Necessary and Sufficient Conditions for Robust Stabilization of an Observer-Based Compensating System Suffering Nonlinear Time-Varying Perturbation

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Abstract—The purpose of this note is to demonstrate that the robust stabilization presented by Chen and You [3] against additive nonlinear time-varying uncertainty is not only just a necessary condition, but also a necessary condition. Besides, one more necessary and sufficient conditions in the case of multiplicative uncertainty are also discussed in this note.

I. INTRODUCTION

The necessary and sufficient conditions for the robust stabilization about a linear time feedback system had been stated in detail in the research of Vidyasagar [1]. But the practical design problem of robust stabilization which control engineers have to confront is to overcome the analytical difficulty caused by nonlinear perturbation. Polua and Ting [2] discussed the necessary and sufficient condition of robustness for a discrete time feedback system with additive nonlinear perturbation via a nonlinear controller. Based on the structure of the observer-based compensating system, Chen and You [3] proposed a sufficient condition of robust stabilization under additive nonlinear perturbation. Thus, whether such a sufficient condition is also a necessary condition or not is another topic left to explore. For convenience of discussion, some notations and descriptions are summarized below.

Mathematical Notations

(1) \[ \|A\|_2 \] denotes the induced \( L_2 \) norm of operator \( A \).
(2) \[ \|A(j\omega)\|_2 \] denotes the spectral norm of \( A(j\omega) \).
(3) \[ \|A(j\omega)\|_2 \] denotes the largest singular value of \( A(j\omega) \).
(4) For any matrices \( U \) and \( V \), \[ \|U\|_2 = \|U^T\|_2 \] .
(5) \[ \|U(j\omega)\|_2 \] denotes the set of matrices whose elements are all proper stable rational functions.

Fig. 1 illustrates the observer-based compensating system \( S_\delta(P + N, T, R, S) \) with additive nonlinear time-varying perturbation system \( N \) where \( T(s), R(s), S(s) \in H[s] \). The control law is given by

\[ u(t) = r(t) - T^{-1}(Ru(t) + S(t)). \] (1)

II. MAIN RESULT

Lemma 1 [3]: The nominal closed-loop system in Fig. 1 will maintain asymptotic stability if and only if \( (T(s) + R(s))A_1(s) + S(s)B_1(s) = L(s) \) where \( L(s) \in H[s] \) has a stable inverse. That is to say, if and only if

\[ T(s) + R(s) = L(s)X_1(s) + M(s)B_1(s), S(s) = L(s)Y_1(s) - M(s)A_1(s) \] (2)

where \( M(s), L^{-1}(s) \in H[s] \). Consider the system \( S_\delta(P + N, T, R, S) \) in Fig. 1; it is seen that [3]

\[ u(t) = A_1((T + R)A_1 + SB_1)^{-1}Tr(t) - A_1((T + R)A_1 + SB_1)^{-1}SNv(t) \] (3)

\[ u(t) = (I + A_1L^{(-1)SN})^{-1}A_1L^{-1}Tr(t). \] (4)

We put forth the following theorem for the robust stabilization of a system suffering additive nonlinear time-varying uncertainty under the assumptions (A1) and (A2).

Theorem 1: Select control parameters \( T(s), R(s), \) and \( S(s) \) as \( \delta \); then the perturbed system \( S_\delta(P + N, T, R, S) \) will keep \( L_2 \) stable if and only if

\[ \delta \|A_1(j\omega)L^{-1}(j\omega)S(j\omega)\|_2 = \delta \|A_1(j\omega)Y_1(j\omega) - K_1(j\omega)A(j\omega)\|_2 < 1 \] (5)

where \( K_1(s) \leq \frac{1}{\delta}S^{-1}(s)M(s) \in H[s] \).


(Only If): By contradiction, suppose (9) is false; we will construct a plant \( P \in S_\delta(P + N, T, R, S) \) which is not stabilized by the compensator.

Suppose \( \delta \|A_1(j\omega)L^{-1}(j\omega)S(j\omega)\|_2 \geq 1 \); then it is clear from (M.3) that

\[ \delta \|A_1L^{-1}S\|_2 \geq 1. \] (6)

Select unitary matrices \( U \) and \( V \) to make the following singular value
decomposition:

\[ A_1 L^{-1} S = U \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_r \end{bmatrix} V \]  

(11)

where \( \sigma_1 \geq \sigma_2 \cdots \geq \sigma_r \) are the singular values of \( A_1 L^{-1} S \).

