Overview

- Direct Current (DC) vs. Alternating Current (AC).
- Sinusoidal AC;
  - AC generators;
  - single-phase and three-phase
- Phasor and impedance;
  - Time domain and phasor domain;
  - Sinusoidal steady state;
  - Definition of impedances;
- Root Mean Square (RMS) Values and AC power calculations.
Direct Current (DC)

DC batteries are first developed by Alessandro Volta in 1799.

DC generators are developed by Thomas A. Edison; First DC distribution network implemented in Manhattan, New York, USA.

Volta’s first battery

Pearl Street Power Station in Manhattan, NYC, NY
Direct Current (DC)

Edison’s DC generators.
Alternating Current (AC)

- AC induction machines are developed in 1888 by Nikola Tesla.
- AC generation and distribution system developed in 1890s by Tesla and Westinghouse.
A winding placed in the rotor to produce flux, and the rotor rotates at the rate of 3600 RPM\((= 60 \times 2\pi = 377\text{ rad/s})\)

Three sets of copper windings are distributed around the stator. The flux linkage of each set of windings are given as:

\[
\phi_a = \Phi_m \sin(\omega t); \quad \phi_b = \Phi_m \sin(\omega t - 2\pi/3); \quad \phi_c = \Phi_m \sin(\omega t + 2\pi/3);
\]
By Farady’s law, the induced voltage across the winding terminals are:

\[ v_a = \frac{d\Phi_a}{dt} = \omega \Phi_m \cos(\omega t); \quad v_b = \frac{d\Phi_b}{dt} = \omega \Phi_m \cos(\omega t - 2\pi/3); \]
\[ v_c = \frac{d\Phi_c}{dt} = \omega \Phi_m \cos(\omega t + 2\pi/3); \]
Sinusoidal Steady State

Assuming the circuit under sinusoidal excitation has reached steady-state:

\[ v(t) = V_m \sin (\omega t + \phi) \]

where

\[ V_m = \text{the amplitude of the sinusoid} \]
\[ \omega = \text{the angular frequency in rad/sec} \]
\[ \phi = \text{the phase angle of the sinusoid} \]

- Periodic signal: \( v(t) = v(t + nT) \), where \( T = \frac{2\pi}{\omega} \), and \( n \) is an integer.
- Constant frequency: \( \omega \) does not vary with time.
Definition of Phasors

Eular’s identity: $e^{j\phi} = \cos \phi + j \sin \phi$

A sinusoidal signal in the time domain can be presented in complex form using Eular’s identity:

$$v(t) = V_m \cos(\omega t + \phi) = \Re[V_m(\cos(\omega t + \phi) + j \sin(\omega t + \phi))] = \Re[V_m e^{j(\omega t+\phi)}]$$

$$= \Re[V_m e^{j\phi} e^{j\omega t}] = \Re[V e^{j\omega t}]$$

where $V = V_m e^{j\phi}$.

In sinusoidal steady state, $\omega$ is a known constant,

Phasor of $v(t) = V_m \cos(\omega t + \phi) \rightarrow V = V_m e^{j\phi}$ or $V_m \angle \phi$

The phasor analysis is developed by Charles P. Steinmetz in 1900s.
### Definition of Phasors

For a steady state signal

\[ v(t) = V_m \cos(\omega t + \phi) \]

\[
\frac{dv}{dt} = -\omega V_m \sin(\omega t + \phi) \\
= \omega V_m \cos(\omega t + \phi + \pi/2) \\
= \Re[\omega V_m e^{j\phi} e^{j\omega t} e^{j\pi/2}] \\
= \Re[j\omega V e^{j\omega t}] \\
\]

\[
\frac{dv}{dt} \rightarrow j\omega V
\]

\[
\int v(t)dt = \frac{1}{\omega} V_m \sin(\omega t + \phi) \\
= \frac{1}{\omega} V_m \cos(\omega t + \phi - \pi/2) \\
= \Re[\frac{1}{\omega} V_m e^{j\phi} e^{j\omega t} e^{j(-\pi/2)}] \\
= \Re[\frac{1}{j\omega} V e^{j\omega t}] \\
\]

\[
\int v(t)dt \rightarrow \frac{1}{j\omega} V
\]
Impedances

\[ i_C = C \frac{dv}{dt} = -\omega CV_m \sin(\omega t + \phi) \]
\[ = \omega CV_m \cos(\omega t + \phi + \frac{\pi}{2}) \]
\[ = \Re[\omega CV_m e^{j\phi} e^{j\omega t} e^{j\pi/2}] \]
\[ = \Re[j\omega CV e^{j\omega t}] \]
\[ I_C = j\omega CV \]
\[ \rightarrow Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C} \]

