1 + c_2\}$$
$$\Omega_1 \triangleq \left\{\overline{x} | \|\overline{x}_1\| \le \sqrt{\frac{2M_0}{\alpha_1}}, \|\overline{x}_2\| \le \sqrt{\frac{2M_0}{\lambda_{a_l}}}\right\}$$

where $M_0 > \max_{\overline{x}\in\Omega_0, q_d, \dot{q}_d\in\Omega_d, \Theta_1^*, \hat{\Theta}_1\in\Omega_{\theta_1}} V(t,\overline{x},\tilde{\Theta}_1)$ and λ_{a_l} denotes the minimum eigenvalue of the matrix $A_L(q^1)$. An adaptive state feedback compensator based on a fuzzy logic system is proposed to solve the tracking control problem of holonomic mechanical systems in the following theorem. We shall show that if $(\overline{x}(0), \hat{\Theta}_1(0)) \in \Omega_0 \times \Omega_{\theta_1}$, then $(\overline{x}(t), \hat{\Theta}_1(t)) \in \Omega_1 \times \Omega_{\theta_1}$ for all $t \ge 0$, i.e., the set $\Omega_1 \times \Omega_{\theta_1}$ is a positively invariant set. Let

$$\lambda_c \triangleq \lambda_d - k_\lambda(\lambda - \lambda_d) \quad \text{and} \quad E \triangleq \begin{bmatrix} I_{(n-m)\times(n-m)} \\ 0_{m\times(n-m)} \end{bmatrix}$$

for some $k_\lambda > 0$.

Theorem 3.1: Consider the equations of motion (3), (4) with plant uncertainties and external disturbances. For a desired reference trajectory $q_d(t)$ and a contact force $\lambda_d(t)$, let an adaptive fuzzy-based controller be given by

$$\dot{\hat{\Theta}}_1 = \begin{cases} -\gamma_1 Y_1^T L \overline{x}_2, & \text{if } \|\hat{\Theta}_1\|^2 < M_\theta \text{ or} \\ & (\|\hat{\Theta}_1\|^2 = M_\theta \text{ and } \overline{x}_2^T L^T Y_1 \hat{\Theta}_1 \ge 0) \\ -\gamma_1 Y_1^T L \overline{x}_2 + \gamma_1 \dfrac{\overline{x}_2^T L^T Y_1 \hat{\Theta}_1}{\|\hat{\Theta}_1\|^2}\,\hat{\Theta}_1, & \\ & \text{if } \|\hat{\Theta}_1\|^2 = M_\theta \text{ and } \overline{x}_2^T L^T Y_1 \hat{\Theta}_1 < 0 \end{cases} \quad (22)$$

$$\tau = Y_1\hat{\Theta}_1 - k_0 E \overline{x}_2 - J^T\lambda_c \quad (23)$$

where k_0 is a constant gain which would be designed later. Then, if the following initial conditions hold: $(q(0), \dot{q}(0)) \in \mathbf{M}_{hol}$, $\|q^1(0) - q_d^1(0)\| \le c_1$, $\|\dot{q}^1(0) - \dot{q}_d^1(0)\| \le c_2$, and $\hat{\Theta}_1(0) \in \Omega_{\theta_1}$, there exists a choice of constant gain k_0 such that the following performance is achieved.

1) $\hat{\Theta}_1(t) \in \Omega_{\theta_1}$, $\overline{x}(t) \in \Omega_1$, and all the variables $q(t), \dot{q}(t)$ and $\tau(t)$ are bounded for all $t \ge 0$.
2) The tracking error is uniformly ultimately bounded.
3) The steady–state force error $\lambda - \lambda_d$ is inversely proportional to the value of $k_\lambda + 1$.

Proof: Taking the minimum approximation error w into account, the error equations (17), (18) can be rewritten as

$$\dot{\overline{x}}_1 = -p\overline{x}_1 + \overline{x}_2 \quad (24)$$

$$A_L\dot{\overline{x}}_2 = -L^T Y_1 \Theta_1^* - L^T C_L \overline{x}_2 + L^T\tau + L^T w. \quad (25)$$

Choose the Lyapunov function candidate as in (21). Taking the derivative of V along the trajectories (24), (25) yields

$$\begin{aligned}\dot{V} =& \alpha_1\overline{x}_1^T(-p\overline{x}_1 + \overline{x}_2) + \frac{1}{2}\overline{x}_2^T\dot{A}_L\overline{x}_2 \\ &+ \overline{x}_2^T(-L^T Y_1\Theta_1^* - L^T C_L\overline{x}_2 + L^T\tau + L^T w) + \frac{1}{\gamma_1}\dot{\hat{\Theta}}_1^T\tilde{\Theta}_1.\end{aligned}$$

Taking into account the control law (23) and using the properties of **PH2** and **PH3**, the derivative of V is obtained as

$$\begin{aligned}\dot{V} =& -\alpha_1 p\overline{x}_1^T\overline{x}_1 - k_0\overline{x}_2^T\overline{x}_2 + \alpha_1\overline{x}_1^T\overline{x}_2 + \overline{x}_2^T L^T w \\ &+ (\overline{x}_2^T L^T Y_1 + \frac{1}{\gamma_1}\,\dot{\hat{\Theta}}_1^T)\tilde{\Theta}_1.\end{aligned} \quad (26)$$

Consider the last term in the above equality. Taking into account the update law (22), which is a standard projection algorithm [29], [30], we can guarantee that $(\overline{x}_2^T L^T Y_1 + \frac{1}{\gamma_1}\dot{\hat{\Theta}}_1^T)\tilde{\Theta}_1 \le 0$ and $\hat{\Theta}_1(t) \in \Omega_{\theta_1}$ for all $t \ge 0$ if $\hat{\Theta}_1(0) \in \Omega_{\theta_1}$. Hence, the equality (26) can be reduced to

$$\dot{V}(t) \le -\alpha_1 p\overline{x}_1^T\overline{x}_1 - k_0\overline{x}_2^T\overline{x}_2 + \alpha_1\overline{x}_1^T\overline{x}_2 + \overline{x}_2^T L^T w. \quad (27)$$

Let λ_L and λ_{A_L} denote the maximum eigenvalues of the matrices $L(q^1)$ and $A_L(q^1)$ on the compact set $\Omega_1 \times \Omega_d$, respectively. Hence, based on the assumption **AH3**, the equality (27) can be bounded as

$$\begin{aligned}\dot{V}(t) \le& -\alpha_1 p\|\overline{x}_1\|^2 - k_0\|\overline{x}_2\|^2 + (\alpha_1 + \lambda_L w_1)\|\overline{x}_1\|\,\|\overline{x}_2\| \\ &+ \lambda_L w_2\|\overline{x}_2\|^2 + \lambda_L w_0\|\overline{x}_2\|.\end{aligned} \quad (28)$$

By completing the squares, if we choose

$$\alpha_1 p > a \quad (29)$$

$$k_0 > \frac{(\alpha_1 + \lambda_L w_1)^2}{4a} + \frac{\lambda_L^2}{4\rho^2} + \lambda_L w_2 \quad (30)$$

for some $a > 0$ and $\rho > 0$, then there exist positive constants η_1 and η_2 such that

$$\begin{aligned}\dot{V}(t) \le& -\eta_1\|\overline{x}_1\|^2 - \eta_2\|\overline{x}_2\|^2 + \rho^2 w_0^2 \\ \le& -c_0 V + \frac{c_0}{2\gamma}\,\tilde{\Theta}_1^T\tilde{\Theta}_1 + \rho^2 w_0^2 \\ \le& -c_0 V + \nu\end{aligned} \quad (31)$$

where $c_0 \triangleq \min(\eta_1, \eta_2)/\max(\alpha_1, \lambda_{A_L})$ and $\nu \triangleq (c_0 M_\theta/2\gamma_1) + \rho^2 w_0^2$. For sufficiently small ρ and sufficiently large M_0, the solution of (31) satisfies the inequality

$$0 \le V(t) \le \beta + (V(0) - \beta)\exp(-c_0 t) \le M_0 \quad (32)$$

where $\beta \triangleq \nu/c_0 = (M_\theta/2\gamma) + (\rho^2 w_0^2)/c_0 > 0$. It is clear that $\|q^1(0) - q_d^1(0)\| \le c_1$ and $\|\dot{q}^1(0) - \dot{q}_d^1(0)\| \le c_2$ imply $\overline{x}(0) \in \Omega_0$. From the definition of Ω_1 we can conclude that if $\overline{x}(0) \in \Omega_0$ and $\hat{\Theta}_1(0) \in \Omega_{\theta_1}$, then $\overline{x}(t) \in \Omega_1$ for all $t \ge 0$. Therefore, the set $\Omega_1 \times \Omega_{\theta_1}$ is a positively invariant set.

Next, from (21) and (32), we obtain that given any $\mu > \sqrt{2\beta/\alpha_1}$ there exists T such that $\|\overline{x}_1(t)\| \leq \mu$ for all $t \geq T$. Finally, from (9) we get

$$\begin{aligned}(k_\lambda + 1)J^T(q)(\lambda - \lambda_d) &= M(q^1)L(q^1)\ddot{q}^1 + C_L(q^1, \dot{q}^1)\dot{q}^1 \\ &\quad + G(q^1) - d - Y_1(x_e)\hat{\Theta}_1 + k_0 E\overline{x}_2.\end{aligned}$$

Since the right-hand side of the above equation is bounded, we can conclude that the steady-state force error $\lambda - \lambda_d$ is inversely proportional to the value of $k_\lambda + 1$. This completes the proof.□

The approximation error w is assumed to be bounded in the above arguments. On the other hand, if w possesses the property of finite energy, then the controller (22), (23) guarantees the following performance.

Corollary 3.1: If $w(\cdot) \in L_2[0, \infty)$, then we can conclude that the following H^∞ performance is achieved [2]

$$\int_0^T \|\overline{x}(t)\|_Q^2 \, dt \leq V(0) + \rho^2 \int_0^T \|w(t)\|^2 \, dt, \quad \text{for all } T \geq 0 \tag{33}$$

where Q denotes the weighting matrix and the constant ρ is the attenuation level. If, in addition, $w(\cdot) \in L_2[0, \infty) \cap L_\infty[0, \infty)$, then we can conclude that

$$\lim_{t\to\infty} (q^1(t) - q_d^1(t)) = 0 \quad \text{and} \quad \lim_{t\to\infty} (\dot{q}^1(t) - \dot{q}_d^1(t)) = 0.$$

Proof: From (27) and by completing the squares, we get

$$\dot{V} \leq -\overline{x}^T Q \overline{x} + \rho^2 \|w\|^2 \tag{34}$$

for some weighting matrix Q and attenuation level ρ. It should be noted that Q can be arbitrarily pre-assigned and ρ can be arbitrarily small by increasing the gain k_0. Integrating the above equation from $t = 0$ to $t = T$ yields

$$\begin{aligned}&V(\overline{x}(T), \tilde{\Theta}(T), T) - V(\overline{x}(0), \tilde{\Theta}(0), 0) \\ &\quad \leq -\int_0^T \|\overline{x}(t)\|_Q^2 \, dt + \rho^2 \int_0^T \|w(t)\|^2 \, dt.\end{aligned}$$

Since $V(\overline{x}(T), \tilde{\Theta}(T), T) \geq 0$, the above inequality leads to (33). That is, the H^∞ performance is achieved.

Moreover, from the closed-loop error system (22)–(25), it is clear that $\dot{\overline{x}}_1(t)$ and $\dot{\overline{x}}_2(t)$ are uniformly bounded. This implies that $\overline{x}_1(t)$ and $\overline{x}_2(t)$ are uniformly continuous. Based on the Barbalat's lemma [13], [29] and the inequality (34), if $w(\cdot) \in L_2[0, \infty) \cap L_\infty[0, \infty)$, we can conclude that $\lim_{t\to\infty} (q^1(t) - q_d^1(t)) = 0$ and $\lim_{t\to\infty} (\dot{q}^1(t) - \dot{q}_d^1(t)) = 0$. □

Remark 3.1: 1) The proposed controller (23) consists of two parts. The first part (i.e., $Y_1\hat{\Theta}_1$) is an adaptive fuzzy logic system and the second part (i.e., $-k_0 E\overline{x}_2 - J^T\lambda_c$) is the robust controller to achieve the desired L_∞, H^∞ and uniformly ultimately bounded performance. Hence, in practice, this controller is a hybrid adaptive-robust controller. 2) The robust controller is used to efficiently eliminate the effect on the tracking error due to the approximation error provided that it satisfies the bounded condition described in **AH3**. The values of w_0, w_1 and w_2 in the assumption **AH3** are not required to be very small in the above analysis. Moreover, the assumption **AH3** can also be relaxed to assume that $w_0(\cdot), w_1(\cdot)$ and $w_2(\cdot)$ depend on the variables of $\overline{x}_1, \overline{x}_2, q_d, \dot{q}_d$, and $\ddot{q}_d$ and are continuous functions on the compact set $\Omega_1 \times \Omega_d$ rather than three fixed constants. □

Remark 3.2: The proposed control scheme (22), (23) can be applied to any initial conditions which satisfy $\overline{x}(0) \in \Omega_0$ and $\hat{\Theta}_1(0) \in \Omega_{\theta_1}$. Both the attraction region Ω_0 and the constraint region Ω_{θ_1} can be not only arbitrarily enlarged, but also explicitly pre-assigned. The region Ω_1 can then be exactly constructed as both regions Ω_0 and Ω_{θ_1} have been assigned. Moreover, from the definitions of $\overline{x}_1$ and $\overline{x}_2$, the region U_{x_e} can also be explicitly computed. All of these regions can be arbitrarily enlarged and consequently a semi-global adaptive fuzzy-based tracking control scheme for the holonomic control system can be obtained.□

