Multirate Modeling of AR/MA Stochastic Signals and Its Application to the Combined Estimation-Interpolation Problem

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Abstract—The use of the Kalman filter is investigated in this work for interpolating and estimating values of an AR or MA stochastic signal when only a noisy, down-sampled version of the signal can be measured. A multirate modeling theory of the AR/MA stochastic signals is first derived from a block state-space viewpoint. The missing samples are embedded in the state vector so that missing signal reconstruction problem becomes a state estimation scheme. Next, Kalman state estimation theory is introduced to treat the combined estimation-interpolation problem. Some extensions are also discussed for variations of the original basic problem. The proposed Kalman reconstruction filter can be also applied toward recovering missing speech packets in a packet switching network with packet interleaving configuration. By analysis of state estimation theory, the proposed Kalman reconstruction filters produce minimum-variance estimates of the original signals. Simulation results indicate that the multirate Kalman reconstruction filters possess better estimation/interpolation performance than a Wiener reconstruction filter under adequate numerical complexity.

I. INTRODUCTION

The process of creating samples of an original physical process from a reduced set of samples is called interpolation. Interpolation has been conventionally used by mathematicians to provide a high degree of accuracy for a useful mathematical function from tabulated (sampled) values of the function [1]. Now, the concepts of digital signal processing are playing an increasingly important role in the area of multirate signal processing, i.e., signal processing algorithms that involve interpolation and decimation [2]–[5].

A few representative examples of multirate digital systems can be found in [5], which contain digital audio systems, subband coding systems, communication systems, analog voice privacy systems, and multirate adaptive filters, etc.

The signal interpolation problem has been viewed from the perspective of filter design. In the filter design viewpoint, a prescribed desired frequency response was approximated by the frequency response of the designed filter and some measure of the error frequency response was selected as an approximation criterion. Many techniques are available for designing such filters, and they are all based on slightly different criteria. A brief discussion of this class of filter design procedures can be found in [5] and [6].

Alternately, design of an interpolation filter may be viewed as a problem of estimating the missing samples of a signal. This problem can be examined using the theory of optimal recovery [7], [8]. In the optimal recovery viewpoint, one looks at optimal recovery of unknown time samples from known samples, and from knowledge that the signal belongs to a filter class that is used to model prior knowledge about the input signals to the interpolation filter.

In previous studies of both filter design techniques [5], [6] and optimal recovery approaches [7], [8], the input signals to the interpolation filters were constrained to be decimated versions of some deterministic signals that have bandlimited frequency characteristics. Oetken et al. [9], [10] have placed an emphasis on designing the interpolation filters for this class of decimated signals and are also of interest.

Lu and Gupta [11] considered the use of Wiener filter theory to the design of an optimum multirate digital filter with emphasis on its application to interpolating missing samples from decimated stochastic signals. The derived Wiener interpolation filter is optimum in the minimum mean-square-interpolation-error sense. The decimated stochastic signals considered in [11] were assumed to not be corrupted by noises; in addition, prior knowledge of the structure of the original signals was the autocorrelation function. In many situations, only a noisy version of the decimated stochastic signals is available for interpolation. Furthermore, the original stochastic signals often possess some kind of specific structure other than autocorrelation functions. Under these cases, the Wiener interpolation filter may not reconstruct the original signals satisfactorily. Thereby, other reconstruction filters must be designed to take the noises into consideration and explore the specific signal structure in a full context.

The signal to be interpolated is assumed here to be produced from an AR or MA stochastic source and additive corrupting noise is also considered. The design of an interpolation filter under corrupting noise may be viewed as a problem of estimating the missing samples of a desired signal sequence by the available decimated, noisy signals. This problem can be solved by employing the multirate version of the optimal state estimation theory developed in this paper. First, a signal sequence with missing samples and corrupted by additive noise is modeled from the Kalman state-space viewpoint. The
resulting block state-space model can be considered as a digital state-space dynamic system with a block input but only with a single measurement. The missing samples are embedded in the state variables of the block state-space model. Hence, the signal reconstruction problem is formulated into a state estimation problem. The Kalman state estimation techniques can be employed to treat this combined “estimation-interpolation” problem.

The paper is organized as follows. First, the combined estimation-interpolation problem is formulated in Section II. The structure of the source signal and its block state-space model are then derived in Section III. Based on Kalman state estimation theory, the problem of optimum estimation-interpolation filter design is solved under the minimum mean-square-error criterion in Section IV. Some extensions are also introduced in Section V to treat variations of the original basic problem. The application of Kalman reconstruction filter toward recovery of missing speech packets is treated in Section VI. Numerical examples of the speech signal estimation/interpolation under corrupting noise are provided in Section VII to illustrate the utility of the proposed Kalman reconstruction filter to realistic AR-type signals. Finally, concluding remarks are made in Section VIII.

II. PROBLEM STATEMENT

Consider a sequence \( y(k), k = 0, L, 2L, \cdots \) which is the result of decimating an unknown discrete-time signal \( x = \{ x(k), k = 0, 1, 2, \cdots \} \) and corrupted by the noise sequence \( n = \{ n(k), k = 0, L, 2L, \cdots \} \), i.e.

\[
y(k) = \begin{cases} x(k) + n(k), & \forall k = iL \\ 0, & \forall k \neq iL \end{cases}
\]

(1)

where the decimation factor \( L > 1 \) is an integer and \( i = 0, 1, 2, \cdots \). Hence, the available signal \( y \) is a noisy, downsampled version of \( x \).

Facing this undesired but more realistic condition, a combined estimation-interpolation filter is designed in this study, which employs the available noisy, decimated sequence \( \{ y(k), k = 0, L, 2L, \cdots \} \) in (1) as input, to recover the complete, clean source \( \{ x(k), k = 0, 1, 2, \cdots \} \). By using prior knowledge of both unknown signal sequence \( \{ x(k), k = 0, 1, 2, \cdots \} \) and additive noise sequence \( \{ n(k), k = 0, L, 2L, \cdots \} \), the desired estimation-interpolation filter would be designed so that the signal sequence \( \{ x(k), k = 0, 1, 2, \cdots \} \) can be both estimated and interpolated as closely as possible.