\[ i_L = \frac{1}{L} \int v(t) dt = \frac{1}{\omega L} V_m \sin(\omega t + \phi) \]
\[ = \frac{1}{\omega L} V_m \cos(\omega t + \phi - \frac{\pi}{2}) \]
\[ = \Re[\frac{1}{\omega L} V_m e^{j\phi} e^{j\omega t} e^{j(-\pi/2)}] \]
\[ = \Re[\frac{1}{j\omega L} V e^{j\omega t}] \]
\[ I_L = \frac{1}{j\omega L} V \rightarrow Z_L = \frac{V_L}{I_L} = j\omega L \]
Impedances

\[ v_c = V_m \cos(\omega t + \phi) \]
\[ i_c = \omega CV_m \cos(\omega t + \phi + \pi/2) \]
\[ V_C = V_m e^{j\phi}; \quad I_C = j\omega CV \]

\[ v_L = V_m \cos(\omega t + \phi) \]
\[ i_L = \frac{1}{\omega L} V_m \cos(\omega t + \phi - \pi/2) \]
\[ V_L = V_m e^{j\phi}; \quad I_L = \frac{1}{j\omega L} V \]
# Impedances

<table>
<thead>
<tr>
<th>Time domain</th>
<th>Phasor domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) = V_m \cos(\omega t + \theta_v) )</td>
<td>( V = V_m e^{j\theta_v} ) or ( V_m \angle \theta_v )</td>
</tr>
<tr>
<td>( i(t) = I_m \cos(\omega t + \theta_i) )</td>
<td>( I = I_m e^{j\theta_i} ) or ( I_m \angle \theta_i )</td>
</tr>
<tr>
<td>( \frac{dv}{dt} )</td>
<td>( j\omega V )</td>
</tr>
<tr>
<td>( \int v , dt )</td>
<td>( V / j\omega )</td>
</tr>
<tr>
<td>( R )</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>( C )</td>
<td>( F )</td>
</tr>
<tr>
<td>( L )</td>
<td>( H )</td>
</tr>
</tbody>
</table>

\( Z_R = R \) \( \Omega \) (resistance)

\( Z_C = \frac{1}{j\omega C} \) \( \Omega \) (reactance)

\( Z_L = j\omega L \) \( \Omega \) (reactance)

- At dc, \( \omega = 0 \), \( Z_L = 0 \), \( Z_C \rightarrow \infty \).

- At high frequency, \( \omega \rightarrow \infty \), \( Z_L \rightarrow \infty \), \( Z_C \rightarrow 0 \).
**Kirchhoff’s Laws for Phasors**

**KVL:** \( v_1 + v_2 + \cdots + v_n = 0 \)

\[
\begin{align*}
&\rightarrow V_{m1}\cos(\omega t + \phi_1) + V_{m2}\cos(\omega t + \phi_2) + \cdots + V_{mn}\cos(\omega t + \phi_n) = 0 \\
&\rightarrow \Re[V_{m1}e^{j\phi_1}e^{j\omega t}] + \Re[V_{m2}e^{j\phi_2}e^{j\omega t}] + \cdots + \Re[V_{mn}e^{j\phi_n}e^{j\omega t}] = 0 \\
&\rightarrow \Re[(V_1 + V_2 + \cdots + V_n)e^{j\omega t}] = 0 \\
&\rightarrow V_1 + V_2 + \cdots + V_n = 0
\end{align*}
\]

**KCL:** \( i_1 + i_2 + \cdots + i_n = 0 \) \( \cdots \rightarrow I_1 + I_2 + \cdots + I_n = 0 \)

\[Z_{\text{ser}} = Z_1 + Z_2 + Z_3 + \cdots\]

\[1/Z_{\text{para}} = 1/Z_1 + 1/Z_2 + \cdots\]
Example 1

\[ v_s(t) = 1000 \cos(\omega t + 30^\circ), \quad \omega = 10 \text{ rad/sec}, \]

- Find the phasors \( I_c, V_c \), and the time domain representation \( i_c(t), v_c(t) \).
- For \( \omega = 100 \text{ rad/sec} \), find the above quantities.
Example 1

The phasor representation of $v_s$ is $V_s = 1000 \angle (30^\circ) \ V$. For $\omega = 10 \ \text{rad/sec}$, the reactance of the capacitor is