Remark 3.3: 1) Lower bound conditions of the values p and k_0 have been proposed in (29) and (30), respectively. Note that for different choice of the value a in (30) we can obtain different controller gain k_0. In general, it is desired to choose the value a such that the value k_0 is as small as possible. For any specified attenuation level ρ and weighting matrix Q, the H^∞ performance in Corollary 3.1 can be achieved just by tuning the gain k_0. There is a design tradeoff between the values ρ and Q as well as the controller gain k_0. 2) The value k_0 also depends upon the bound values w_1 and w_2 in the assumption **AH3** with respect to the approximation error. According to the universal approximation theorem [29], [30], [32] these two bound values can be made as small as possible by increasing the number of fuzzy IF–THEN rules. On the other hand, if there are some good linguistic descriptions about the unknown dynamics $F(x_e)$ such that the initial fuzzy approximator $\hat{F}(x_e, \hat{\Theta}_1)$ can be close to $F(x_e)$, then the approximation error should be small and so the bound values w_0, w_1 and w_2 also are small. Therefore, a well-performing fuzzy logic system may cause a small approximation error and so a smaller controller gain k_0 is required. 3) It is clear that the uniformly ultimate bound μ can be made as small as possible by increasing the gains γ_1 and k_0. 4) A conservative design has been obtained since the controller gain k_0 should be chosen to treat the worst situation of the values w_1, w_2 and λ_L. Even without knowledge of these bound values, the closed-loop system can still achieve the same performance as in Theorem 3.1 and Corollary 3.1 just by selecting k_0 to be sufficiently large. However, from a practical viewpoint, increasing the gain k_0 would result in variation of a high-gain control scheme which may excite unmodelled dynamics of the system and cause the actuator saturation problem. For many mechanical control problems, the states $q, \dot{q}$ and control τ are required to be constrained within certain regions. Recall that the set $\Omega_1 \times \Omega_{\theta_1}$ is shown to be a positively invariant set. For given constraints we can specify the design parameters $c_1, c_2, p, M_\theta, \alpha_1, \rho, Q$ and μ, which can be pre-assigned by the designer such that the variables $q, \dot{q}$ and τ are within the constraint sets. On the other hand, one may also expect to use the well-known saturation function $sat(\tau)$ to solve the actuator saturation problem and this needs further mathematical analysis in the future research. □

Remark 3.4: A standard adaptation rule with a projection modification in (22) has been employed to solve the tracking design of perturbative constrained robot systems. This parameter projection results in an adaptation rule with discontinuous right-hand side. On the other hand, the smoothed projection law

[13] can also be used to solve this problem. Suppose $\Omega_{\theta_0} = \{\hat{\Theta}_1: \hat{\Theta}_1^T \hat{\Theta}_1 \leq M_\theta\}$ and let $\Omega_{\theta_1} = \{\hat{\Theta}_1 : \hat{\Theta}_1^T \hat{\Theta}_1 \leq M_\theta + \delta\}$ where $\delta > 0$. Then the parameter update laws in (22) can be modified as

$$\dot{\hat{\Theta}}_1 = \begin{cases} -\gamma_1 Y_1^T L \overline{x}_2, & \text{if } \|\hat{\Theta}_1\|^2 \leq M_\theta \text{ or} \\ & (\|\hat{\Theta}_1\|^2 > M_\theta \text{ and } \overline{x}_2^T L^T Y_1 \hat{\Theta}_1 \geq 0) \\ -\gamma_1 Y_1^T L \overline{x}_2 + \gamma_1 \dfrac{(\|\hat{\Theta}_1\|^2 - M_\theta) \overline{x}_2^T L^T Y_1 \hat{\Theta}_1}{\delta \|\hat{\Theta}_1\|^2} \hat{\Theta}_1, & \\ & \text{otherwise.} \end{cases} \tag{35}$$

It can be verified that if $\hat{\Theta}_1(0) \in \Omega_{\theta_0}$, then $\hat{\Theta}_1(t) \in \Omega_{\theta_1}$, forall $t \geq 0$ and the inequality $(\overline{x}_2^T L^T Y_1 + (1/\gamma_1)\dot{\tilde{\Theta}}_1^T)\tilde{\Theta}_1 \leq 0$ still holds. Hence, a smooth adaptive fuzzy-based controller is constructed such that the performance described in Theorem 3.1 and Corollary 3.1 can also be guaranteed. □

Remark 3.5: Generally speaking, most of the conventional adaptive control design developed so far for constrained robot system (see, for example, [9], [19], [28]) always assume that the uncertain dynamics $F(x_e)$ in (18) can be expressed as $F(x_e) = Y_1(q^1, \dot{q}^1, \dot{q}_d^1 - p\overline{x}_1, \ddot{q}_d^1 - p\dot{\overline{x}}_1)\Theta_1^*$, where Y_1 is a regressor matrix of known functions and the parameter Θ_1^* is a constant vector with components depending on the manipulator parameters (such as link masses, moments of inertia, etc.) and then the adaptive controller

$$\hat{F}(x_e) = Y_1(q^1, \dot{q}^1, \dot{q}_d^1 - p\overline{x}_1, \ddot{q}_d^1 - p\dot{\overline{x}}_1)\hat{\Theta}_1 \tag{36}$$

where $\hat{\Theta}_1$ denotes the estimated value of Θ_1^* is used to compensate this uncertain dynamics. It is obvious that the mathematical structure of the linearly parametrized models (20) as well as (36), respectively, in the intelligent fuzzy-based method as well as in the conventional adaptive method strongly resembles. Hence, the analysis and design developed in this study can also be directly applied to the constrained robot systems with linear parametrization property. A globally stable result may be concluded in the conventional adaptive control design because the entire structure of the uncertain parameter matrices $M(q), C(q, \dot{q})$ and $G(q)$ is assumed to be well-known and so the basis function $Y_1(q^1, \dot{q}^1, \dot{q}_d^1 - p\overline{x}_1, \ddot{q}_d^1 - p\dot{\overline{x}}_1)$ is sufficient to achieve a globally exact match to the plant dynamics. However, there are some drawbacks for the conventional adaptive control scheme, for example, assumption of an explicitly linear parametrization of the uncertain dynamics is required and the relatively large amount of prior information about the parameter matrices $M(q), C(q, \dot{q})$ and $G(q)$, which is very hard to obtain for many practical mechanical systems, is required for the computation of the regressor matrix $Y_1(q^1, \dot{q}^1, \dot{q}_d^1 - p\overline{x}_1, \ddot{q}_d^1 - p\dot{\overline{x}}_1)$. In contrast, implementation of the adaptive fuzzy-based control scheme developed in this study does not require any precise knowledge of either the mathematical model or linear parametrization of the mechanical dynamics. Therefore, the proposed adaptive fuzzy-based control scheme can be employed to treat a larger class of mechanical systems that may be uncertain in terms of plant structures, whereas the conventional adaptive control scheme can be employed only to deal with parameter-uncertain systems in which the unknown parameter must be assumed to be constant or slow varying. Moreover, according to the framework in (2) the adaptive fuzzy controller provides a systematic and efficient manner such that the linguistic fuzzy information from a human expert can be directly incorporated into the controller, whereas the conventional adaptive control scheme cannot. □

Remark 3.6: From a practical point of view, the contact between the robot end-effector and the environment is a very complicated issue. Some kinds of the environment assumptions used in the early literature have been described in Spong *et al.* [24]. Complete knowledge of the constraint surface has been employed in this study to solve the adaptive fuzzy-based tracking control design that is similar to the assumption in many existing (adaptive) control schemes [9], [11], [15], [18], [19], [27], [28], etc. The constraint surface is assumed here to be fixed, nondeformable, and frictionless. However, relaxing this assumption is very important for the application of the proposed control scheme to the practical robot. □

Suppose that there are uncertainties on the constraint surface. The constraint function in (4) is modified as

$$\phi(q) = \Delta\phi(t) \tag{37}$$

where $\Delta\phi(t)$ represents the time-varying $m \times 1$ vector of constraint uncertainty. It is worth noting that the desired reference trajectories $q_d(t)$ and $f_d(t)$ are assumed to be consistent with the constraints and they define a constrained equilibrium corresponding to $\Delta\phi(t) = 0$ [18]. Moreover, this constraint uncertainty is assumed to be small such that according to the implicit function theorem the constraint equation (37) can always be expressed explicitly as $q^2 = \sigma(q^1) + \Delta\sigma(t)$ where $\Delta\sigma(t)$ denotes the small perturbation due to the time-varying constraint uncertainty $\Delta\phi(t)$. After some simple manipulations, the reduced-form dynamic equation in (9) can be recomputed as

$$\begin{aligned} M(q^1)L(q^1)\ddot{q}^1 + C_L(q^1, \dot{q}^1)\dot{q}^1 + G(q^1) \\ = \tau + J^T(q)\lambda + \Delta F(t, x_e) + d \end{aligned} \tag{38}$$

where $\Delta F(t, x_e) \triangleq \Delta M(t, q^1)L(q^1)\ddot{q}^1 + \Delta C_L(t, q^1, \dot{q}^1)\dot{q}^1 + \Delta G(t, q^1) + \Delta f(t, q^1)$ denotes the additional perturbation term due to the effect of $\Delta\sigma(t)$.

Recall that the fuzzy logic system possesses universal approximation property. The fuzzy logic system $\hat{F}(x_e, \hat{\Theta}_1)$ proposed in (20) can be reconstructed here to learn the behavior of the uncertain dynamics $F(x_e) - \Delta F(t, x_e)$. Moreover, the performance in Theorem 3.1 still holds, provided that the augmented error $w \triangleq Y_1(x_e)\Theta_1^* - F(x_e) + \Delta F(t, x_e) + d$ satisfies the assumption **AH3**.

Corollary 3.2: Consider the constrained robot system (3) and (37) in which the constraint surface is perturbed by a small time-varying perturbation. Consequently, the performances proposed in Theorem 3.1 can also be guaranteed just by tuning the controller gain k_0. □

IV. Adaptive Fuzzy-Based Tracking Control Design for Nonholonomic Mechanical Systems

An adaptive fuzzy-based tracking control problem for a large class of nonholonomic mechanical systems (for example, nonholonomic Caplygin systems [4]) with plant uncertainties and external disturbances is considered and solved in this section.

A. Problem Formulation for a Class of Nonholonomic Control Systems

Consider the constrained dynamic equation (3) together with m independent nonholonomic constraints (6). Since the equality constraint equation (6) is embedded into the dynamic equation (3) of nonholonomic systems, (12) is suitable for control design purposes and forms the basis of the subsequent development.

Using (11), $\dot{z}$ can be obtained from q and $\dot{q}$ as follows:

$$\dot{z} = [R^T(q)R(q)]^{-1}R^T(q)\dot{q}.$$

For simplicity of design, the following assumptions are required throughout this section.

AN1: There exists an $(n-m)$-vector $z \triangleq z(q)$, which depends only on the configuration position q, but not on the velocity $\dot{q}$ [27]. Moreover, the matrices $R(q), B(q), M(q), C(q,\dot{q})$, and $G(q)$ are functions of both variables z and $\dot{z}$ only.

AN2: Let $r = n - m$ and the matrix $R^T(z)B(z)$ be of full rank. This assumption guarantees that all $n-m$ degrees of freedom can be independently actuated [4]. □

These assumptions always hold for a large class of nonholonomic mechanical systems such as nonholonomic Caplygin systems (which include a vertical wheel rolling without slipping on a plane surface, a mobile wheeled robot moving on a horizontal plane, and a knife edge moving in point contact on a plane surface, etc. In these systems, the internal state $\dot{z}(t)$ and variable $z(q)$ possess practical physical meanings.) Give a desired joint trajectory $z_d(t) \in \Omega_d$, which is assumed to be bounded and has bounded derivatives up to the second order. An adaptive tracking control problem for perturbative nonholonomic mechanical systems based on fuzzy approaches is formulated as follows.

