Solutions for Quiz 2 (10pts)

1. (1+1+1pts)
   (a) direct proof (p → q)
   Show that if n is an even number, then \( n^2 \) is also an even number.
   Let \( n = 2k \) is an even number, where \( k \) is an integer
   then \( n^2 = (2k)^2 = 4k^2 = 2(2k^2) \) is also an even number

   (b) indirect proof (\(~q \rightarrow ~p~\))
   Show that if \( n^2 \) is an odd number, then \( n \) is also an odd number.
   Let \( n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 \) is an odd number, where \( k \) is an integer
   then \( n = (4k^2 + 4k + 1)^{1/2} = 2k + 1 \) is an odd number
   so, if \( n \) is an even number, then \( n^2 \) is also an even number.

   (c) a proof by contradiction
   Let \( P \) be the statement that the square of an even number is an odd number, and \( P \) is true.
   Set \( n = 2k + 1 \) is an odd number, where \( k \) is an integer
   then \( n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 \) is an odd number,
   which contradicts to the statement \( P \).
   So, the square of an even number is an even number is true.

2. (1+1+1+1pts)
   (a) Let \( P(x) \) denotes \( x \) in this class likes mathematics
   Negation: \(~ (\forall x P(x)) \equiv \exists x \sim P(x)\)
   At least one student in this class does not like mathematics.

   (b) Let \( P(x) \) denotes \( x \) in this class who has ever seen a computer
   Negation: \(~ (\exists x \sim P(x)) \equiv \forall x P(x)\)
   Every student in this class has ever seen a computer.

   (c) \( P(x,y) \): \( x \) in this class who has taken \( (y \) course) offered at this school
   Negation: \(~ (\exists x \forall y P(x,y)) \equiv \forall x \exists y \sim P(x,y)\)
   Every student in this class hasn’t taken at least one mathematic course
   offered at this school.
(d) $P(x,y,z)$: x in this class who has been in (y room) of (z building) on campus

Negation: $\sim (\exists x \exists y \forall z P(x,y,z)) \equiv \forall x \forall y \exists z \sim P(x,y,z)$

Every student in this class hasn’t been in every room of some building on campus.

3. 

\[
A \oplus B = (A - B) \cup (B - A) \\
= A' B' + A B' \\
= (\overline{A} \cap B) \cup (A \cap \overline{B}) \\
= \{x | x \notin A \land x \in B\} \cup \{x | x \in A \land x \notin B\} \\
= (B - A) \cup (A - B)
\]