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 10 \times 10^{-3}} = -j100 \ \Omega$$

$$I = \frac{V_s}{R + Z_C} = \frac{1000 \angle (30^\circ)}{100 - j100} = \frac{1000 \angle (30^\circ)}{100\sqrt{2} \angle (-45^\circ)} = \frac{10 \angle (75^\circ)}{\sqrt{2}}$$

$$V_C = I \left( \frac{1}{j\omega C} \right) = \frac{10 \angle (75^\circ)}{\sqrt{2}} (-j100) = \frac{1000 \angle (-15^\circ)}{\sqrt{2}}$$

The corresponding time-domain representation of $i$ and $v_c$:

$$i(t) = 7.07 \cos (10t + 75^\circ); \quad v_C(t) = 707 \cos (10t - 15^\circ)$$
Example 1

The phasor representation of $v_s$ is $V_s = 1000\angle(30^\circ)$ V. For $\omega = 100 \text{ rad/sec}$, the reactance of the capacitor is

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 100 \times 10^{-3}} = -j10 \Omega$$

$$I = \frac{V_s}{R + Z_C} = \frac{1000\angle(30^\circ)}{100 - j10} = \frac{1000\angle(30^\circ)}{100.5\angle(-5.7^\circ)} = 9.95\angle(35.7^\circ)$$

$$V_C = I\left(\frac{1}{j\omega C}\right) = 9.95\angle(35.7^\circ)(-j10) = 99.5\angle(-54.3^\circ)$$

The corresponding time-domain representation of $i$ and $v_c$:

$$i(t) = 9.95 \cos(100t + 35.7^\circ); \quad v_C(t) = 99.5 \cos(100t - 54.3^\circ)$$
Example 2

\[ v_s(t) = 1000 \cos(\omega t + 30^\circ), \omega = 10 \text{ rad/sec}, \]

- Find the phasors \( I_c, V_c \), and the time domain representation \( i_c(t), v_c(t) \).
- For \( \omega = 100 \text{ rad/sec} \), find the above quantities.
Example 2

The phasor representation of $v_s$ is $V_s = 1000\angle(30^\circ)$ V. For $\omega = 10 \text{ rad/sec}$, the reactance of the inductor is

$$Z_L = j\omega L = j \times 10 \times 10 = j100 \Omega$$

$$I = \frac{V_s}{R + Z_L} = \frac{1000\angle(30^\circ)}{100 + j100} = \frac{1000\angle(30^\circ)}{100\sqrt{2}\angle(45^\circ)} = 7.07\angle(-15^\circ)$$

$$V_C = I(j\omega L) = 7.07\angle(-15^\circ)(j100) = 707\angle(75^\circ)$$

The corresponding time-domain representation of $i$ and $v_c$:

$$i(t) = 7.07 \cos(10t - 15^\circ); \quad v_C(t) = 707 \cos(10t + 75^\circ)$$
Root Mean Square (RMS)

The average power consumption of a resistor $R$ can be calculated as:

$$P_{AV} = \frac{1}{T} \int_{0}^{T} \frac{v^2(t)}{R} dt = \frac{1}{R} \frac{1}{T} \int_{0}^{T} v^2(t) dt$$

Define $V_{RMS} = \sqrt{\frac{1}{T} \int_{0}^{T} v^2(t) dt}$ . The average power delivered by $v(t)$ can be expressed as $P_{AV} = \frac{V_{RMS}^2}{R}$.

For a sinusoidal voltage $v(t) = V_m \cos(\omega t + \theta_v)$,

$$V_{RMS} = \sqrt{\frac{1}{T} \int_{0}^{T} V_m^2 \cos^2(\omega t + \theta_v) dt} = \sqrt{\frac{V_m^2}{T} \int_{0}^{T} \frac{1 + \cos(2\omega t + \theta_v)}{2} dt}$$

$$= \sqrt{\frac{V_m^2}{T} \cdot \frac{T}{2}} = \frac{V_m}{\sqrt{2}}$$
Conclusion

Sinusoidal AC sources;

Phasors: sinusoidal steady-state, periodic signal of a single frequency.

$R$, $L$, and $C$ in phasor domain calculation.

KCL and KVL in the phasor domain.

RMS values.

Future Extension:

- Circuit Analysis (EE2220): DC circuit analysis; AC circuits analysis.
- Signal and Systems (EE3610): Continuous-time and Discrete-time system theories.
- Power Electronics (EE4815): Switch-mode power conversion circuits.
- Electric Machinery (EE4840): Transformers and various electric machines.