Problem 2: Consider the nonholonomic mechanical system (3), (6) with plant uncertainties and external disturbances. Develop an adaptive fuzzy-based controller of the form

$$\dot{\theta} = \alpha_3(t,\theta,z,\dot{z}) \tag{39}$$

$$\tau = \alpha_4(t,\theta,z,\dot{z}) \tag{40}$$

such that for all $(q(0),\dot{q}(0)) \in \mathbf{M}_{noh}$, all the variables of the closed-loop system (12), (39), (40) are bounded for all $t \geq 0$ and the trajectory errors $z - z_d$ and $\dot{z} - \dot{z}_d$ should be as small as possible. □

B. Controller Design and Stability Analysis for Nonholonomic Control Systems

Premultiplying both sides of (12) by $R^T(z)$, we get

$$\begin{aligned} A_R(z)\ddot{z} + R^T(z)C_R(z,\dot{z})\dot{z} + R^T(z)G(z) \\ = R^T(z)(B(z)\tau + d). \end{aligned} \tag{41}$$

Let $\overline{x}_1(t) \triangleq z(t) - z_d(t)$ and define the filtered link tracking error $\overline{x}_2(t)$ as $\overline{x}_2(t) = (\dot{z}(t) - \dot{z}_d(t)) + p(z(t) - z_d(t))$ for some $p > 0$. (For simplicity of notations and explanations, some representations of the variables same as those in the study of holonomic control designs in the last section will be employed in this section also.) Then, the error dynamic equations with respect to $\overline{x}_1$ and $\overline{x}_2$ are obtained as

$$\dot{\overline{x}}_1 = -p\overline{x}_1 + \overline{x}_2 \tag{42}$$

$$\begin{aligned} A_R(z)\dot{\overline{x}}_2 = & -R^T(z)H(z_e) - R^T(z)C_R(z,\dot{z})\overline{x}_2 \\ & + R^T(z)(B(z)\tau + d) \end{aligned} \tag{43}$$

where $z_e \triangleq [z^T\ \dot{z}^T\ z_d^T\ \dot{z}_d^T\ \ddot{z}_d^T]^T$ and $H(z_e) \triangleq M(z)R(z)(\ddot{z}_d - p\dot{\overline{x}}_1) + C_R(z,\dot{z})(\dot{z}_d - p\overline{x}_1) + G(z)$. A fuzzy logic system $\hat{H}(z_e,\hat{\Theta}_2)$ with input vector $z_e \in U_{z_e}$ is proposed here to approximate the uncertain term $H(z_e)$, where $\hat{\Theta}_2$ is a vector containing the tunable approximation parameters similar to the arguments in Section III. Assume that $\hat{H}(z_e,\hat{\Theta}_2)$ can be expressed as

$$\hat{H}(z_e,\hat{\Theta}_2) = Y_2(z_e)\hat{\Theta}_2$$

where Y_2 denotes a basis matrix. Consider the constraint region of the parameter $\hat{\Theta}_2$ to be defined as

$$\Omega_{\theta_2} \triangleq \left\{\hat{\Theta}_2 | \hat{\Theta}_2^T\hat{\Theta}_2 \leq M_\theta, M_\theta > 0\right\}.$$

The following assumptions are used in this section.

AN3: Assume that there exists a finite parameter value $\Theta_2^* \in \Omega_{\theta_2}$, known as the optimal approximation parameter, such that $\hat{H}(z_e,\Theta_2^*)$ can approximate the uncertain dynamics $H(z_e)$ as best as possible.

AN4: Let $w \triangleq Y_2(z_e)\Theta_2^* - H(z_e) + d$. There exist three positive constants w_0, w_1 and w_2 such that $\|w\| \leq w_0 + w_1\|\overline{x}_1\| + w_2\|\overline{x}_2\|$. □

Choose the Lyapunov function candidate as

$$V(t,\overline{x},\tilde{\Theta}_2) = \frac{\alpha_2}{2}\,\overline{x}_1^T\overline{x}_1 + \frac{1}{2}\,\overline{x}_2^TA_R\overline{x}_2 + \frac{1}{2\gamma_2}\,\tilde{\Theta}_2^T\tilde{\Theta}_2 \tag{44}$$

for some $\alpha_2 > 0$ and $\gamma_2 > 0$, where $\tilde{\Theta}_2 \triangleq \hat{\Theta}_2 - \Theta_2^*$. Consider positive constants c_3 and c_4. Let

$$\begin{aligned} \Omega_2 &\triangleq \{\overline{x} | \|\overline{x}_1\| \leq c_3, \|\overline{x}_2\| \leq pc_3 + c_4\} \\ \Omega_3 &\triangleq \left\{\overline{x} | \|\overline{x}_1\| \leq \sqrt{\frac{2M_2}{\alpha_2}}, \|\overline{x}_2\| \leq \sqrt{\frac{2M_2}{\lambda_{a_r}}}\right\} \end{aligned}$$

where $M_2 > \max_{\overline{x}\in\Omega_2, z_d, \dot{z}_d \in \Omega_d, \Theta_2^*, \hat{\Theta}_2 \in \Omega_{\theta_2}} W(t,\overline{x},\tilde{\Theta}_2)$ and λ_{a_r} denotes the minimum eigenvalue of the matrix $A_R(z)$.

Since both error dynamic equations (17), (18) as well as (42), (43) strongly resemble and possess similar properties **PH1-PH3** and **PN1-PN3**, respectively, the same procedure of analysis as in Theorem 3.1 can be employed and the following result can be obtained.

Theorem 4.1: Consider the equations of motion (3), (6) with plant uncertainties and external disturbances. For a desired reference trajectory $z_d(t)$, let an adaptive fuzzy-based controller be given by

$$\dot{\hat{\Theta}}_2=\begin{cases}-\gamma_2 Y_2^T R\overline{x}_2, & \text{if } \|\hat{\Theta}_2\|^2<M_\theta \text{ or } (\|\hat{\Theta}_2\|^2=M_\theta \text{ and } \overline{x}_2^T R^T Y_2\hat{\Theta}_2\geq 0)\\ -\gamma_2 Y_2^T R\overline{x}_2+\gamma_2\dfrac{\overline{x}_2^T R^T Y_2\hat{\Theta}_2}{\|\hat{\Theta}_2\|^2}\hat{\Theta}_2, & \text{if } \|\hat{\Theta}_2\|^2=M_\theta \text{ and } \overline{x}_2^T R^T Y_2\hat{\Theta}_2<0\end{cases} \tag{45}$$

$$\tau=(R^T B)^{-1}(R^T Y_2\hat{\Theta}_2-k_0\overline{x}_2). \tag{46}$$

Then, if the following initial conditions hold: $(q(0),\dot{q}(0))\in\mathbf{M}_{noh}$, $\|z(0)-z_d(0)\|\leq c_3$, $\|\dot{z}(0)-\dot{z}_d(0)\|\leq c_4$, and $\hat{\Theta}_2(0)\in\Omega_{\theta_2}$, there exists a choice of constant gain k_0 such that the following performance is achieved.

1) $\hat{\Theta}_2(t)\in\Omega_{\theta_2}$, $\overline{x}(t)\in\Omega_3$, and all the variables $z(t)$, $\dot{z}(t)$, $\tau(t)$, and $f(t)$ are bounded for all $t\geq 0$.
2) The tracking error can be as small as desired by increasing the gains k_0 and γ_2.
3) If $w(\cdot)\in L_2[0,\infty)\cap L_\infty[0,\infty)$, then we can conclude that $\lim_{t\to\infty}(z(t)-z_d(t))=0$ and $\lim_{t\to\infty}(\dot{z}(t)-\dot{z}_d(t))=0$.

Proof: This proof is similar to that of Theorem 3.1. □

The fuzzy logic system $\hat{H}(z_e,\hat{\Theta}_2)$ with the projection update law (45) has been constructed to learn the behavior of uncertain dynamics $H(z_e)$, which has n outputs. If the fuzzy system $\hat{H}(z_e,\hat{\Theta}_2)$ is reconstructed to learn the behavior of $R^T H(z_e)$, then only a universal approximator with $(n-m)$ outputs must be constructed rather than n outputs proposed in Theorem 4.1. In fact, by a little modification of the control law in (45), (46), the following result can be obtained.

Corollary 4.1: Consider the equations of motion (12). Let an adaptive fuzzy-based controller be given by

$$\dot{\hat{\Theta}}_2=\begin{cases}-\gamma_2 Y_2^T\overline{x}_2, & \text{if } \|\hat{\Theta}_2\|^2<M_\theta \text{ or } (\|\hat{\Theta}_2\|^2=M_\theta \text{ and } \overline{x}_2^T Y_2\hat{\Theta}_2\geq 0)\\ -\gamma_2 Y_2^T\overline{x}_2+\gamma_2\dfrac{\overline{x}_2^T Y_2\hat{\Theta}_2}{\|\hat{\Theta}_2\|^2}\hat{\Theta}_2, & \text{if } \|\hat{\Theta}_2\|^2=M_\theta \text{ and } \overline{x}_2^T Y_2\hat{\Theta}_2<0\end{cases} \tag{47}$$

$$\tau=(R^T B)^{-1}(Y_2\hat{\Theta}_2-k_0\overline{x}_2). \tag{48}$$

Then, the same performance as proposed in Theorem 4.1 can also be achieved. □

Now, let us consider an idealized case of no fuzzy functional reconstruction error and no external disturbances, i.e., $w=0$, in the nonholonomic mechanical dynamics.

Corollary 4.2: Consider the equations of motion (3), (6). Suppose the fuzzy approximation error and external disturbance are equal to zero. Let an adaptive controller be given by

$$\dot{\hat{\Theta}}_2=-\gamma_2 Y_2^T R\overline{x}_2 \tag{49}$$

$$\tau=(R^T B)^{-1}(R^T Y_2\hat{\Theta}_2-k_0\overline{x}_2). \tag{50}$$

Then, the tracking error goes to zero and the weight estimate $\hat{\Theta}_2$ is bounded. □

Remark 4.1: A smooth control algorithm, as described in Corollary 4.2, has also been proposed in [27] to ensure that the nonholonomic mechanical control system satisfying the assumption of linear parametrization and being free of external disturbances can be stabilized to a desired manifold. However, only parametric uncertainties are considered in their designs, while the external disturbances are neglected. □

V. Construction of Partitioned Fuzzy Logic Systems

The fuzzy logic systems $\hat{F}(x_e,\hat{\Theta}_1)$ and $\hat{H}(z_e,\hat{\Theta}_2)$ are employed in Sections III and IV to approximate the uncertain terms $F(x_e)$ and $H(z_e)$, respectively. A partitioned procedure motivated from [17] is proposed in this section such that both fuzzy logic approximators $\hat{F}(\cdot)$ and $\hat{H}(\cdot)$ can be easily implemented.

Consider the uncertain term $F(x_e)$ in the holonomic control design, i.e., $F(x_e)\triangleq M(q^1)L(q^1)(\ddot{q}_d^1-p\dot{\overline{x}}_1)+C_L(q^1,\dot{q}^1)(\dot{q}_d^1-p\overline{x}_1)+G(q^1)$. For simplicity of notations, define

$$\zeta_1(t)=[\zeta_{11}(t)\cdots\zeta_{1\kappa}(t)]^T\triangleq\ddot{q}_d^1-p\dot{\overline{x}}_1$$

$$\zeta_2(t)=[\zeta_{21}(t)\cdots\zeta_{2\kappa}(t)]^T\triangleq\dot{q}_d^1-p\overline{x}_1$$

where $\kappa=n-m$. It is clear that the terms ζ_1 and ζ_2 can be exactly known. In the following, three individual fuzzy logic subsystems $\hat{M}_L(q^1,\hat{\Theta}_{ML})$, $\hat{C}_L(q^1,\dot{q}^1,\hat{\Theta}_{CL})$, and $\hat{G}_L(q^1,\hat{\Theta}_{GL})$ are proposed, respectively, to approximate the uncertain terms $ML(q^1)$, $C_L(q^1,\dot{q}^1)$, and $G(q^1)$. First, consider the uncertain term $ML(q^1)(\ddot{q}_d^1-p\dot{\overline{x}}_1)$. Express $ML(q^1)$ as

$$ML(q^1)=[M_{l1}(q^1)\quad M_{l2}(q^1)\quad\cdots\quad M_{l\kappa}(q^1)]$$

where $M_{li}(q^1)$ for $i=1,\cdots,\kappa$ is the ith column of $ML(q^1)$. The fuzzy logic system $\hat{M}_{li}(q^1,\hat{\Theta}_{ML})$ is proposed to approximate $M_{li}(q^1)$, respectively, for $i=1,\cdots,\kappa$, where $\hat{M}_{li}(q^1,\hat{\Theta}_{ML})$ denotes the ith column of $\hat{M}_L(q^1,\hat{\Theta}_{ML})$. Let

$$\hat{M}_{li}(q^1,\hat{\Theta}_{ML})\triangleq Y_{ml_i}(q^1)\hat{\Theta}_{ml_i}.$$

Hence, the uncertain term $ML(q^1)\zeta_1(t)$ can be approximated by

$$\begin{aligned}\hat{M}_L(q^1,\hat{\Theta}_{ML})\zeta_1(t)&=\zeta_{11}(t)Y_{ml_1}(q^1)\hat{\Theta}_{ml_1}+\cdots+\zeta_{1\kappa}(t)Y_{ml_\kappa}(q^1)\hat{\Theta}_{ml_\kappa}\\&=\left[\zeta_{11}(t)Y_{ml_1}(q^1)\quad\cdots\quad\zeta_{1\kappa}(t)Y_{ml_\kappa}(q^1)\right]\cdot\begin{bmatrix}\hat{\Theta}_{ml_1}\\\vdots\\\hat{\Theta}_{ml_\kappa}\end{bmatrix}\\&\triangleq Y_{ML}(t,q^1)\hat{\Theta}_{ML}\end{aligned} \tag{51}$$

where $Y_{ML}\triangleq[\zeta_{11}Y_{ml_1}\cdots\zeta_{1\kappa}Y_{ml_\kappa}]$ and $\hat{\Theta}_{ML}\triangleq\left[\hat{\Theta}_{ml_1}^T\cdots\hat{\Theta}_{ml_\kappa}^T\right]^T$. Similarly, the uncertain term $C_L(q^1,\dot{q}^1)\zeta_2(t)$ can be approximated by

$$\begin{aligned}\hat{C}_L(q^1,\dot{q}^1,\hat{\Theta}_{CL})\zeta_2(t)&=\left[\zeta_{21}(t)Y_{cl_1}(q^1,\dot{q}^1)\cdots\zeta_{2\kappa}(t)Y_{cl_\kappa}(q^1,\dot{q}^1)\right]\hat{\Theta}_{CL}\\&\triangleq Y_{CL}(t,q^1,\dot{q}^1)\hat{\Theta}_{CL}\end{aligned} \tag{52}$$

and the uncertain term $G(q^1)$ can be approximated by

$$\hat{G}_L(q^1,\hat{\Theta}_{GL})=Y_{GL}(q^1)\hat{\Theta}_{GL}. \tag{53}$$

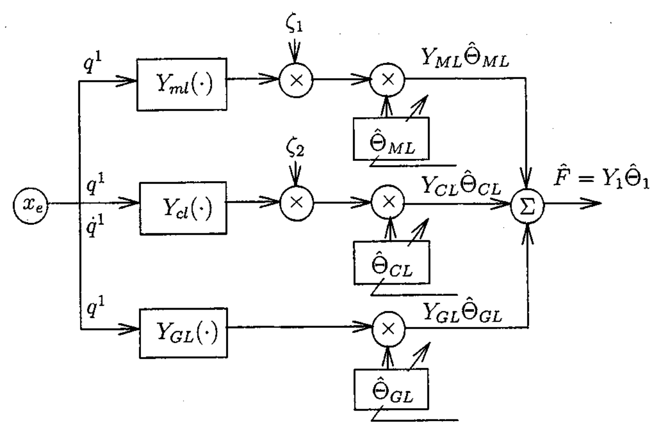


Fig. 2. The structure of fuzzy logic approximator $\hat{F}(x_e, \hat{\Theta}_1)$ in the holonomic control system.