III. SIGNAL GENERATION MODEL

The term model is used for any hypothesis that may be applied to account for or describe the hidden laws that are assumed to govern or constrain the generation of some data of interest. It is crucial to our interpolation problem since the missing samples are recovered from noisy version of the downsampled signals by utilizing the information of these hidden laws.

A. AR Signal-Generation Model

In the field of statistical signal processing, the most popular of the time series modeling approaches to highly correlated signals is the autoregressive (AR) signal generator, which assumes that the signal sequence \( \{ x(k), k = 0, 1, 2, \cdots \} \) is produced by driving a white sequence \( \{ v(k), k = 0, 1, 2, \cdots \} \) as follows:

\[
x(k) = \sum_{i=1}^{N} a_i x(k-i) + v(k)
\]

(2)

where the driving source \( v(k) \) is a zero-mean, white Gaussian noise with time-varying covariance \( E[v(k)v(i)] = Q(k) \delta(k-i) \) and the \( N \) dimensional initial signal vector \( x_o = [x(-N) \cdots x(-2) x(-1)]^T \) is known to have mean \( E[x_o] = \bar{x}_o \) and covariance matrix \( E[(x_o - \bar{x}_o)(x_o - \bar{x}_o)^T] = P_o \), respectively. Furthermore, the corrupting noise \( n(k) \) in (1) is assumed to be a zero-mean, white Gaussian noise with time-varying covariance \( E[n(k)n(i)] = R(k) \delta(k-i) \) and uncorrelated with the driving noise \( v(k) \).

It is well known that the AR model generation can be easily transformed to the state-space model description, which has the Kalman filter as the minimum-variance state estimator under the criterion of minimizing the state error covariance. The state of a system at time \( k \) is the minimum set of internal variables that represents the effect of all past excitations and is fundamental in determining the future evolution of the system. The concept of state is important for conventional linear systems; here it is crucial to the missing-sample system since it can be defined to contain the missing samples. The generation of the signal sequence \( \{ x(k), k = 0, 1, 2, \cdots \} \) in (2) can be modeled by the following state-space signal generation model:

\[
z(k+1) = A z(k) + B v(k+1)
\]

\[
x(k) = C z(k), \quad k = 0, 1, 2 \cdots
\]

(3)

where the state vector \( z(k) = [x(k-N+1) x(k-N+2) \cdots x(k)]^T \) and the parameter matrix/vectors \( A, B, \) and \( C \), respectively, are as follows:

\[
A = \begin{bmatrix} 0 & \vdots \\ \vdots & \ddots \vdots \\ 0 & \cdots & a_N \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}
\]

(4)

where \( I \) is a \( (N-1) \times (N-1) \) identity matrix.

B. Block State-Space Model

Since the available noisy sequence \( \{ y(k), k = 0, L, 2L, \cdots \} \) in (1) and the desired signal sequence \( \{ x(k), k = 0, 1, 2, \cdots \} \) in (3) are now evolving in a different time scale, the signal generation model (3) is expressed in the following in terms of the state values at times \( k = iL, i = 0, 1, 2, \cdots \). From (3), a representation of the signal sequence \( \{ x(k), k = 0, 2, 4, \cdots \} \) can be obtained for decimation factor \( L = 2 \) as follows:

\[
z(k+2) = A^2 z(k) + [AB \quad B] \begin{bmatrix} v(k+1) \\ v(k+2) \end{bmatrix}
\]

\[
x(k) = C z(k), \quad k = 0, 2, 4, \cdots
\]

(5)
By a similar procedure, a generation model of the signal sequence \( \{ x(k), \ k = 0, L, 2L, \cdots \} \) with general decimation factor \( L \) is obtained as follows:

\[
  z(k + L) = A^L z(k) + B_L v_L(k)
\]

\[
  x(k) = C^L z(k), \quad k = 0, L, 2L, \cdots \quad (6)
\]

where the vector \( v_L(k) \) of the blocked driving noise and the parameter matrix \( B_L \), respectively, are denoted as follows:

\[
  v_L(k) = [v(k + 1), v(k + 2), \cdots, v(k + L)]^T
\]

\[
  B_L = [A^{L-1} B, A^{L-2} B, \cdots, B]. \quad (7)
\]

The representation of the decimated signal \( x(k) \) in (6) is a block state-space model since the desired signal \( x(k) \) in (6) is driven by \( L \) different inputs at \( L \) sequential time points; however, only a single output \( x(k) \) is generated at the last time point.

Finally, combining the representation of the decimated signal \( x(k) \) in (6) with the corrupting model of the noisy signal \( y(k) \) in (1), the noisy, decimated signal \( y(k) \) can be described by the following block state-space model:

\[
  z(k + L) = A^L z(k) + B_L v_L(k)
\]

\[
  y(k) = C^L z(k) + n(k), \quad k = 0, L, 2L, \cdots. \quad (8)
\]

It will be noted that in the above block representation (8), the transition of the state vector \( z(k) \) and generation of the noisy signal \( y(k) \) take place at time instances \( k = 0, L, 2L, \cdots \) only, which match the prior assumptions on the available noisy measurements \( y(k) \). Furthermore, the representation (8) is of the well-known Kalman state-space form [12], [13].

The statistical framework of the block driving noise \( v_L(k) \) in (8) is investigated in the following. By taking the expectation on outer product of the block driving noise \( v_L(k) \) at different time points and using the zero-mean, white noise characteristic of the driving noise \( v(k) \) in (2), we have the following white and diagonal covariance structure about the block driving noise \( v_L(k) \):

\[
  E[ v_L(k) v_L^T(l) ] = \begin{bmatrix}
  Q(k+1) & O & \cdots \\
  O & Q(k+2) & \cdots \\
  \vdots & \vdots & \ddots \\
  \end{bmatrix} \delta(k-l)
\]

\[
  \Delta = Q_L(k) \delta(k-l); \quad k, l = 0, L, 2L, \cdots. \quad (9)
\]

Furthermore, since the corrupting noise \( n(k) \) is assumed to be uncorrelated with driving noise \( v(k) \), it remains uncorrelated with the block driving noise \( v_L(k) \).