In view of the reason that the value of \( \| A_1 L^{-1} S \|_r \) is the largest singular value of \( A_1 L^{-1} S \) and the fact that (M.4) holds, it is seen from (10) and (11) that

\[ \delta \sigma_i \geq \varepsilon. \]  

(12)

Now, the remaining proof left is to seek a permissible additive plant uncertainty (i.e., (3) still holds) which satisfies (12), but violates the stability of the perturbed system. Suppose we select

\[ N = -V^{-1} \begin{bmatrix} 1/\sigma_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} U^{-1} N_0 \]  

(13)

where \( N_0 \) is the nonlinear time-varying operator which lies within the conic sector \([-1, 1]\) and touches the line of slope +1 at \( t = t_0 \) (see Fig. 2).

Thus, with the selection of \( N \) in (13), for any test signal \( x(t) \in L^2_t \), the following manipulations hold in succession:

\[ \| N x(t) \|_2(1/\sigma_i) \| N_0 x(t) \|_2 \leq \| x(t) \|_2 \leq \delta \| x(t) \|_2 \]  

by (12)

and

\[ \| N(x, t) \|_2 = \| V^{-1} \begin{bmatrix} 1/\sigma_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} U^{-1} N_0 \|_2 = 0, \]

for \( t \in [0, \infty) \).

So, the selection of \( N \) in (13) satisfies (3) and (4) and is a permissible one.

Once the nonlinear time-varying operation \( N_0 x(t) \) touches the line of slope +1 at \( t = t_0 \), the operation \((I + A_1 L^{-1} S N)x(t)\) becomes

\[ (I + A_1 L^{-1} S N)x(t_0) = (I - U \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_r \end{bmatrix} V) \begin{bmatrix} 1/\sigma_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} U^{-1} N_0 x(t_0) \]

\[ = x(t_0) - U \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} U^{-1} x(t_0) \]

\[ = (U \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} U^{-1} x(t_0) \]

(since \( N_0 x(t_0) = x(t_0) \)).

It reflects the fact that \((I + A_1 L^{-1} S N)^{-1}\) does not exist at \( t = t_0 \). From (8), it is evident that the value of \( u(t_0) \) approaches infinity. So, the phenomenon of \( u(t_0) \) being infinite violates the stability of the system. This accomplishes the proof. Q.E.D.

With respect to the multiplicative case, we replace the true plant with \( P(I + N) \) and this perturbed system is named \( S_n(P(I + N), T, R, S) \). Proceeding with the same procedure as in the additive case, we introduce the following theorem and omit the details.

Theorem 2: Consider the system of \( S_n(P(I + N), T, R, S) \); suppose we choose control parameters \( T(x), R(s), \) and \( S(s) \) according to Lemma 1, and suppose for any test signal \( x(t) \in L^2_t \), \( N_0 x(t) \leq \delta \| x(t) \|_2 \). Then \( S_n(P(I + N), T, R, S) \) will keep \( L^2_t \) stable if and only if

\[ \delta \| I - A_1(jw) L^{-1} (jw)(T(jw) + R(jw)) \|_2 \]

\[ = \| I - A_1(jw)(X(jw) + K_0(jw)B(jw)) \|_2 < 1. \]  

(14)

III. CONCLUSION

Considering the observer-based compensating linear multivariable system, the main result of this note provides an effective manipulation to derive the necessary and sufficient conditions of robust stabilization for perturbed systems suffering additive or multiplicative nonlinear time-varying plant uncertainty.

REFERENCES


On the Robustness of Sampled-Data Control to Unmodeled High-Frequency Dynamics

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Abstract—In this paper, robustness of sampled-data control designs to unmodeled high-frequency dynamics is studied using singular perturbation theory. It is argued that when the plant is preceded by a zero-order hold, a direct transmission term of the reduced-order model, which results from neglecting high-frequency dynamics, should be modeled as a delay element in order to ensure robustness.

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