Hence, the totally fuzzy logic approximator with respect to the uncertain term $F(x_e)$ can be constructed as

$$\begin{aligned}\hat{F}(x_e, \hat{\Theta}_1) &= Y_{ML}\hat{\Theta}_{ML} + Y_{CL}\hat{\Theta}_{CL} + Y_{GL}\hat{\Theta}_{GL} \\ &= [Y_{ML} \quad Y_{CL} \quad Y_{GL}] \begin{bmatrix} \hat{\Theta}_{ML} \\ \hat{\Theta}_{CL} \\ \hat{\Theta}_{GL} \end{bmatrix} \\ &\triangleq Y_1(x_e)\hat{\Theta}_1.\end{aligned}$$

This structure is shown in Fig. 2. Further, it can be verified that these three individual partitioned fuzzy logic subsystems can be separately tuned. Indeed, let the constraint regions of parameters $\hat{\Theta}_{ML}, \hat{\Theta}_{CL},$ and $\hat{\Theta}_{GL}$ be defined as $\Omega_{\theta_{1m}} \triangleq \left\{\hat{\Theta}_{ML} | \, \|\hat{\Theta}_{ML}\|^2 \leq M_{\theta_m}, \ M_{\theta_m} > 0\right\}$, $\Omega_{\theta_{1c}} \triangleq \left\{\hat{\Theta}_{CL} | \, \|\hat{\Theta}_{CL}\|^2 \leq M_{\theta_c}, M_{\theta_c} > 0\right\}$, and $\Omega_{\theta_{1g}} \triangleq \left\{\hat{\Theta}_{GL} | \, \|\hat{\Theta}_{GL}\|^2 \leq M_{\theta_g}, M_{\theta_g} > 0\right\}$, respectively. Thus, according to these constraint regions, the parameter update laws with respect to $\hat{\Theta}_{ML}, \hat{\Theta}_{CL},$ and $\hat{\Theta}_{GL}$ can be obtained, respectively, as

$$\dot{\hat{\Theta}}_{ML} = Proj[-\gamma_{1m} Y_{ML}^T L \overline{x}_2] \tag{54}$$

$$\dot{\hat{\Theta}}_{CL} = Proj[-\gamma_{1c} Y_{CL}^T L \overline{x}_2] \tag{55}$$

$$\dot{\hat{\Theta}}_{GL} = Proj[-\gamma_{1g} Y_{GL}^T L \overline{x}_2] \tag{56}$$

where $Proj[\cdot]$ denotes the nonsmoothed projection algorithm as defined in (22) or the smoothed projection algorithm as defined in (35) for some $\delta > 0$ and $\gamma_{1m}, \gamma_{1c},$ and γ_{1g} are the adaptive gains.On the other hand, consider the uncertain term $H(z_e)$ in the nonholonomic control design, i.e.,

$$H(z_e) \triangleq M(z)R(z)(\ddot{z}_d - p\dot{\overline{x}}_1) + C_R(z, \dot{z})(\dot{z}_d - p\overline{x}_1) + G(z).$$

By defining $\zeta_3(t) = \ddot{z}_d - p\dot{\overline{x}}_1$ and $\zeta_4(t) = \dot{z}_d - p\overline{x}_1$, the uncertain term $MR(z)\zeta_3(t)$ can be approximated by

$$\hat{M}_R(z, \hat{\Theta}_{MR})\zeta_3(t) \triangleq Y_{MR}(t, z)\hat{\Theta}_{MR}$$

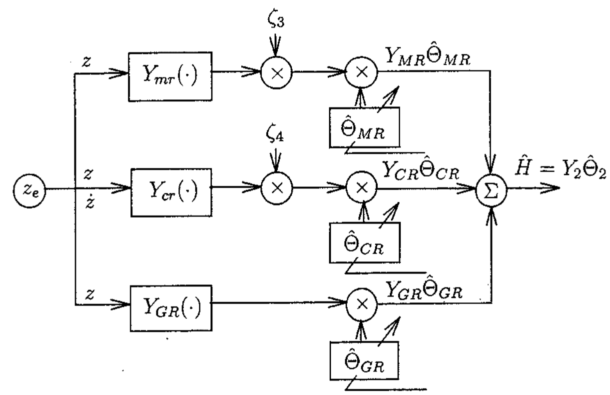


Fig. 3. The structure of fuzzy logic approximator $\hat{H}(z_e, \hat{\Theta}_2)$ in the nonholonomic control system.

where $Y_{MR} \triangleq [\zeta_{31} Y_{mr_1} \cdots \zeta_{3\kappa} Y_{mr_\kappa}]$ and $\hat{\Theta}_{MR} \triangleq \left[\hat{\Theta}_{mr_1}^T \cdots \hat{\Theta}_{mr_\kappa}^T\right]^T$. The uncertain term $C_R(z, \dot{z})\zeta_4(t)$ can be approximated by

$$\begin{aligned}\hat{C}_R(z, \dot{z}, \hat{\Theta}_{CR})\zeta_4(t) &= [\zeta_{41} Y_{cr_1} \quad \cdots \quad \zeta_{4\kappa} Y_{cr_\kappa}]\hat{\Theta}_{CR} \\ &\triangleq Y_{CR}(t, z, \dot{z})\hat{\Theta}_{CR}\end{aligned}$$

and the uncertain term $G(z)$ can be approximated by

$$\hat{G}_R(z, \hat{\Theta}_{GR}) = Y_{GR}(z)\hat{\Theta}_{GR}.$$

Hence, the totally fuzzy logic approximator with respect to the uncertain term $H(z_e)$ can be constructed as

$$\begin{aligned}\hat{H}(z_e, \hat{\Theta}_2) &= Y_{MR}\hat{\Theta}_{MR} + Y_{CR}\hat{\Theta}_{CR} + Y_{GR}\hat{\Theta}_{GR} \\ &\triangleq Y_2(z_e)\hat{\Theta}_2.\end{aligned}$$

This structure is shown in Fig. 3. Further, for given three constraint regions $\Omega_{\theta_{2m}}$ $\Omega_{\theta_{2c}}$ and $\Omega_{\theta_{2g}}$ the parameter update laws with respect to $\hat{\Theta}_{MR}, \hat{\Theta}_{CR},$ and $\hat{\Theta}_{GR}$ can be obtained, respectively, as

$$\dot{\hat{\Theta}}_{MR} = Proj[-\gamma_{2m} Y_{MR}^T R \overline{x}_2] \tag{57}$$

$$\dot{\hat{\Theta}}_{CR} = Proj[-\gamma_{2c} Y_{CR}^T R \overline{x}_2] \tag{58}$$

$$\dot{\hat{\Theta}}_{GR} = Proj[-\gamma_{2g} Y_{GR}^T R \overline{x}_2] \tag{59}$$

where $Proj[\cdot]$ denotes the projection algorithm as defined in (45) and γ_{2m}, γ_{2c} and γ_{2g} are the adaptive gains.

It is clear that structures of both the fuzzy logic approximators for the uncertain terms $F(x_e)$ and $H(z_e)$ are similar. The main advantages for this partitioned procedure are the following.

1) The basis functions for each individual partitioned fuzzy logic subsystem can be separately determined. It is a much simplified problem to choose the suitable basis functions.
2) The approximation parameter in each fuzzy logic subsystem can be separately tuned, enabling a faster weight update procedure.

3) The linguistic descriptions are dependent only on the variables q^1 and $\dot{q}^1$ as well as z and $\dot{z}$ for the holonomic design and nonholonomic design, respectively. Thus, the number of fuzzy rule bases can be greatly reduced.

Remark 5.1: From the universal approximation theorem the fuzzy logic reconstruction error can be made as small as desired by increasing the number of fuzzy IF–THEN rules. However, from the viewpoint of practical implementations, the number of fuzzy IF–THEN rules has to be finite, and consequently the reconstruction error may not be very small. Fortunately, two hybrid adaptive-robust controllers have been constructed such that there exists a trade-off between the number of fuzzy IF–THEN rules and the controller gain k_0 in the robustifying PD algorithm. This implies that even with finite number of fuzzy IF–THEN rules, the proposed controller can be easily implemented from the practical control design point of view. □

Remark 5.2: Although the linearly parametrized fuzzy model is employed in the above arguments for approximating the uncertain dynamics $F(x_e)$ and $H(z_e)$, a similar procedure of analysis can be extended to treat more complicated nonlinearly parametrized approximators by using Taylor's theorem. In such cases, the fuzzy logic reconstruction error w will be augmented with an extra term that represents the higher order component of the approximator's Taylor series expansion with respect to the adjustable weight. The analyses can still hold, provided that the augmented error w satisfies the assumptions **AH3** or **AN4**. □

Remark 5.3: The controller proposed in this study may be categorized into the indirect adaptive fuzzy controller because the controller uses the fuzzy logic system as the model of the unknown dynamics. Though this paper is developed through the same general philosophy as many existing adaptive fuzzy-based (or neural network-based) control schemes, e.g., [6], [8], [17], [21], [25], [26], [29], [31], [33] (in which a fuzzy logic system is employed to learn the behavior of unknown mechanical dynamics by using an adaptive algorithm and then based on such fuzzy approximator the adaptive fuzzy controller is developed to guarantee a satisfactory tracking performance), our indirect adaptive controllers do possess the following different properties: 1) a partitioned procedure with respect to the developed adaptive fuzzy logic approximators is introduced such that the number of fuzzy IF–THEN rules is significantly reduced and the developed control schemes can be easily implemented from the viewpoint of practical applications; 2) most of the developed indirect adaptive fuzzy-based (or neural network-based) control schemes consider a class of nonlinear plants, which must be expressed as a feedback linearizable form with state-dependent input gain [8], [25], [29] or fixed constant input gain [21], [26], [31], [33]. In this situation, the developed fuzzy controllers are based on the well-known certainty equivalence principle and, therefore, two fuzzy logic systems should be employed to learn the behavior of the unknown plant dynamics [8], [25], [29]. Particularly, one fuzzy logic system must be constructed to approximate the uncertain state-dependent input gain. For example, consider the reduced-form dynamic equations (9) and (12), which can also be rewritten as a feedback linearizable. By the method proposed in [8], [25], [29], fuzzy logic systems must be employed to learn the behavior of the inverse dynamics of the state-dependent input gains $A_L(q^1)$ and $A_R(z)$, respectively, in the holonomic control system and in the nonholonomic control system. However, such fuzzy logic systems must be designed to bound away from zero and this is a very complicated work. In contrast, according to the method proposed in this study only one fuzzy logic approximator $\hat{F}(x_e, \hat{\Theta}_1)$ in the holonomic control system or one $\hat{H}(z_e, \hat{\Theta}_2)$ in the nonholonomic control system is required to design the adaptive fuzzy controller. This important advantage is based on the fact that the well-known skew-symmetric properties **PH2** and **PN2** which are inherent properties of the mechanical systems have been used in this study to design the controllers; 3) in [6], [8], [17], and [29]–[31], the convergence of the tracking error is guaranteed by assuming that the approximation error is square integrable. This, however, is difficult to show for any given mechanical plant. In contrast, we simple require knowledge of the bounds on the approximation error; 4) from a practical perspective, knowledge of the compact region over which the input membership functions must be defined is very important in the construction of the on-line approximation model. In this work, once the attraction regions Ω_0, Ω_2 and the constraint regions $\Omega_{\theta_1}, \Omega_{\theta_2}$ are pre-assigned just by setting the constants c_1, c_2, c_3, c_4, p, and M_θ, both regions Ω_1 and Ω_3, which are shown to be invariant sets can be exactly constructed. Since the input region U_{x_e} for the fuzzy approximator $\hat{F}(x_e, \hat{\Theta}_1)$ is constructed from Ω_d and Ω_1 and the input region U_{z_e} for the fuzzy approximator $\hat{H}(z_e, \hat{\Theta}_2)$ is constructed from Ω_d and Ω_3, these two input regions can also be exactly computed. Therefore, the input membership functions can be easily defined to cover all possible plant states. □