Since the state vector \( z(k) \) in (8) at \( k = iL \) also contains the missing samples \( x(k) \), \( k \neq iL \) as entries, the estimation-interpolation filter design problem can be alternately described as follows: given the available noisy, decimated sequence \( \{ y(k), k = 0, L, 2L, \cdots \} \) as measurements and the block state-space model (8) as information, the estimation and interpolation of the desired signal sequence \( \{ x(k), k = 0, 1, 2, \cdots \} \) will be obtained by estimating the state vector \( z(k) \) that would be optimally estimated in some statistical sense. The problem of estimating missing samples is now converted to a problem of estimating the state vector.

Remark 1: Making a comparison of the representation (8) with results from the previous work [14] would be worthwhile. Kalman and Bertram [14] considered several different types of sampling systems in a unified approach from a state-space viewpoint, in which the multirate sampling systems are closely related, in concept, to the block state-space model (8) of the noisy, decimated AR signals of this paper. Their work applied the concept of state and transition matrix toward describing the evolution of a dynamic system with different sampling operations in the internal nodes of the system. The generation of the decimated AR signals is considered here as a dynamic system with different sampling rates in the input and output nodes. The evolution of the state through time is described as the least common period of the different sampling operations in both of the multirate sampling systems [14] and the block state-space model (8). The emphasis of their work [14] is placed on the unified representation of a multirate sampling system with continuous and discrete dynamic elements, while the block state-space model (8) of this paper is specific to the dynamic representation of the noisy, decimated AR signals.

IV. KALMAN RECONSTRUCTION FILTER

Based on the block state-space model (8), a Kalman reconstruction filter is derived in this section to solve the combined estimation-interpolation problem. The Wiener solution is also introduced to serve as a reference.

A. Signal Reconstruction Filter

In view of the definition of the state vector \( z(k) \) in (3) and the block state-space model (8), the following important result of this paper is obtained: if the model order \( N \) of the desired signal \( x(k) \) in (2) is greater than or equal to the decimation factor \( L \) in (1), then the set of all states \( z(k) = [x(k-N+1), x(k-N+2), \cdots, x(k)]^T \) in (8) at time \( \tau = iL, i = 0, 1, 2, \cdots \) contains not only the samples \( x(k) \) with \( k = iL \) but also the missing samples \( x(k) \) with \( k \neq iL \). After each of the state estimates \( \hat{z}(k) \), \( k = 0, L, 2L, \cdots \) is obtained, the estimated and interpolated samples of the desired signal \( x(k) \) can be obtained as follows:

\[
  \begin{bmatrix}
    \hat{x}(k-s-L+1) \\
    \vdots \\
    \hat{x}(k-s)
  \end{bmatrix} = \begin{bmatrix}
    O_{L \times (N-L-s)} & I_{L \times L} & O_{L \times s}
  \end{bmatrix} \hat{z}(k)
\]

\[
  (10)
\]

where \( s \geq 0 \) is a smoothing lag factor. The performance of the fixed-lag estimates \( \hat{x}(k) \) in (10) will be better than the filtering case, i.e., \( s = 0 \). Furthermore, unlike the general fixed-lag smoother that must augment the dimension of the state vector, the fixed-lag estimates \( \hat{x}(k) \) in (10) retain the dimension of the state vector \( z(k) \) unchanged since a former element of the state vector \( z(k) \) has been a delayed version of its immediately next element. As the smoothing lag increases, the estimation error variance decreases due to the information provided by the additional data. In practice, a nearly optimal performance may be achieved by making the smoothing lag equal to two or three times the dominant time constants of the block state-space model (8).
B. Kalman State Estimator

The solution (10) for interpolating and estimating values of an AR stochastic signal provides the minimum variance estimates if the estimated states \( \hat{z}(k) \) are optimum in the mean-square-error sense. Since the representation (8) is in the well-known Kalman state-space form, the following Kalman filter equations provide the optimum state estimates [12, 13].

\[
\hat{z}(k + L) = [I - K(k + L)C]A^L \hat{z}(k) + K(k + L)y(k + L)
\]

(11)

where \( I \) is an identity matrix with adequate dimension and the time scale evolves by \( k = 0, L, 2L, \ldots \). The Kalman gain \( K(k) \) in (11) with time scale \( k = 0, L, 2L, \ldots \) will be recursively updated as follows:

\[
\begin{align*}
K(k + L) &= P(k + L|k)C^T \\
&\cdot [CP(k + L|k)C^T + R(k + L)]^{-1}
\end{align*}
\]

\[
P(k + L|k) = A^L P(k|k)A^{LT} + BQ_L(k)B_k^T
\]

(12)

where \( P(k + L|k) \) and \( P(k|k) \) are prediction and filtering state error covariance matrices, respectively. They could provide performance measures of the state estimator (11). Since the estimates \{\( \hat{z}(k), k = 0, L, 2L, \ldots \)\} in (11) are the minimum variance state estimates based on the available noisy signals \{\( y(k), k = 0, L, 2L, \ldots \)\}, the estimates \{\( \hat{x}(k), k = 0, 1, 2, \ldots \)\} in (10) provide the minimum variance estimation and interpolation of the original AR sequence \{\( x(k), k = 0, 1, 2, \ldots \)\} (i.e., \( \hat{x}(i) = E[x(i)|y(0), y(L), \ldots, y(k)] \); \( i = k - s - L + 1, k - s - L + 2, \ldots, k - s; k = 0, L, 2L, \ldots \)).

Now, the initial state error \( \hat{z}(0) \) and initial state error covariance matrix \( P(0|0) \) must be set up by another initial-setting filter as follows:

\[
\hat{z}(0) = [I - K(0)C]A\hat{z}(-1) + K(0)y(0)
\]

\[
P(0|0) = [I - K(0)C]P(0|-1)
\]

(13)

where the optimal initial guess \( \hat{z}(-1) = E[x(-N) \ldots x(-2)x(-1)] = \bar{x}_a \) and the optimal initial Kalman gain \( K(0) \) must be computed based on the prior knowledge of the covariance matrix \( P_0 \) of the initial signal vector \( x_a \) as follows:

\[
K(0) = P(0|-1)C^T [CP(0|-1)C^T + R(0)]^{-1}
\]

\[
P(0|-1) = AP(-1)A^T + BQ(0)B^T,
\]

\[
P(-1|0) = P_0.
\]

(14)

The initial-setting filter (13)–(14) is important to recovery of missing speech packets (see Section VI) since the missing packets are often a segment of a high-energy voiced speech and the overall recovery performance is highly sensitive to the initial estimate.