VI. Simulation Examples

Example 1—Constrained Robot: A two-link robotic manipulator with a circular path constraint shown in Fig. 4 [28], is used to verify the validity of the adaptive fuzzy-based control algorithm proposed in Section III. The constrained dynamic equation in the form of (3) can be written as

$$\begin{bmatrix} M_{11}(q_2) & M_{12}(q_2) \\ M_{12}(q_2) & M_{22}(q_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -C_{12}(q_2)\dot{q}_2 & -C_{12}(q_2)(\dot{q}_1+\dot{q}_2) \\ C_{12}(q_2)\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1(q_1,q_2) \\ G_2(q_1,q_2) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{60}$$

where $M_{11}(q_2) = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos q_2, M_{12}(q_2) = m_2 l_2^2 + m_2 l_1 l_2 \cos q_2, M_{22}(q_2) = m_2 l_2^2, C_{12}(q_2) = m_2 l_1 l_2 \sin q_2, G_1(q_1, q_2) = (m_1 + m_2) l_1 g \cos q_1 + m_2 l_2 \cos(q_1 + q_2)$, and $G_2(q_1, q_2) = m_2 l_2 g \cos(q_1 + q_2)$. The constraint is a circle in the work space (the (x, y) plane) whose center coincides with the axis of rotation of the first link. The constraint, when expressed in terms of joint space, is

$$\phi(q) = l_1^2 + l_2^2 + 2l_1 l_2 \cos q_2 - r^2 = 0 \tag{61}$$

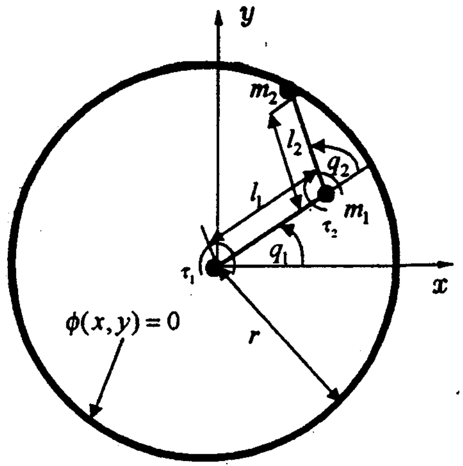


Fig. 4. A two-link robotic manipulator with a circular path constraint.

which has a unique constant solution for q_2

$$q_2 = \cos^{-1}\left[\frac{r^2-(l_1^2+l_2^2)}{2l_1l_2}\right] \triangleq q_2^*.$$

Hence, the Jacobian matrix is $J(q) = [0\ -2l_1l_2\sin q_2]$ and the matrix $L(q^1) = [1\ 0]^T$. The constrained robot can be expressed in the reduced form (9) as

$$\begin{bmatrix} M_{11}(q_2^*) \\ M_{12}(q_2^*) \end{bmatrix}\ddot{q}_1 + \begin{bmatrix} 0 \\ C_{12}(q_2^*)\dot{q}_1 \end{bmatrix}\dot{q}_1 + \begin{bmatrix} G_1(q_1,q_2^*) \\ G_2(q_1,q_2^*) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2l_1l_2\sin q_2^* \end{bmatrix}\lambda + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where $q^1(t) \triangleq q_1(t)$ and $q^2(t) \triangleq q_2(t)$. For the convenience of simulation, the nominal parameters of the robot system are taken as $m_1 = 1$(kg), $m_2 = 2$(kg), $l_1 = l_2 = 1$(m), $r = \sqrt{2}$(m), and $g = 9.8$(m/s^2), the initial conditions are taken as $q_1(0) = 0$ and $\dot{q}_1(0) = 0$, and the desired reference trajectories are taken as $q_d^1(t) = \sin(t)$ and $\lambda_d(t) = 10$. The exogenous disturbances d_1 and d_2 are considered to be square waves with period $T = 2\pi$, i.e.,

$$d_1 = \begin{cases} 0.5 & 0 \le t < \pi \\ 0 & \pi \le t < 2\pi \end{cases}, \quad d_2 = \begin{cases} 0.25 & 0 \le t < \pi \\ -0.25 & \pi \le t < 2\pi. \end{cases}$$

By assuming the parameter matrices $M(q), C(q,\dot{q})$ and $G(q)$ in the constrained robot system are totally unknown, the intelligent adaptive fuzzy-based control law with the smoothed projection law proposed in Theorem 3.1 and Remark 3.4 is employed to treat this robotic trajectory planning problem.

A fuzzy logic system $\hat{F}(x_e,\hat{\Theta}_1)$ is proposed here to approximate the uncertain term $F(x_e)$ defined in (18). Since the terms $\ddot{q}_d^1 - p\dot{\bar{x}}_1$ and $\dot{q}_d^1 - p\bar{x}_1$ are known exactly, a simplified and partitioned fuzzy logic system proposed in Section V is employed to learn the behavior of $F(x_e)$.

First, construct the fuzzy logic subsystem $\hat{M}_L(q^1,\hat{\Theta}_{ML})$ to approximate the uncertain term $ML(q^1) = [M_{11}(q_2^*)\ M_{12}(q_2^*)]^T$, which is a column vector and depends only on the position measurement. Define five fuzzy sets with labels A_1^1 (negative large), A_1^2 (near -1), A_1^3 (near 0), A_1^4 (near 1), and A_1^5 (positive large), which are characterized by the following membership functions, respectively,

$$\begin{aligned} \mu_{A_1^1}(q^1) &= 1/(1+\exp(5(q^1+2))) \\ \mu_{A_1^2}(q^1) &= \exp(-2(q^1+1)^2) \\ \mu_{A_1^3}(q^1) &= \exp(-2(q^1)^2) \\ \mu_{A_1^4}(q^1) &= \exp(-2(q^1-1)^2), \quad \text{and} \\ \mu_{A_1^5}(q^1) &= 1/(1+\exp(-5(q^1-2))) \end{aligned}$$

which are shown in Fig. 5(a). Take the following fuzzy IF–THEN rules from the experts

$$R_{f_M}^i: If\ q^1\ is\ A_1^i,\ Then\ f_M\ is\ B_1^i$$

for $i = 1,\cdots,5$. Hence, according to (51) the term $M(q^1)L(q^1)(\ddot{q}_d^1 - p\dot{\bar{x}}_1)$ is approximated by the fuzzy logic system $Y_{ML}\hat{\Theta}_{ML}$ where

$$Y_{ML} \triangleq [Y_m\ 0\ 0\ Y_m], \qquad \hat{\Theta}_{ML} \triangleq \left[\hat{\theta}_1\ \cdots\ \hat{\theta}_{10}\right]^T$$

with

$$\begin{aligned} Y_m &\triangleq (\ddot{q}_d^1 - p\dot{q}^1 + p\dot{q}_d^1)\,[y_{m1}\ y_{m2}\ y_{m3}\ y_{m4}\ y_{m5}] \\ y_{m1}(q^1) &\triangleq \frac{\mu_{A_1^1}(q^1)}{D_M}, \quad y_{m2}(q^1) \triangleq \frac{\mu_{A_1^2}(q^1)}{D_M} \\ y_{m3}(q^1) &\triangleq \frac{\mu_{A_1^3}(q^1)}{D_M}, \quad y_{m4}(q^1) \triangleq \frac{\mu_{A_1^4}(q^1)}{D_M} \\ y_{m5}(q^1) &\triangleq \frac{\mu_{A_1^5}(q^1)}{D_M}, \quad D_M \triangleq \sum_{i=1}^{5} \mu_{A_1^i}(q^1). \end{aligned}$$

Second, design the fuzzy logic subsystem $\hat{C}_L(q^1,\dot{q}^1,\hat{\Theta}_{CL})$ to approximate the uncertain term $C_L(q^1,\dot{q}^1)$. In addition to the fuzzy sets for q^1 defined above, take three fuzzy sets for $\dot{q}^1$ with labels A_2^1 (negative), A_2^2 (near 0), and A_2^3 (positive), which are characterized by the following membership functions, respectively,

$$\begin{aligned} \mu_{A_2^1}(\dot{q}^1) &= 1/(1+\exp(5(\dot{q}^1+1))) \\ \mu_{A_2^2}(\dot{q}^1) &= \exp(-(\dot{q}^1)^2), \quad \text{and} \\ \mu_{A_2^3}(\dot{q}^1) &= 1/(1+\exp(-5(\dot{q}^1-1))) \end{aligned}$$

which are shown in Fig. 5(b). Since $C_L(\cdot)$ depends upon not only q^1 but also $\dot{q}^1$, fifteen fuzzy IF–THEN rules from the experts are included

$$\begin{aligned} R_{f_C}^{i1}&: \text{If } q^1 \text{ is } A_1^i \text{ and } \dot{q}^1 \text{ is } A_2^1,\ \text{Then } f_C \text{ is } B_2^{i1} \\ R_{f_C}^{i2}&: \text{If } q^1 \text{ is } A_1^i \text{ and } \dot{q}^1 \text{ is } A_2^2,\ \text{Then } f_C \text{ is } B_2^{i2} \\ R_{f_C}^{i3}&: \text{If } q^1 \text{ is } A_1^i \text{ and } \dot{q}^1 \text{ is } A_2^3,\ \text{Then } f_C \text{ is } B_2^{i3}. \end{aligned}$$

Finally, construct the fuzzy logic subsystem $\hat{G}_L(q^1,\hat{\Theta}_{GL})$ to approximate the uncertain term $G(q^1)$. Since $G(q^1)$ also depends only on q^1, according to the fuzzy sets with labels A_1^i the following fuzzy rules are included:

$$R_{f_G}^i: If\ q^1\ is\ A_1^i,\ Then\ f_G\ is\ B_3^i$$

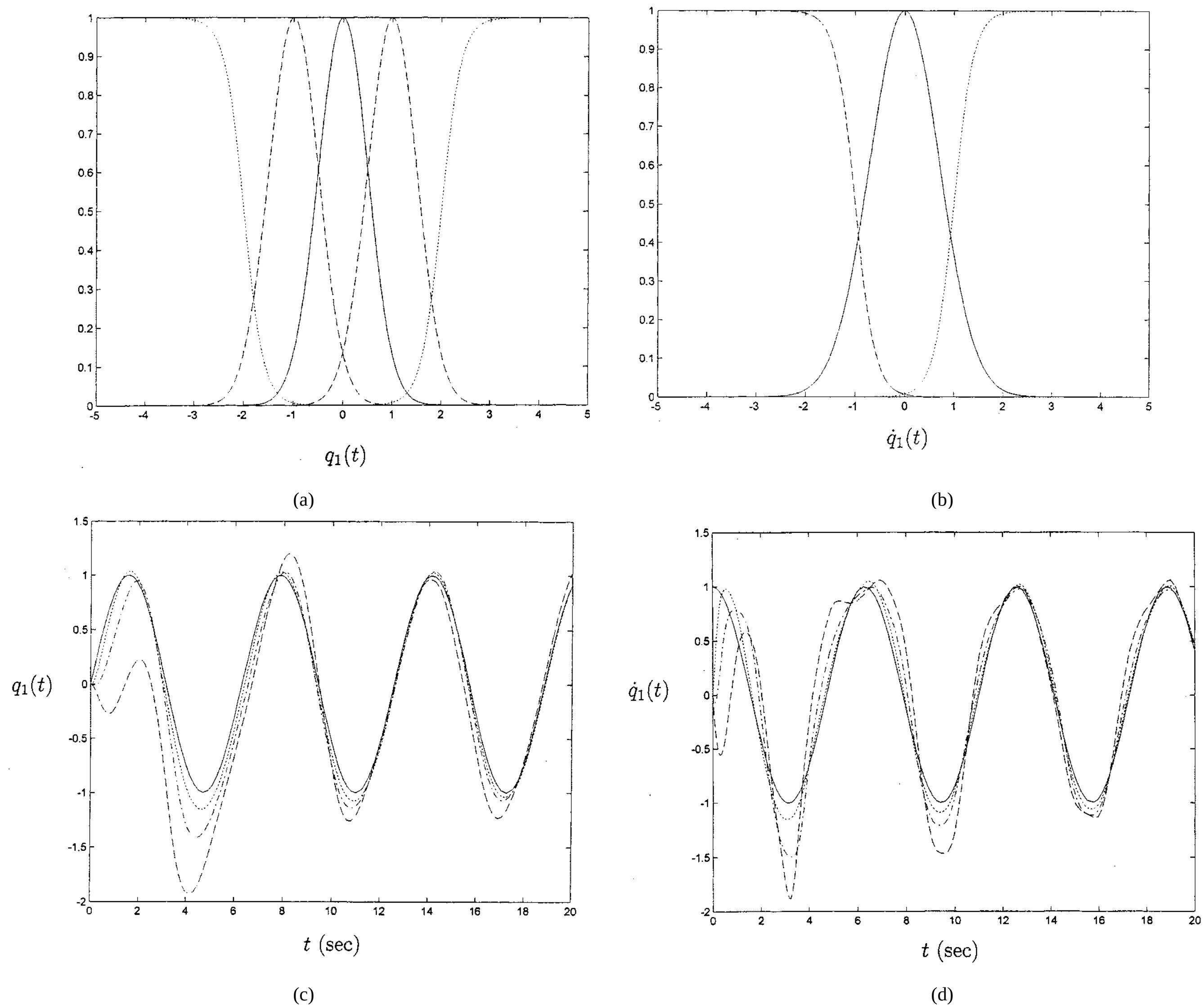


Fig. 5. (a) Fuzzy membership functions with respect to $q_1(t)$. (b) Fuzzy membership functions with respect to $\dot{q}_1(t)$. (c) The angular position $q_1(t)$. (Desired: "—"; Case 1: "- - -"; Case 2: "-·-·-·"; Case 3: "· · ·".) (d) The angular velocity $\dot{q}_1(t)$. (Desired: "—"; Case 1: "- - -"; Case 2: "-·-·-·"; Case 3: "· · ·".)

for $i = 1, \cdots, 5$. Hence, according to (53), the term $G(q^1)$ is approximated by the fuzzy logic system $Y_{GL}\hat{\Theta}_{GL}$ where

$$Y_{GL} \triangleq \begin{bmatrix} Y_g & 0 \\ 0 & Y_g \end{bmatrix}, \quad \hat{\Theta}_{GL} \triangleq \begin{bmatrix} \hat{\theta}_{41} & \cdots \hat{\theta}_{50} \end{bmatrix}^T$$

with

$$Y_g \triangleq [y_{m1} \quad y_{m2} \quad y_{m3} \quad y_{m4} \quad y_{m5}].$$

Consequently, the fuzzy logic system $\hat{F}(x_e, \hat{\Theta}_1)$ to approximate the uncertain term $F(x_e)$ is constructed as

$$Y_1(x_e)\hat{\Theta}_1$$

where

$$Y_1 \triangleq [Y_{ML} \quad Y_{CL} \quad Y_{GL}] \in R^{2\times 50} \quad \text{and}$$
$$\hat{\Theta}_1 \triangleq \begin{bmatrix} \hat{\Theta}_{ML}^T & \hat{\Theta}_{CL}^T & \hat{\Theta}_{GL}^T \end{bmatrix}^T \in R^{50\times 1}.$$

The adaptive fuzzy-based control algorithm described in (23) and (54)–(56) is used to solve this problem. For the convenience of simulation, choose $\hat{\Theta}_1(0) = 0$, $\gamma_{1m} = \gamma_{1c} = \gamma_{1g} = 10$, $M_{\theta_m} = 200$, $M_{\theta_c} = 400$, $M_{\theta_g} = 200$ and $\delta = 10$. Recall that $L(q^1) = [1\ 0]^T$, $E = [1\ 0]^T$, $q_2^* = (\pi/2)$, and $J(q_2^*) = [0\ {-2}]$. Thus, obtain the adaptive fuzzy-based control law with a smoothed projection update law

$$\begin{aligned}
\tau &= Y_{ML}\hat{\Theta}_{ML} + Y_{CL}\hat{\Theta}_{CL} + Y_{GL}\hat{\Theta}_{GL} - \begin{bmatrix} k_0\overline{x}_2 \\ -2\lambda_c \end{bmatrix} \\
\dot{\hat{\Theta}}_{ML} &= Proj[-10Y_{ML}^T L\overline{x}_2] \\
\dot{\hat{\Theta}}_{CL} &= Proj[-10Y_{CL}^T L\overline{x}_2] \\
\dot{\hat{\Theta}}_{GL} &= Proj[-10Y_{GL}^T L\overline{x}_2] \\
\overline{x}_2 &= \dot{q}^1 - \dot{q}_d^1 + p(q^1 - q_d^1) \\
\lambda_c &= \lambda_d - k_\lambda(\lambda - \lambda_d)
\end{aligned}$$

where the controller gains p, k_0 and k_λ are defined below. In order to illustrate the tracking capability with respect to

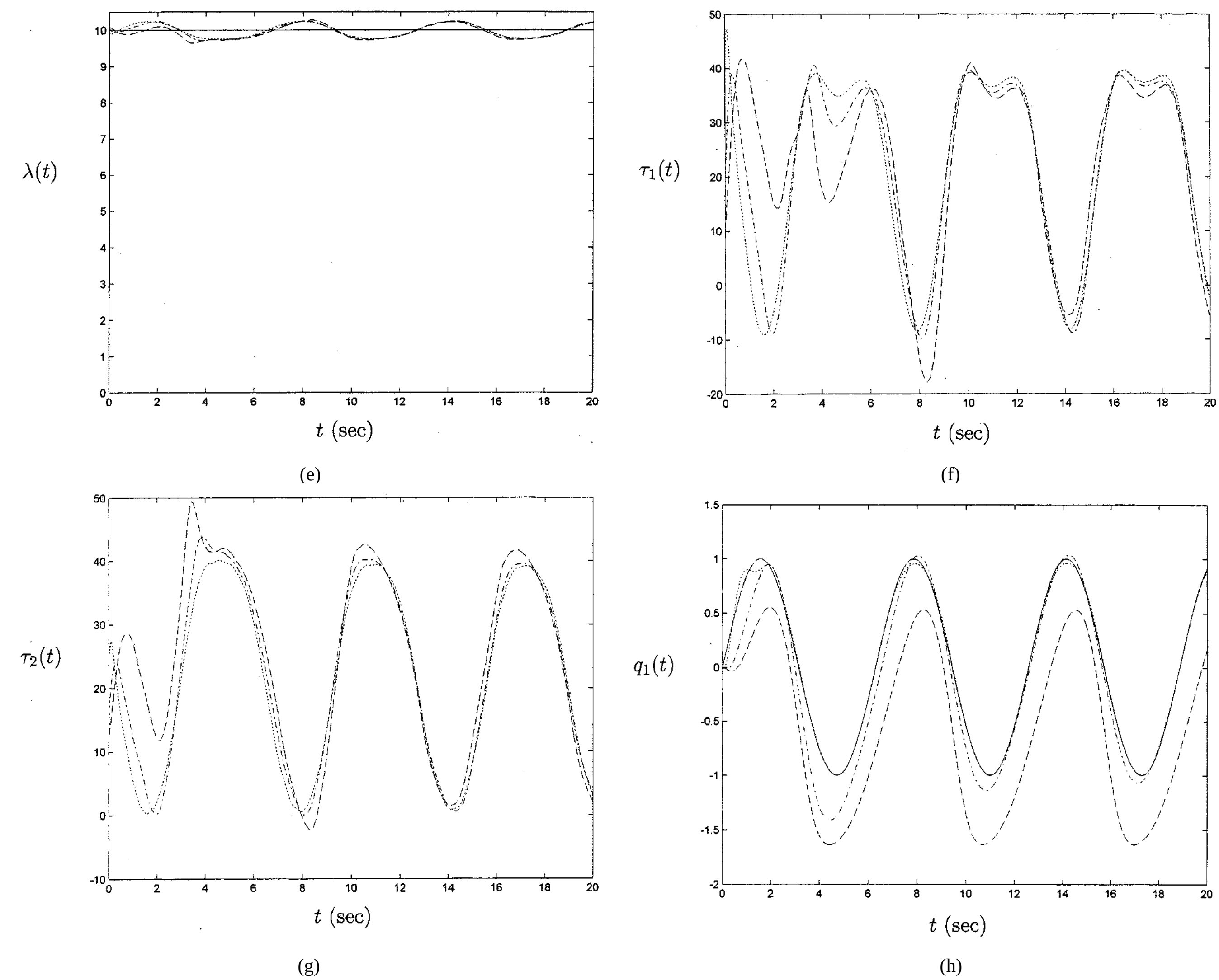


Fig. 5. (*Continued.*) (e) The contact force $\lambda(t)$. (Desired: "—"; Case 1: "- - -"; Case 2: "-·-·-·"; Case 3: "· · ·".) (f) The applied torque $\tau_1(t)$. (Case 1: "- - -"; Case 2: "-·-·-·"; Case 3: "· · ·".) (g) The applied torque $\tau_2(t)$. (Case 1: "- - -"; Case 2: "-·-·-·"; Case 3: "· · ·".) (h) The angular position $q_1(t)$. (Desired: "—"; Case 2: "-·-·-·"; Case 4: "- - -"; Case 5: "· · ·".)

different controller gains, we deliberately design the control laws to achieve the following three different levels as:

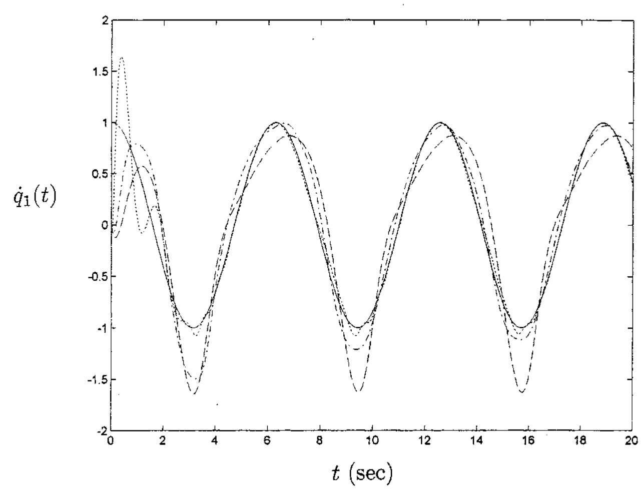
Case 1: Set $\rho = 0.8$, and $w_0 = w_1 = w_2 = 4$. Then, $p = 1$, $k_0 = 10$, and $k_\lambda = 40$.

Case 2: Set $\rho = 0.5$, and $w_0 = w_1 = w_2 = 4$. Then, $p = 2$, $k_0 = 20$, and $k_\lambda = 40$.

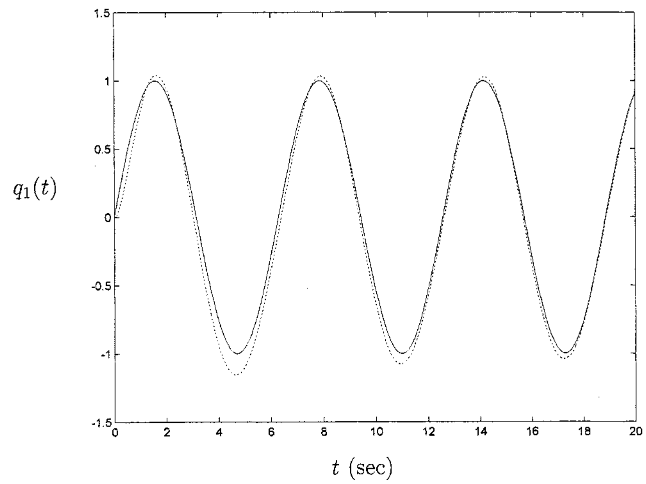
Case 3: Set $\rho = 0.2$, and $w_0 = w_1 = w_2 = 4$. Then, $p = 4$, $k_0 = 40$, and $k_\lambda = 40$.

The simulation results are shown in Fig. 5(c)–(g). The angular position $q_1(t)$ and velocity $\dot{q}_1(t)$ are represented in Fig. 5(c) and (b), respectively. The contact force $\lambda(t)$ is shown in Fig. 5(e). The applied torques $\tau_1(t)$ and $\tau_2(t)$ are plotted in Fig. 5(f) and (g), respectively. From the simulation results of the above three cases, it can be seen that the desired tracking properties of the proposed designs have been achieved. The tracking performance in Case 3 with larger gains is much better than that in Case 1 with smaller gains. This approach can hence be used to diminish the effects due to plant uncertainty and external disturbance in holonomic mechanical control systems.