Remark 2: Lu and Gupta [11] considered a noncausal Wiener filtering approach for interpolation when the decimated version of a stochastic signal can be measured. The decimated measurements of [11] were assumed to be not noisy, and the prior knowledge of the stochastic signals is their autocorrelation function. For the combined estimation-interpolation problem (1) of this work, the solution of the paper [11] must be modified to consider the effect of the corrupting noises. An illustrative example of the Wiener estimation-interpolation solution is provided in the appendix with the second-order FIR filter for decimation factor \( L = 2 \). Similar results can be derived for the higher order Wiener solution and general decimation factor \( L \). A noteworthy result is that a \( N_w \)th-order Wiener reconstruction filter will need \( N_w \times L \) coefficients to perform reconstruction; \( N_w \) of which are used to perform estimation and the others are used to perform interpolation. In addition, the Wiener–Hopf equation (A.4) does not have an efficient algorithm to solve it. The Kalman filter and Wiener solution for reconstructing an AR stochastic signal from its noisy, decimated measurements will be simulated and compared in Section VII.

V. Extensions

The estimation-interpolation problem considered above is only a basic version of our work. As a result of the powerful adaptability of Kalman filter to variations of the previous basic state-space model, many restrictive assumptions on original estimation-interpolation problem can be released.

A. Model Order \( N < L \)

In some situations, the model order \( N \) of the AR signal generator (2) will be less than decimation factor \( L \). Under this case, there will be \( L - N \) desired signal estimates \( \hat{x}(k - L + 1) \) \( \hat{x}(k - L + 2) \) \( \cdots \) \( \hat{x}(k - N) \) that can not be obtained from \( \hat{z}(k) \). This is because there is not enough correlation between \( x(k) \) and its past samples under the case \( N < L \). This kind of problem can be solved by the concept of augmented state. An augmented state vector \( \hat{z}(k) \) of the original state vector \( z(k) \) can be defined as follows:

\[
\hat{z}(k) = \begin{bmatrix} \hat{z}^T(k) & \hat{z}^T(k+1) \end{bmatrix}^T
\]

(15)

where \( \hat{z}(k) = [x(k - L + 1) \ x(k - L + 2) \ \cdots \ x(k - N)]^T \) is an additional state vector, its elements contain the set of the missing samples that will be not estimated from the original state estimate \( \hat{z}(k) \). By the above definition of the new state vector \( \hat{z}(k) \), we obtain an augmented state-space representation of the original signal generation model (3) as follows:

\[
\begin{align*}
\hat{z}(k + 1) &= \bar{A}\hat{z}(k) + \bar{B}u(k + 1)
\end{align*}
\]

\[
x(k) = \bar{C}\hat{z}(k), \quad k = 0, 1, 2, \ldots
\]

(16)

where the parameter matrix/vectors \( \bar{A}, \bar{B}, \bar{C} \) are, respectively, as follows:

\[
\bar{A} = \begin{bmatrix} 0 & 1 & \cdots \ 0 & & 0 & 1 \end{bmatrix}
\]

\[
\bar{B} = \begin{bmatrix} O_{N \times (L-N)} \\ O_{(L-N) \times 1} \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} O_{1 \times (L-N)} & C \end{bmatrix}.
\]

(17)
It should be noted here that, except for different parameter matrix/vectors and differently defined state vectors, the above augmented state-space model possesses the same structure as the original state-space model (3). Hence, similar results can be derived for the augmented state-space model (16), e.g., the block state-space representation (8) and the minimum-variance estimator (10)–(14). Hence, the optimal estimation-interpolation problem can also be solved under the case $N < L$ by the state augmenting technique.

B. A More Sophisticated Model

In the problem formulation (1) and (2), the driving noise $v(k)$ in (2) and the corrupting noise $n(k)$ in (1) are assumed to be both zero-mean, white Gaussian and mutually uncorrelated, and no available signals $y(k)$ are noise-free (i.e., pure interpolation problem). However, the following cases frequently occur in practice:

1) either nonzero-mean noise processes or known bias functions or both in the signal generation model (1) or (2)
2) correlated noise processes
3) colored noise processes
4) some perfect available signals $y(k)$.

In those cases where some or all of them occur together, the structure of the state-space model (3) (and hence the associated block state-space model (8)) can be adequately modified [13]. Therefore, the new state-space model conforms to the basic assumptions of the original problem.

For the case of time-varying AR signal, i.e., parameters $a_i$ in (2) depend on time index $k$, all previous results are not changed except that the parameter matrices $A_L$ and $B_L$ must be replaced by $A_L(k)$ and $B_L(k)$ respectively, which are defined as follows:

$$A_L(k) = A(k + L - 1)A(k + L - 2) \cdots A(k)$$
$$B_L(k) = [A_{L-1}(k + 1)B \cdots A_1(k + L - 1)B \quad B].$$

Furthermore, even when the Gaussian assumption of the driving and/or corrupting noises is removed, the Kalman reconstruction filter (10)–(14) remains a linear minimum variance estimator.

In some situations, the decimated signal $y(k)$ is not only corrupted by additive noise $n(k)$, but also a smoothing version of the desired signals $x(k)$, i.e.,

$$y(k) = \sum_{l=0}^{N-1} c_l x(k - l) + n(k), \quad \forall k = iL.$$  \hspace{1cm} (19)

This kind of degradation phenomenon is referred to as smoothing, blurring, or intersymbol interference (ISI) in digital filtering, image systems, and communication channels, respectively. Under this condition, the parameter vector $C$ (defined in (4)) of the block representation (8) must be replaced by $C = [c_{N-1} \quad c_{N-2} \cdots c_0]$ and other results are the same as that of previous section without changing. Therefore, the Kalman reconstruction filter (10)–(14) is optimum for not only signal interpolation and noise cancellation, but also restoration of smoothed signals. If the upper limit of the summation over index $l$ was greater than $N - 1$, then another augmented state vector $\tilde{x}(k)$ must be used to replace the original state vector $x(k)$.

C. MA Signal-Generation Model

Another frequently adopted signal generation model is the moving average (MA) signal generator, which is quite valuable when the power spectral density (PSD) of the desired signals is characterized by broad peaks and/or sharp nulls. The MA signal sequence $\{x(k), k = 0, 1, 2, \cdots\}$ with model order $M$ is produced by driving a white sequence $\{u(k), k = 0, 1, 2, \cdots\}$ as follows:

$$x(k) = \sum_{i=0}^{M-1} h_i u(k - i)$$  \hspace{1cm} (20)

where the driving signal $u(k)$ is a zero-mean, white Gaussian noise with time-varying covariance $E[u(k)u^T(k)] = \Gamma(k)\delta(k - i)$ and uncorrelated with corrupting noise $n(k)$.

The concept of state can also be applied here to contain the driving noises $u(k)$ since the desired signals $x(k)$ are a linear combination of them. Hence, the generation of the signal sequence $\{x(k), k = 0, 1, 2, \cdots\}$ in (20) can be modeled by the following state-space model:

$$z(k + 1) = Fz(k) + Gu(k + 1)$$
$$x(k) = Hz(k), \quad k = 0, 1, 2, \cdots$$  \hspace{1cm} (21)

where the state vector $z(k) = [u(k) \cdots u(k - M - L + 2)]^T$ and the parameter matrix/vectors $F$, $G$, and $H$ are, respectively, as follows:

$$F = \begin{bmatrix} 0 & 1 & & & 0 \\ & & \ddots & & \vdots \\ & & & 0 & 1 \\ & & & & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix}, \quad H = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \end{bmatrix}$$  \hspace{1cm} (22)

where $h_i = 0, i = M, M + 1, \cdots, M + L - 2$. The MA state-space model (21) possesses the same structure as the AR state-space model (3), hence similar results, such as the block state-space representation (8) and the minimum-variance state estimator (11)–(14), can also be derived for the MA sequence (20).

However, owing to differently defined state vectors in (3) and (21), another method must be applied to the MA sequence to obtain the desired signals $\{x(k), k = 0, 1, 2, \cdots\}$ from the estimated state vectors $\{\tilde{x}(k), k = 0, L, 2L, \cdots\}$. Without loss of generality, the model order $M$ of the MA signal generator (20) is assumed here to be greater than the decimation factor $L$. After each of the state estimates $\tilde{x}(k), k = 0, L, 2L, \cdots$ is obtained, the estimate of the desired
signal $x(k)$ can be obtained as follows:

$$
\begin{bmatrix}
\hat{x}(k) \\
\hat{x}(k-1) \\
\vdots \\
\hat{x}(k-L+1)
\end{bmatrix} = 
\begin{bmatrix}
h_0 & h_1 & \cdots & h_{M-1} & 0 \\
h_0 & h_1 & \cdots & h_{M-1} & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & h_0 & h_1 & \cdots & h_{M-1}
\end{bmatrix}
\begin{bmatrix}
\hat{z}(k)
\end{bmatrix}.
$$

(23)

The estimates $\{\hat{x}(k), k = 0, 1, 2, \cdots\}$ in (23) are the minimum-variance estimates of the original MA signal sequence (20).

VI. APPLICATION TO RECOVERING MISSING SPEECH PACKETS

Packetized speech will find application in telecommunication systems with combined voice and data services [15], [16]. In a packet switching network, packet discarding can occur on the transmitting side if a number of packets are generated in excess of the transmission capacity, or on the receiving side if a packet has not been received within the delay time of the buffer memory. Missing packets are a major cause of impairment in packet voice networks. Whenever a packet discarding occurs, the missing speech packet has to be recovered somehow if the quality of the speech is not to be sacrificed.

A variety of missing packet recovery techniques for PCM coded speech have been investigated in [17]–[22] to reduce degradation by the missing speech segments. These packet recovery techniques produce different types of distortion. Odd-even sample interpolation [17]–[19] produces aliasing distortion, the waveform substitution [20], [21] produces beep and chirp-like distortion, and LSB-dropping [22] produces amplitude-modulated quantization noises.

In addition to different types of distortion, the reported maximum tolerable missing packet rates do not have an obvious difference between the techniques. Advanced efforts must be taken with each of these techniques to find an optimum solution among them, or at least possible candidates from them. The following development is devoted to improve the packet recovery performance of the sample-interpolation procedure.

A. Wiener Sample-Interpolation Procedure

In ordinary packet voice communications, one packet loss causes the omission of consecutive $B$ samples, where $B$ is the packet length in terms of the number of samples. This absence of samples leads to speech clipping. Interleaving methods [23] are well-known digital transmission techniques for converting burst errors to separate errors by reordering a digital signal sequence.

For the odd-even packet interleaving procedure, a speech segment with $2B$ samples is interleaved into the odd and even packets, each of which has $B$ samples. Let $S(j)$ be the receiving factor that represents the value of the received packets in the $j$th speech segment. Clearly the constraint $0 \leq S(j) \leq 2$ holds for all $j$. If both of the odd and even packets in the $j$th speech segment are received, i.e., $S(j) = 2$, they are already to be depacketized. Whenever both of the odd and even packets are lost, i.e., $S(j) = 0$, we assume the zero-amplitude stuffing for the entire segment of $2B$ samples. If one of the odd or even packet is lost, i.e., $S(j) = 1$, the sample-interpolation method will be used to recover the missing speech packet by using the remaining samples in the arrived packet. This is a special case of the problem (1) with $n(k) = 0$ and $L = 2$.

Jayant and Christensen’s sample interpolation procedure [17] used an adaptive second-order Wiener interpolation filter and its interpolation coefficients are based on the first- and second-order autocorrelation functions of the original speech packets, i.e.

$$
\hat{x}(k) = \alpha x(k-1) + \beta x(k+1)
$$

$$
\alpha = \beta = \frac{\phi_{xx}(1)}{1 + \phi_{xx}(2)}
$$

(24)

where $\phi_{xx}(m)$ is the normalized autocorrelation function of the original speech segment, i.e., $\phi_{xx}(m) = E[x(k)x(k + m)]/E[x^2(k)]$. The higher order Wiener-type interpolation filter in Remark 2 can also be applied to treat the missing packet recovery. There are two shortcomings inherent in the odd-even sample-interpolation procedure [17]. First, the forward adaptation of the interpolation coefficients $\alpha$ and $\beta$ needs extra bits in the packet headers to send this side information from transmitter to receiver. Second, the autocorrelation functions are not enough to describe the specific structure of the speech signals in a full context.