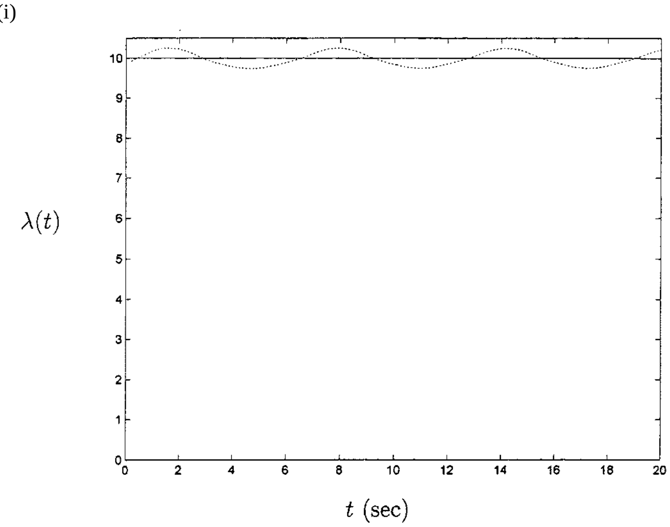
Remark 6.1: 1) The number of fuzzy sets and the number of fuzzy rules in the fuzzy rule bases heavily influence the complexity of a fuzzy logic system [26], [29]. In general, the larger the number, the more complex is the fuzzy logic system and the higher is the expected performance of the fuzzy logic system. Hence, there is always a tradeoff between complexity and accuracy in the choice of the numbers of fuzzy sets and fuzzy rules. Their choice is usually quite subjective and based on some experiences. In the above design, five fuzzy sets for q^1 are chosen and the centers of the corresponding membership functions are selected as $-2, -1, 0, 1, 2$, respectively, and three fuzzy sets for $\dot{q}^1$ are chosen and the centers of the corresponding membership functions are selected as $-1, 0, 1$, respectively. Since the value of the desired trajectories $q_d^1 = \sin(t)$ and $\dot{q}_d^1 = \cos(t)$ belongs to the neighborhood $[-1, 1]$ and one is interested in driving the state variables q^1 and $\dot{q}^1$ to track q_d^1 and $\dot{q}_d^1$, it is intuitively evident that the centers should be clustered around zero. 2) On the other hand, the weighting coefficients in the membership functions heavily influence the smoothness of the input/output (I/O) surface determined by the fuzzy logic system [26], [29]. In gen-



(i)



(j)



(k)

Fig. 5. (*Continued.*) (i) The angular velocity $\dot{q}_1(t)$. (Desired: "—"; Case 2: "-·-·-·"; Case 4: "- - -"; Case 5: "· · ·".) (j) The angular position $q_1(t)$ for the holonomic system with a time-varying perturbation in the constraint surface. (k) The contact force $\lambda(t)$ for the holonomic system with a time-varying perturbation in the constraint surface.

eral, the sharper the membership functions, the less smooth is the I/O surface. The choice of the weighting coefficients (for example, the width in the Gaussian membership function) is also subjective and based on some experiences. Here, these weighting coefficients are selected such that the membership functions are smooth and so our proposed control law can also be guaranteed to be smooth. 3) Moreover, since the effect on the tracking error due to the approximation error has been efficiently compensated by the robust controller (i.e., $-k_0 E\overline{x}_2 - J^T\lambda_c$), only a finite number of fuzzy rules is chosen in this design. Based on the partitioned procedure proposed in Section V, only 25 fuzzy rules are required to construct the fuzzy logic system $\hat{F}(x_e, \hat{\Theta}_1)$. The number of fuzzy IF–THEN rules has been greatly decreased. The values of $B_1^i, B_2^{i1}, B_2^{i2}, B_2^{i3}$, and B_3^i for $i = 1, \cdots 5$ in the linguistic descriptions are not required here since the exact Θ_1^* is not required in the implementation of control law. However, if there exist some good linguistic descriptions about robotic dynamics such that the values of $B_1^i, B_2^{i1}, B_2^{i2}, B_2^{i3}$, and B_3^i can be exactly computed by evaluating $ML(q^1), C_L(q^1, \dot{q}^1)$ and $G(q^1)$, respectively, at the centers of membership functions, then we can expect that the adaptation procedure will converge faster because the initial fuzzy approximator constructed from good linguistic descriptions should be close to the true robotic dynamics [29].□

Next, in order to illustrate the effectiveness of the proposed adaptive fuzzy logic approximator, we also simulate the following different cases.

Case 4: A convenient control law $\tau = -k_0 E\overline{x}_2 - J^T\lambda_c$ is employed with p, k_0, and k_λ as in Case 2. That is, the fuzzy logic approximator is turned off.

Case 5: The classical adaptive control law described in Remark 3.5 is employed with p, k_0, and k_λ as in Case 2. Assume the masses m_1 and m_2 to be unknown. Define $\Theta_1^* \triangleq [m_1 \;\; m_2]^T$. With this parametrization, the regressor matrix is equal to

$$Y_1 = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{13} & Y_{14} \end{bmatrix}$$

where $Y_{11} = \ddot{q}_d^1 - p\dot{q}^1 + p\dot{q}_d^1 + g\cos q_1, Y_{12} = 2(1+\cos q_2^*)(\ddot{q}_d^1 - p\dot{q}^1 + p\dot{q}_d^1) + g\cos q_1 + g\cos(q_1 + q_2^*), Y_{13} = 0$, and $Y_{14} = (1 + \cos q_2^*)(\ddot{q}_d^1 - p\dot{q}^1 + p\dot{q}_d^1) + \sin q_2^* \dot{q}_1(\dot{q}_d^1 - pq^1 + pq_d^1) + g\cos(q_1 +$

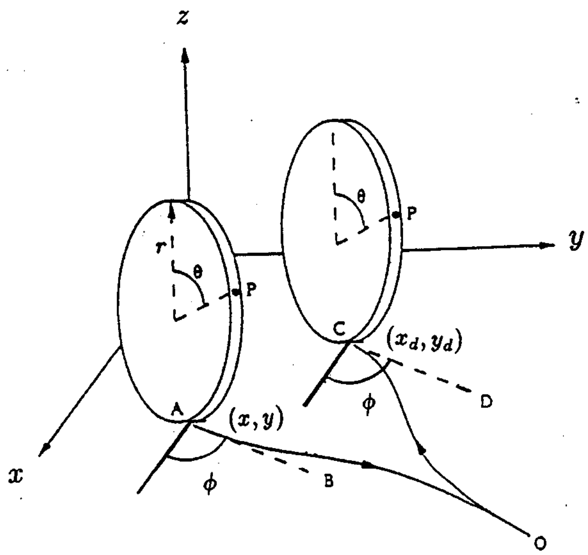


Fig. 6. A vertical wheel rolling without slipping on a plane surface.

q_2^*). The structure of control algorithm is described as in (23) and (35) with $\hat{\Theta}_1(0) = 0, \gamma_1 = 10, M_\theta = 200$, and $\delta = 10$. The angular position $q_1(t)$ and velocity $\dot{q}_1(t)$ are shown in Fig. 5(h) and (i), respectively. It is obvious that the speed of convergence in Case 5 of conventional adaptive state feedback control is faster than the others. It is reasonable since the structures of parametric matrices have been assumed to be known exactly. However, in practical applications, this may be impossible and, therefore, the use of intelligent adaptive fuzzy logic approximator is of interest. The tracking performance in Case 2 with adaptive fuzzy-based control is much better than Case 4 without fuzzy-based control.

Finally, in order to highlight the robustness performance of the proposed control scheme with respect to uncertainties on the constraint surface, we make the following simulation. Suppose that there is a time-varying perturbation $\Delta\phi(t) = 0.2\sin 3t$ on the constraint surface; that is, the constraint function in (61) is modified as

$$\phi(q) = l_1^2 + l_2^2 + 2l_1 l_2 \cos q_2 - r^2 - 0.2\sin 3t = 0$$

which has a unique solution for $q_2 = \cos^{-1}[0.1\sin 3t]$. According to the result in Corollary 3.2, the fuzzy logic approximator proposed above can also be used to compensate the effect due to this constraint perturbation. The adaptive fuzzy-based control law with p, k_0 and k_1 as in Case 2 is employed to solve this robotic trajectory planning problem. The simulation results are shown in Fig. 5(j) and (k). It is clear that a satisfactory tracking performance is achieved and so the proposed control scheme is also robust with respect to the time-varying constraint perturbation except plant uncertainty and external disturbance.

Example 2—Vertical Wheel: Consider the control system of a vertical wheel rolling without slipping on a plane surface [5], [7], [35] shown in Fig. 6. Let x and y denote the coordinates of the point of contact of the vertical wheel on the plane. Let ϕ denote the heading angle of the vertical wheel (measured from the x-axis), and θ denote the rotational angle of the vertical wheel due to rolling (measured from a fixed reference). Then, the dynamic equations of the vertical wheel are

$$m\ddot{x} = \lambda_1, \quad m\ddot{y} = \lambda_2, \quad I_\phi\ddot{\phi} = \tau_2$$
$$I_\theta\ddot{\theta} = \tau_1 - \lambda_1 \cos\phi - \lambda_2 \sin\phi$$

where m denotes the mass of the wheel, I_θ and I_ϕ denote the moment of inertia around the fixed reference axis and x-axis, respectively, τ_1 denotes the control torque about the rolling axis of the wheel, τ_2 denotes the control torque about the vertical axis through the point of the contact, λ_1 and λ_2 denote the forces of the constraint that arise from the nonholonomic constraints of the vertical wheel $\dot{x} = \dot{\theta} \cos\phi$ and $\dot{y} = \dot{\theta} \sin\phi$ where, for simplicity, the radius r is set to unity. Let $q \triangleq [x\ y\ \theta\ \phi]^T$ and choose the variable $z(q) \triangleq [\theta\ \phi]^T$. The matrix $R(q)$ is computed as

$$R(q) = \begin{bmatrix} \cos\phi & \sin\phi & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

so that the relation $\dot{q} = R(q)\dot{z}$ is satisfied. It is clear that the variable $z(q)$ possesses practical physical meanings. Hence, the reduced form in (12) is equal to

$$\begin{bmatrix} m\cos\phi & 0 \\ m\sin\phi & 0 \\ I_\theta & 0 \\ 0 & I_\phi \end{bmatrix}\ddot{z} + \begin{bmatrix} -m\dot{\phi}\sin\phi & 0 \\ m\dot{\phi}\cos\phi & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\dot{z} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\tau + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\cos\phi & -\sin\phi \\ 0 & 0 \end{bmatrix}\lambda + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}.$$

Let us consider the motion tracking control problem with the desired reference trajectories as $\theta_d(t) = \sin(t)$ and $\phi_d(t) = \cos(t)$. For the convenience of simulation, the nominal parameters of the system are taken as $m = 2(kg)$ and $I_\theta = I_\phi = 1$ and the initial conditions are taken as $\theta(0) = \phi(0) = \dot{\theta}(0) = \dot{\phi}(0) = 0$. Suppose the exogenous disturbances d are square waves with period $T = 2\pi$, i.e., d_1 and d_2 are defined as in Example 1, $d_3 = -d_2$ and $d_4 = -d_1$. The intelligent adaptive fuzzy-based control law proposed in Corollary 4.1 is employed to treat this vertical-wheel trajectory planning problem. A fuzzy logic system $\hat{H}(z_e, \hat{\Theta}_2)$ is proposed here to approximate the uncertain term $R^T H(z_e)$. First, construct the fuzzy logic subsystem $\hat{M}_R(\phi, \hat{\Theta}_{MR})$ to approximate the uncertain term $R^T MR(\phi)$. Define three fuzzy sets with labels A_3^1 (negative), A_3^2 (near zero), and A_3^3 (positive) which are characterized by the following membership functions, respectively, $\mu_{A_3^1}(\phi) = 1/(1 + \exp(5(\phi + 1)))$, $\mu_{A_3^2}(\phi) = \exp(-(\phi)^2)$, and $\mu_{A_3^3}(\phi) = 1/(1+\exp(-5(\phi-1)))$. Take the following fuzzy IF–THEN rules

$$R_{h_M}^i: If\ \phi\ is\ A_3^i,\ Then\ h_M\ is\ B_4^i$$

for $i = 1, 2, 3$. Hence, the fuzzy logic system $Y_{MR}\hat{\Theta}_{MR}$ is constructed as

$$Y_{MR} \triangleq \left[(\ddot{\theta}_d - p\dot{\theta} + p\dot{\theta}_d)Y_m \quad (\ddot{\phi}_d - p\dot{\phi} + p\dot{\phi}_d)Y_m\right]$$
$$\hat{\Theta}_{MR} \triangleq \left[\hat{\theta}_1 \quad \hat{\theta}_2 \quad \hat{\theta}_3 \quad \cdots \quad \cdots \quad \hat{\theta}_{10} \quad \hat{\theta}_{11} \quad \hat{\theta}_{12}\right]^T$$

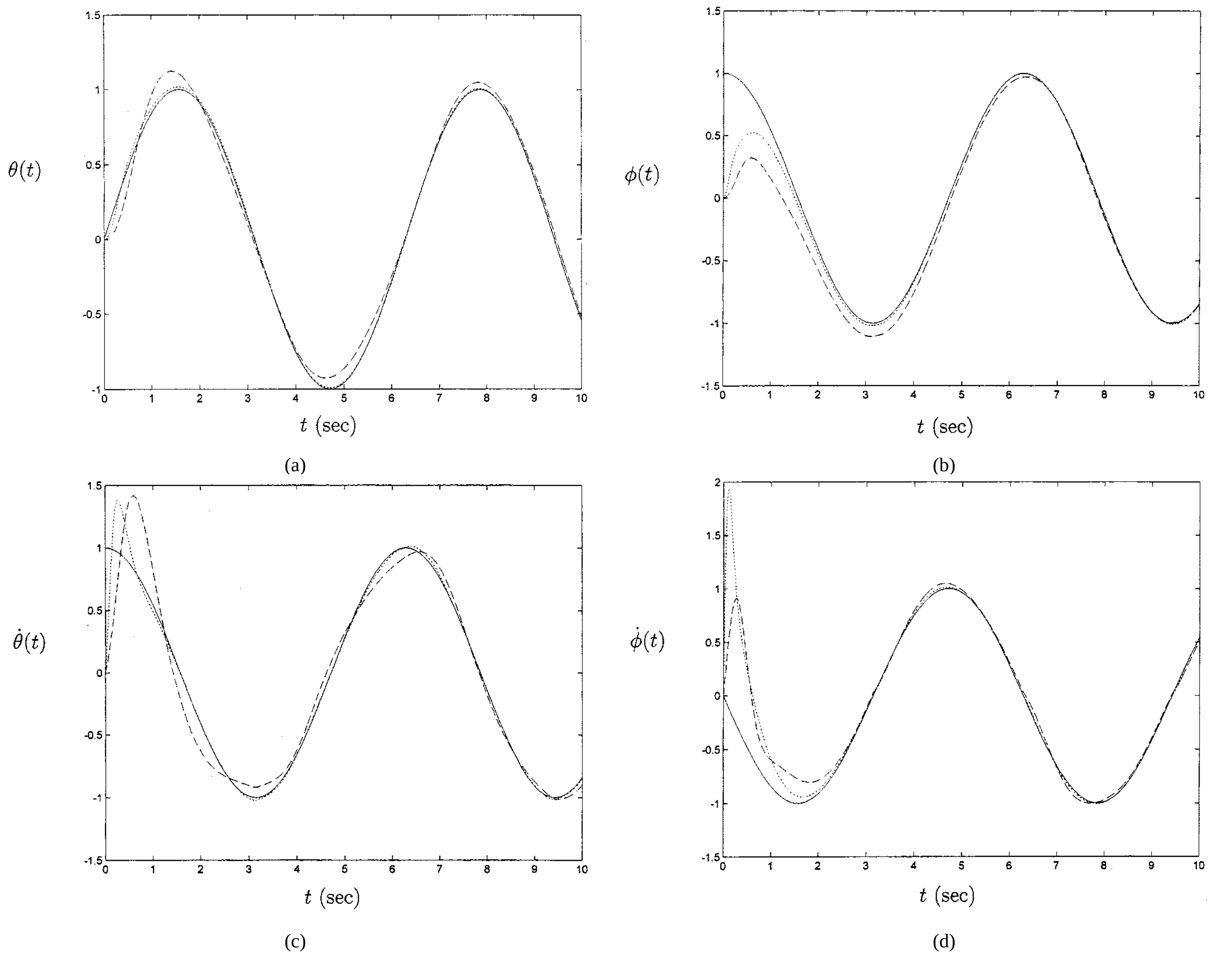


Fig. 7. (a) The angular position $\theta(t)$. (Desired: "—"; Case 6: "'- - -"; Case 7: "· · ·".) (b) The angular position $\phi(t)$. (Desired: "—"; Case 6: "- - -"; Case 7: "· · ·".) (c) The angular velocity $\dot{\theta}(t)$. (Desired: "—"; Case 6: "- -"; Case 7: "· · ·".) (d) The angular velocity $\dot{\phi}(t)$. (Desired: "—"; Case 6: "- - -"; Case 7:"· · ·".)

with

$$Y_m \triangleq \begin{bmatrix} y_{m1} & y_{m2} & y_{m3} & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{m1} & y_{m2} & y_{m3} \end{bmatrix}$$
$$y_{m1}(\phi) \triangleq \frac{\mu_{A_3^1}(\phi)}{D}, \quad y_{m2}(\phi) \triangleq \frac{\mu_{A_3^2}(\phi)}{D}$$
$$y_{m3}(\phi) \triangleq \frac{\mu_{A_3^3}(\phi)}{D}, \quad D \triangleq \sum_{i=1}^{3} \mu_{A_3^i}(\phi).$$

Second, design the fuzzy logic subsystem $\hat{C}_R(\phi, \dot{\phi}, \hat{\Theta}_{CR})$ to approximate the uncertain term $R^T C_R(\phi, \dot{\phi})$. In addition, take two fuzzy sets for $\dot{\phi}$ with labels A_4^1 (negative) and A_4^2 (positive) which are characterized by the following membership functions, respectively, $\mu_{A_4^1}(\dot{\phi}) = 1/(1+\exp(\dot{\phi}+0.5))$ and $\mu_{A_4^2}(\dot{\phi}) = 1/(1+\exp(-(\dot{\phi}-0.5)))$. Six fuzzy IF–THEN rules are included

$$R_{h_C}^{i1}: If\ \phi\ is\ A_3^i\ and\ \dot{\phi}\ is\ A_4^1, Then\ h_C\ is\ B_5^{i1}$$
$$R_{h_C}^{i2}: If\ \phi\ is\ A_3^i\ and\ \dot{\phi}\ is\ A_4^2,\ Then\ h_C\ is\ B_5^{i2}$$

for $i = 1, 2, 3$. Hence, the fuzzy logic system $Y_{CR}\hat{\Theta}_{CR}$ is constructed as

$$Y_{CR} \triangleq \left[(\dot{\theta}_d - p\theta + p\theta_d)Y_c \quad (\dot{\phi}_d - p\phi + p\phi_d)Y_c\right]$$
$$\hat{\Theta}_{CR} \triangleq \left[\hat{\theta}_{13} \quad \hat{\theta}_{14} \quad \hat{\theta}_{15} \quad \cdots \quad \cdots \quad \hat{\theta}_{34} \quad \hat{\theta}_{35} \quad \hat{\theta}_{36}\right]^T$$

with

$$Y_c \triangleq \begin{bmatrix} Y_{ce} & 0 \\ 0 & y_{ce} \end{bmatrix}, \quad y_{ci1}(\phi, \dot{\phi}) \triangleq \frac{\mu_{A_3^i}(\phi)\mu_{A_4^1}(\dot{\phi})}{D_C}$$
$$y_{ci2}(\phi, \dot{\phi}) \triangleq \frac{\mu_{A_3^i}(\phi)\mu_{A_4^2}(\dot{\phi})}{D_C}$$
$$D_C \triangleq \sum_{i=1}^{3}\sum_{j=1}^{2} \mu_{A_3^i}(\phi)\mu_{A_4^j}(\dot{\phi})$$

for $i = 1, 2, 3$. Consequently, the fuzzy logic system $\hat{H}(z_e, \hat{\Theta}_2)$ to approximate the uncertain term $R^T H(z_e)$ is constructed as

$$Y_2(z_e)\hat{\Theta}_2$$

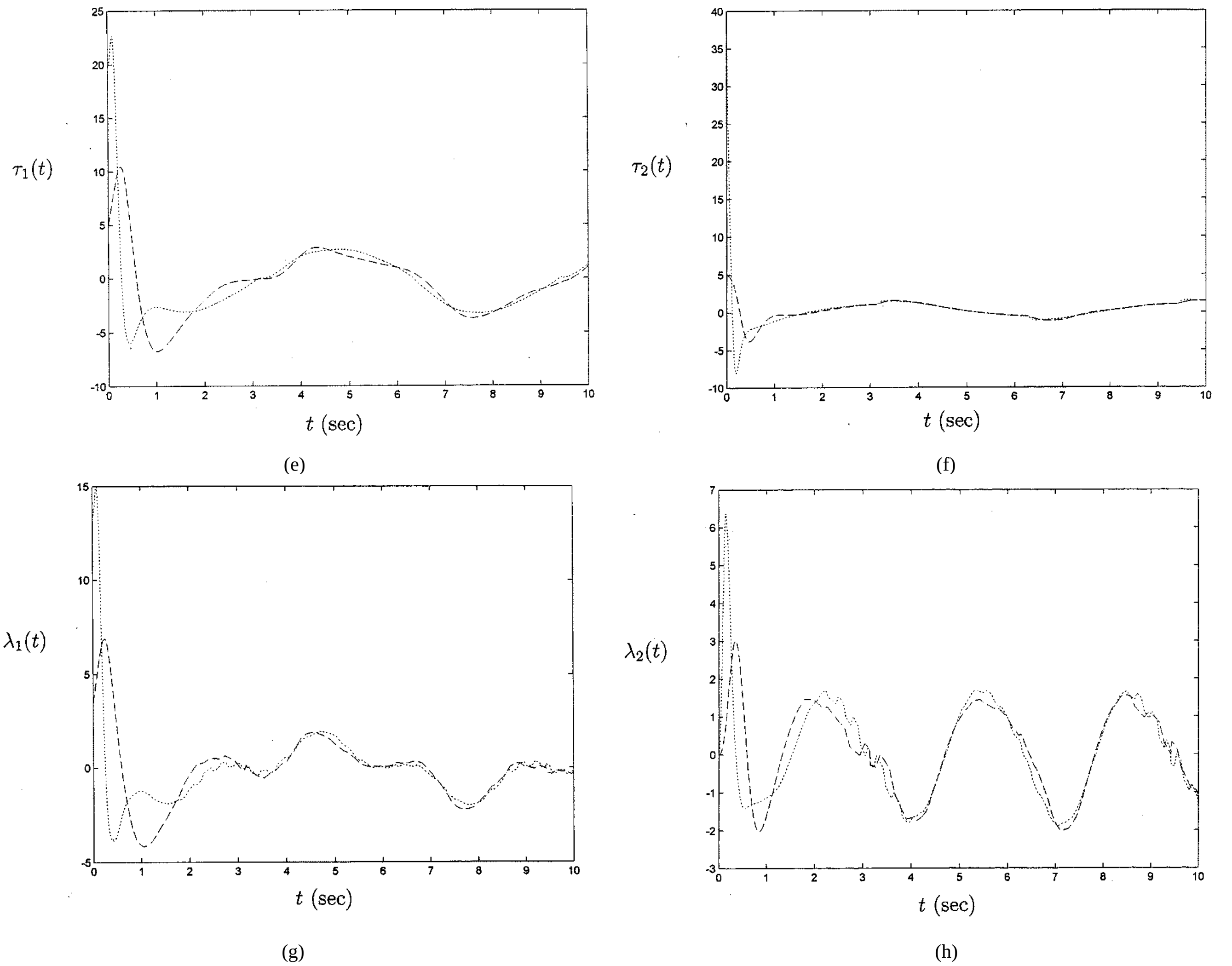


Fig.7. (*Continued.*) (e) The applied torque $\tau_1(t)$. (Case 6: "- - -"; Case 7: "· · ·".) (f) The applied torque $\tau_2(t)$. (Case 6: "- - -"; Case 7: "· · ·".) (g) The contact force $\lambda_1(t)$. (Case 6: "- - -"; Case 7: "· · ·".) (h) The contact force $\lambda_2(t)$. (Case 6: "- - -"; Case 7: "· · ·".)

where $Y_2 \triangleq [Y_{MR} \quad Y_{CR}]$ and $\hat{\Theta}_2 \triangleq \left[\hat{\Theta}_{MR}^T \quad \hat{\Theta}_{CR}^T\right]^T$. The adaptive fuzzy-based control algorithm described in (47), (48) is used to solve this problem. For the convenience of simulation, choose $\hat{\Theta}_2(0) = 0$, $\gamma_2 = 10$, and $M_\theta = 1600$. Note that $R^T B = I_{2\times 2}$ Thus, obtain the adaptive fuzzy-based control law

$$\begin{aligned} \tau &= Y_2\hat{\Theta}_2 - k_0\overline{x}_2 \\ \dot{\hat{\Theta}}_2 &= Proj[-10Y_2^T\overline{x}_2] \\ \overline{x}_2 &= \begin{bmatrix} \dot{\theta} - \dot{\theta}_d + p(\theta - \theta_d) \\ \dot{\phi} - \dot{\phi}_d + p(\phi - \phi_d) \end{bmatrix} \end{aligned}$$

with the controller gains p and k_0 defined below. Similarly, in order to illustrate the tracking capability with respect to different controller gains, we deliberately design the control laws to achieve the following two different cases.

Case 6: Set $\rho = 0.5$ and $w_0 = w_1 = w_2 = 2$. Then $p = 1$, $k_0 = 5$.

Case 7: Set $\rho = 0.2$ and $w_0 = w_1 = w_2 = 2$. Then $p = 2$, $k_0 = 20$. The simulation results are shown in Fig. 7(a)–(h). The angular positions $\theta(t)$ and $\phi(t)$ are shown in Fig. 7(a) and (b), respectively. The angular velocities $\dot{\theta}(t)$ and $\dot{\phi}(t)$ are represented in Fig. 7(c) and (d), respectively. The applied torques τ_1 and τ_2 are plotted in Fig. 7(e) and (f), respectively. The contact forces λ_1 and λ_2 are shown in Fig. 7(g) and (h), respectively. From the simulation results of the above two cases, it is clear that the desired tracking properties of the proposed designs have been achieved. This approach can hence be used to diminish the effects due to plant uncertainty and external disturbance in nonholonomic mechanical systems.

VII. Conclusions

The adaptive fuzzy-based tracking control problems of both holonomic constrained robot systems and a large class of nonholonomic constrained mechanical systems (e.g., nonholonomic Caplygin systems) with plant uncertainties and external disturbances have been proposed and solved from a unified and systematic procedure. By using the proposed adaptive tracking designs via fuzzy approaches, the hybrid adaptive-robust controllers have been constructed for both holonomic and nonholonomic control systems, respectively. Fuzzy logic approximator has been used to learn the behavior of uncertain mechanical dynamics by employing an adaptive algorithm. Next, a robustifying algorithm has been employed to attenuate the effect of learning error on the tracking error. The resulting closed-loop mechanical systems guarantee a satisfactorily transient and asymptotic performance (e.g., L_∞ bounded, H^∞ performance, uniformly ultimately bounded). Moreover, the developed control schemes are very simple and computationally efficient since they do not require a knowledge of either the mathematical model or the parametrization of the mechanical dynamics.

On the other hand, a simplified and partitioned procedure has also been introduced such that the developed adaptive fuzzy logic approximators can be easily implemented. Notably, there exists a tradeoff between the number of fuzzy IF–THEN rules and the controller gain in the robustifying algorithm. Consequently, the number of fuzzy IF–THEN rules can be decreased significantly. This result is very useful from the practical control design point of view.

Finally, simulation examples are presented to illustrate the tracking performance of a two-link constrained robot manipulator and a vertical wheel rolling on a plane surface. From the simulation results in various situations, it is concluded that the proposed design achieves the desired results.

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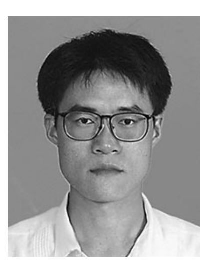

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