B. Kalman Sample-Interpolation Procedure

For the speech signals, the composite spectrum effects of radiation, vocal tract, and glottal excitation can be represented by a slowly time-varying AR generation model. Also, the frame-based linear predictive (LP) techniques can be employed to analyze the evolution of the speech production structure with time [2]. For the packet switching network with adequate packet length (4–64 ms), a packet of speech can be regarded as a realization of an order $N$ stationary AR process in (2). The practical value of the autoregressive order $N$ can range from 1–16, depending on the application.

When the odd-even packet interleaving technique is used on the transmitter and the speech packets are lost by one out of two, the block state-space model (8) will be suitable for describing such missing-speech-packet condition. The samples in the lost packets can be interpolated through the Kalman reconstruction filter (10)–(14) by using the remaining samples in the arrived packets. Theoretical derivation of the Kalman reconstruction filter (10)–(14) relies on the prior knowledge of the AR speech process (2). The problem associated with the Kalman sample-interpolation technique is how to obtain the autoregressive parameters $a_i$ and the covariances $Q$ of the driving noise $\eta(k)$. Theoretically, the optimum solution to this problem would be to compute these parameters from the original speech segments and to include them into the packet headers as part of side information. However, such forward
adaptation of these parameters needs extra bits. Another practical solution is the backward computation of these parameters from the received incomplete speech packets. A simple but effective algorithm for the design of a Kalman interpolation filter with backward parameter adaptation will be described in the following. Similar design rules can be applied to the construction of a Wiener interpolation filter with backward parameter adaptation.

Step 1: If both of the odd and even packets in the previous speech segment are received, then the samples of the previous speech segment (which are completely available) are used to estimate the autoregressive parameters \( a_i \) of the present speech segment. This is reasonable since the speech signal is a short-time stationary process [2]. If the odd and even packets of the previous speech segment are not both received, then the linearly interpolated speech samples of the present missing-packet segment are used to estimate the AR parameters \( a_i \). This estimate is generally effective and especially suitable for the highly correlated signal, such as the voiced speech segments.

Step 2: By using autocorrelation method of the frame-based linear predictive (LP) techniques [2], the desired autoregressive parameters \( a_i \) of AR speech process (2) can be estimated by solving the following Yule–Walker equation (\( \Phi \alpha = \varphi \)):

\[
\begin{bmatrix}
\phi(0) & \cdots & \phi(N - 1) \\
\vdots & \cdots & \vdots \\
\phi(N - 1) & \cdots & \phi(0)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
\vdots \\
a_N
\end{bmatrix}
= 
\begin{bmatrix}
\phi(1) \\
\vdots \\
\phi(N)
\end{bmatrix}
\tag{25}
\]

where \( \phi(k) \) is the estimated autocorrelation function of the present speech segment, i.e.

\[
\phi(k) = \frac{1}{2B} \sum_{k=0}^{2B} \overline{x}(k - i)\overline{x}(k)
\tag{26}
\]

where the quantity \( \overline{x}(k) \) is the speech samples decided in Step 1. Finally, the estimated covariance \( Q \) of the driving noise \( v(k) \) is obtained by

\[
Q = \phi(0) - \alpha^T \varphi.
\tag{27}
\]

The autocorrelation parameter estimation method of Step 2 will generate the AR parameters \( a_i \) that guarantee the stability of the AR speech process [2]. Hence, the Kalman state estimator (11) is asymptotically stable [13] and the stability of the interpolation estimate (10) is assured. Furthermore, since the autocorrelation matrix \( \Phi \) in (25) is a symmetric Toeplitz matrix, the Durbin’s recursive procedure [2] can be used to efficiently solve the Yule–Walker equation. Finally, the efficient implementation of the Kalman interpolation procedure (10)–(14) will be discussed in more detail in the next section.

### VII. NUMERICAL EXAMPLES

The variations of the combined estimation-interpolation problem for the AR/MA stochastic signals have been analytically solved by the Kalman reconstruction filters. Their application to recovery of missing speech packets has also been proposed in the last section. In what follows, attention is focused on the numerical analysis of the estimation/interpolation performances in different test cases.

#### A. Reconstruction of the Speech Signals

In order to examine the utility of the Kalman reconstruction filter to the realistic AR type signals, four sentence-length Mandarin utterances were used throughout the following experiments. Each sentence of speech signals was first lowpass filtered to 3.4 KHz (3 dB), then sampled at the rate of \( f_s = 8000 \) times/s. These speech signals were then decimated by a factor \( L = 2 \) and corrupted by additive noise \( n(k) \) with different variances \( R \) to obtain various noisy cases.

Three kinds of methods were used to reconstruct the original speech from the noisy, decimated measurements. The linear interpolator is a straightforward scheme that will be used as a reference of the simulations. The Wiener reconstruction filter in Remark 2 provides more sophistication than the simple linear interpolator and a eighth-order FIR scheme with forward parameter adaptation was used to make a comparison with the Kalman reconstruction filter. The estimation and interpolation coefficients of the Wiener reconstruction filter were updated for every 32 ms to track the nonstationary characteristics of the speech signals. The Kalman reconstruction filter (10)–(14) provides the optimum reconstruction performance for the AR speech processes. The autoregressive order \( N \) was selected to be four as a compromise of the computational complexity and reconstruction performance. The AR model parameters were obtained by solving the Yule–Walker equations through the frame-based autocorrelation method of linear predictive techniques with frame-length 32 ms [2]. The smoothing lag \( s \) in (10) was set to two to obtain the smoothed estimates.

Various performance measures of the estimation/interpolation operations are introduced in the following. The quantities \( SNRA \), \( SNRE \), \( SNRI \) and \( SNRT \) stand for the ratios (in decibels) of decimated-signal to additive-noise, decimated-signal to estimation-error, missing-signal to interpolation-error and complete-signal to total-average-error respectively, i.e.

\[
\begin{align*}
SNRA &= 10 \log \frac{\sum_{k=L}^{L} x^2(k)}{\sum_{k=L}^{L} n^2(k)} \\
SNRE &= 10 \log \frac{\sum_{k=L}^{L} x^2(k)}{\sum_{k=L}^{L} e^2(k)} \\
SNRI &= 10 \log \frac{\sum_{k \neq L}^{L} x^2(k)}{\sum_{k \neq L}^{L} e^2(k)} \\
SNRT &= 10 \log \frac{\sum_{k} x^2(k)}{\sum_{k} e^2(k)}
\end{align*}
\tag{28}
\]

where the error quantity \( e(k) \) is defined to be \( e(k) = \hat{x}(k) - x(k) \) and the integer index ranges from 0 to the final sample of the test sequence. Notably, the ratio of decimated-
signal to additive-noise SNR is the same as the ratio of
decimated-signal to estimation-error SNR for the linear
interpolator.

Different noisy cases were simulated in the following ex-
periments for the decimated speech sequences and the recon-
struction performances are discussed among different methods.
Simulation results of the reconstruction schemes are illus-
trated in Fig. 1. Fig. 1(a)–(c) refer, respectively, to the estima-
tion, interpolation, and the total reconstruction performances.
Fig. 1(a) indicates that the estimation performance (SNRE)
improvements of the proposed Kalman reconstruction filters
over the Wiener reconstruction methods are about 1–2.5 dB
for different noisy index SNR. For the seriously corrupted
speech (SNR = 5 dB) the improvements of the estimation
performance are significantly over linear interpolator for both
of Kalman (6 dB) and Wiener (5 dB) reconstruction filters.
For the slightly corrupted speech (SNR ≥ 20 dB), the
improvement of the estimation performance is negligible for
the Wiener reconstruction filter; meanwhile, the Kalman re-
construction filter retains adequate improvement over linear
interpolator (2.5–3 dB).

Fig. 1(b) shows the interpolation performances (SNRI) of
the three reconstruction methods under different noisy cases.
The interpolation performances of the linear interpolator are
the worst of the three reconstruction methods since the linear
interpolator is in no way optimum for the speech signals.
The Wiener reconstruction filter improves the interpolation
performances up 1–2 dB over linear interpolator due to a signal
model (auto correlation function model) that is somewhat
related to the speech signals. The SNRI improvements of
the Kalman reconstruction filters over the Wiener recon-
struction methods are about 1–2 dB. This is owing to the
exact interpolation of the Kalman reconstruction according to
the autoregressive structure of the speech signals. Fig. 1(c)
illustrates the SNR improvements of the total reconstruction
performance among Kalman, Wiener, and linear interpolator.
The Kalman reconstruction filter possesses the optimum re-
construction performances (3–4 dB over linear interpolator
and 1–2 dB over Wiener reconstruction filter); the improve-
ments are due to the combined effect of precise estimation and
exact interpolation.

Different decimation factors (L = 2–5) were also ap-
plied to speech signals with fixed noisy index SNR = 20 dB.
The three performance indices are plotted in Fig. 2(a)–(c)
respectively. The Kalman reconstruction filter has the optimum
reconstruction performance among the three methods. The
SNR gains are 1–2 dB over Wiener reconstruction and 2–3 dB over linear interpolation.

B. Recovery of Missing Speech Packets

Kalman sample-interpolation procedure has been derived in the last section to recover the missing speech packets within packet-interleaving networks. The parameters used in the packet interleaving configuration would be the packet length $B$ and the packet loss ratio $P_L$. The packet length used in the simulated odd-even interleaving configuration was $B = 16$ ms (128 samples), thus the decoding delay was $2B = 32$ ms. The simulated packet loss ratio $P_L$ ranged from 0.05–0.2 and five sample-interpolation methods were used to recover the missing speech packets: 1) linear interpolator, 2) eighth-order Wiener interpolation filter with backward parameter adaptation, 3) eighth-order Wiener interpolation filter with forward parameter adaptation, 4) fourth-order Kalman interpolation filter with backward parameter adaptation, and 5) fourth-order Kalman interpolation filter with forward parameter adaptation. There were not obvious silent gaps in the simulated utterances and hence the speech/silence discrimination was not put into the simulations. The purpose of the experiments is to study the reconstructed speech quality as a function of the packet loss ratio $P_L$ with the above five sample-interpolation-based packet recovery schemes. The results are illustrated in terms of signal-to-noise ratio, waveform reconstruction plots, and summaries of subjective listening tests.

The objective measure $SNRL$, the signal-to-noise ratio obtained by averaging $SNR$ (dB) values over lost packets, are illustrated in Fig. 3 for five sample-interpolation procedures. The linear and forward Kalman interpolation give the worst and best $SNRL$ values, respectively, and the $SNRL$ differences between them are about 3–4 dB. For the practical network applications, the configurations of backward parameter adaptation are desired since no additional side information is needed in the packet header. The Kalman interpolation with backward parameter adaptation possesses about 1.5–2.5 dB of $SNRL$ improvements over Wiener interpolation with backward parameter adaptation. An interesting observation is that the Kalman interpolation with backward parameter adaptation performs better than the Wiener interpolation with forward parameter adaptation, which justifies the importance of an adequate signal model for the interpolation problem.

Fig. 4(a)–(d) demonstrate the benefits of Kalman sample interpolation by means of waveform plots and segment-specific $SNR$ values (in dB). The speech packet length used in these examples was $B = 7.5$ ms and each waveform in the illustrations was $2B = 15$ ms (120 samples, at 8 kHz). Fig. 4(a) illustrates the waveform of the original $2B$-long speech segment. Fig. 4(b)–(d) refers, respectively, to reconstructions based on linear, backward Wiener, and backward Kalman interpolation. The Kalman interpolation performs better than the other two interpolation methods; this is reflected by segment-specific $SNR$ values and waveform details in the illustrations. The abrupt changes of the original speech signals are retained in the reconstruction of Kalman interpolation, but have been smoothed in that of linear and Wiener interpolations. Subjective effect of the smoothed reconstructions is a deeper and distorted tone, which is more sensible for the female speech. Based on subjective listening tests of the authors and their colleagues, the results of Figs. 3 and 4 are well confirmed by differences in perceived quality of the corresponding output samples.
C. Discussion

A comparison is made of the Kalman and Wiener reconstruction filters in what follows in terms of reconstruction performance and numerical complexity. From computer simulations of the previous subsections, there are always 1–2.5 dB SNR improvements obtained by the Kalman reconstruction filters over the Wiener reconstruction methods in both forward and backward parameter adaptation configurations, and in different noisy and missing cases. These observations justify the importance of an adequate signal modeling for the estimation/interpolation problem.

Although the convolution operation (or FIR filtering) of a \( N_w \times N_r \times L \times L \) multiplications and additions, the \( N_w \times L \) estimation/interpolation coefficients must be computed from \( N_w(L + 1)/2 \) estimated autocorrelation functions (see (A.4) or refer to (11, (4))), which will be a computational burden when it is applied to realistic nonstationary environments. For the \( N \times N \) Kalman reconstruction filter, the update of the AR parameters needs only \( N + 1 \) estimated autocorrelation functions (see (25)). If Kalman and Wiener reconstruction filters have the same order (i.e., \( N = N_w \)), then \( N + 1 \) is always less than \( N_w(L + 1)/2 \) since the decimation factor \( L \geq 2 \). Furthermore, unlike the Yule–Walker (25), the Wiener–Hopf (A.4) has not an efficient algorithm such as the Durbin’s recursive procedure to solve it.

At the first glance, the Kalman filter equations (11) and (12) involve matrix/vector multiplications/additions and seem complex. However, this is just an illusion due to their expanded profiles. First, in view of the sparse structure of the coefficient matrices/vectors \( A, B, C \) (see (4)) and \( Q_L \) (see (9)), many multiplication and addition operations are automatically eliminated in the computation of (11) and (12). Particularly, the computation of the term \( [CP(k + L)k]C + R(k + L)]^{-1} \) needs only a scalar addition and a scalar inverese. Second, the square matrix \( ALP(k)kA^L \) of (12) is symmetric and, hence, a half of its elements would not be required to compute. Third, the identity matrix that conceals itself in the parameter matrix \( A \) (see (4)) results in that most elements of the square matrix \( ALP(k)kA^L \) are merely a shift of the matrix \( P(k)k \) and only a few elements of the square matrix \( ALP(k)kA^L \) need to be calculated. All of the above observations will be beneficial to realistic implementation of the Kalman reconstruction filter.

Notably, the orders used in the previous Kalman and Wiener reconstruction filters are four and eight, respectively. Thus, the AR parameters \( a_i \) of the Kalman reconstruction filter are obtained by computing \( N + 1 = 5 \) autocorrelation functions and by solving a \( 4 \times 4 \) Yule–Walker (25). Meanwhile the interpolation coefficients of the Wiener reconstruction filter are obtained by computing \( N_w(L + 1)/2 = 4(L + 1) \) autocorrelation functions and by solving a \( 8 \times 8 \) Wiener–Hopf equation.

Based on the above discussions regarding the implementation time-accuracy trade off between the Kalman and Wiener reconstruction filters, the conclusion is that the Kalman reconstruction filter would be the recommendation of this work.

In addition, the powerful adaptability of Kalman filter to variations of the basic estimation-interpolation problem (see Section V) results in that the Kalman reconstruction methods can be applied to more broad applications than the Wiener reconstruction schemes.

VIII. Conclusions

The combined estimation-interpolation problem has been treated in this study from the block state-space modeling viewpoint. The input signals to the proposed Kalman reconstruction filters can be a noisy decimated version of an AR or MA signal, which are not constrained to have lowpass or bandpass frequency characteristics. Furthermore, the additive corrupting noises are also considered in the estimation-interpolation problem and hence conform to the realistic noisy environments.

The proposed Kalman estimation-interpolation filters operate on noisy decimated measurements, and produce good estimates of the complete clean source. By analysis of state estimation theory, the Kalman reconstruction filters can produce the minimum-variance estimates of the original signals. From investigation of the numerical examples, the Kalman reconstruction filters have possessed better estimation/interpolation performances than a Wiener reconstruction filter under adequate numerical complexity.

The proposed Kalman reconstruction method can be also applied to recover missing speech packets in a realistic packet switching network. Detailed packet recovery configurations and subjective listening considerations are currently being investigated by the authors. Finally, adaptability of the Kalman reconstruction method to variations of the original problem results in that it has potential applicability to other multirate applications. Another direction of further work would be to apply the multirate Kalman reconstruction techniques to noisy subband coding systems to obtain the optimal reconstruction of the input signals.

APPENDIX

The second-order Wiener estimation-interpolation filter is derived in the following for decimation factor \( L = 2 \). Assume \( y(k) \) is the present noisy measurement; then the interpolation and estimation filters would be as follows:

\[
\begin{align*}
\hat{x}(k-1) &= \alpha y(k-2) + \beta y(k) \\
\hat{x}(k) &= \gamma y(k-2) + \lambda y(k)
\end{align*}
\]

(A-1)

where the interpolation coefficient \((\alpha, \beta)\) and the estimation coefficients \((\gamma, \lambda)\) must be decided to minimize the variance of the interpolation and estimation error

\[
\begin{align*}
i(k) &= \hat{x}(k-1) - x(k-1) \\
e(k) &= \hat{x}(k) - x(k)
\end{align*}
\]

(A-2)
respectively. The error variances are minimized when the following equations hold

\[
\begin{align*}
\frac{\partial E[e^2(k-1)]}{\partial \alpha} &= \frac{\partial E[e^2(k-1)]}{\partial \beta} = 0 \\
\frac{\partial E[e^2(k)]}{\partial \gamma} &= \frac{\partial E[e^2(k)]}{\partial \lambda} = 0
\end{align*}
\]

(A-3)
which lead to the following Wiener–Hopf equation

\[
\begin{bmatrix}
\phi_{yy}(0) & \phi_{yy}(2) \\
\phi_{yy}(2) & \phi_{yy}(0)
\end{bmatrix}
\begin{bmatrix}
\beta \\
\gamma
\end{bmatrix}
=
\begin{bmatrix}
\phi_{xy}(1) \\
\phi_{xy}(2)
\end{bmatrix}
\tag{A-4}
\]

where

\[
\begin{align*}
\phi_{yy}(i) &= E[y(k-i)y(k)] \\
\phi_{xy}(i) &= E[x(k-i)y(k)] \\
\phi_{xx}(i) &= E[x(k-i)x(k)]
\end{align*}
\tag{A-5}
\]

The resultant Wiener interpolation and estimation coefficients can be obtained by solving the Wiener–Hopf (A-4). Unlike the Yule–Walker (25), the Wiener–Hopf equation (A-4) does not have an efficient algorithm to solve it.

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