Constant Turning Force Adaptive Controller Design

In the Constant Turning Force Adaptive Control system, the cutting process is nonlinear time-varying; besides, the stability cannot be assured by classical control theory since the cutting tools usually cut a workpiece at various cutting depths. In this paper, based on the small gain theorem, we propose a new method to design a PI controller with high robustness to stabilize the force feedback control system against the nonlinear time-varying gain perturbation in the cutting process. A simple design procedure will be presented and several illustrative simulation results are given. The practical experimental results of a converted lathe with the PI controller designed with this method also show a good robustness and good reliability.

1 Introduction

The CNC machine tools cannot automatically work unless the cutting parameters are prescribed by the part-programmers. Consequently, the machining results such as machining time, surface roughness of the workpiece, etc., depend on a part-programmer's knowledge and experience.

A junior part-programmer usually prescribes conservative parameters to avoid the tool breakage and the machining time increases. Therefore, a new concept of adaptive control (AC) for machine tools is proposed to maintain the production rate independent of one's knowledge and experience.

To bring the machine tool to its appropriate cutting condition during the cutting process, the typical adaptive control for machine tools is classified into two styles [1-18]:

(a) Adaptive control with constraints (ACC): During the cutting process, the controller automatically regulates the cutting parameters to maintain the maximum working ability of a machine or tool, such as cutting force [2-9], spindle power [10], tool tip temperature [11], workpiece accuracy [12, 13] and machining stability [14, 15].

(b) Adaptive control with optimization (ACO): During the cutting process, the controller regulates the cutting parameters to maintain the optimized index of performance set by certain economic criteria subject to machining constraints [16-18].

In this paper, the force feedback control system can automatically maintain the cutting force by varying the feed rate.

With the increase of the cutting force owing to the large cutting depth and workpiece hardness, the heat generation [19] and the machining chatter [14, 15] will rise such that the workpiece quality is reduced. Because the force can be maintained by the force feedback controller, the heat generation and the cutting stability can be approximately maintained.

During the cutting process, to avoid the fracture of the cutting tool tip, the main cutting force density per unit length

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on the force feedback control system, it is necessary to find a convenient and reliable stability analysis method for the controller design. After treating the nonlinear time-varying perturbation as a sector, the robustness stability can be easily analyzed by small gain theorem [22, 23]. Based on it, the paper will offer a good rationale for selecting an acceptable gain space within which the simulated and actual system responses are relatively well damped.

2 Modelling

As pointed out in the previous investigations [3–7], the force feedback control system shown in Fig. 1 is composed of the controller, the servosystem, the feed dynamics, the cutting process, and the sensing element. This paper uses the PI controller, whose transfer function can be expressed as:

\[ U_c = \left( \frac{g}{s} + h \right) (F_r - F_d) \]  \hspace{1cm} (1)

We will propose a design procedure to find the suitable gain parameters, g and h, for this force feedback control system. The servosystem is simply constructed of a motor and an inertial load of the feed screw:

\[ V_f = \frac{K_m}{s^2 + 2\xi\omega_n s + \omega_n^2} U_r \]  \hspace{1cm} (2)

Because the feed is defined as the moving distance of a tool per revolution of the spindle, the dynamic relation between feed rate and feed can be obtained as follows:

\[ f(t) = \int_0^t V_f(z) dz - \int_0^{t-T} V_f(z) dz \]  \hspace{1cm} (3)

where \( z \) and \( t \) are the time variables inside and outside the integral operator respectively. \( T \) can be expressed as:

\[ T = \frac{60}{n} \hspace{1cm} (4) \]

The Laplace transform of equation (3) can be expressed as:

\[ f(s) = \frac{1 - e^{-st}}{s} V_f(s) \]  \hspace{1cm} (5)

It is difficult to analyze the dynamic equation with time delay term \( e^{-st} \). By the Pade approximant, \( e^{-st} \) can be approximated as [20]:

\[ e^{-st} = \frac{1 - (Ts/2)}{1 + (Ts/2)} \]  \hspace{1cm} (6)

Then equation (5) can be approximately expressed as:

\[ f(s) = \frac{T}{T + 1} V_f(s) \]  \hspace{1cm} (7)

Because of the robustness of the control system, the cutting force model need not be constructed in an exact form. Consequently, the cutting force formula is approximately expressed as the following by the least square error fitting [4–7]:

\[ F_c = aK_f f^{n_p} = (aK_f f^{n-1}) f \]  \hspace{1cm} (8)

A dynamometer with good response is equipped to detect the cutting force which is received through an A/D converter installed on the computer. The relation between \( F_c \) and \( F_d \) is expressed as:

\[ F_d = K_d F_c \]  \hspace{1cm} (9)

The command \( F_c \) of the system can be obtained subject to the machining ability of the tool or the lathe.

3 Cutting Force Model

Because the cutting process in the force feedback control system is a time-varying (owing to \( a \) and nonlinear (owing to

### Nomenclature

- \( a \) = depth of cut, mm
- \( b \) = maximum slope of the nonlinear time-varying sector, mm/rev
- \( d \) = minimum slope of the nonlinear time-varying sector, mm/rev
- \( f \) = feed of the cutting tool, mm/rev
- \( F \) = indicating element \( F \)
- \( F_c \) = cutting force, N
- \( F_r' \) = implicit cutting force \( F_c/K_t \), mm
- \( F_d \) = measured cutting force
- \( g \) = integral gain of the PI controller
- \( h \) = proportional gain of the PI controller
- \( K_d \) = conversion factor of the dynamometer, N\(^{-1}\)
- \( K_f \) = specific cutting force of the cutting process, N/mm\(^2\)
- \( K_t \) = constant of the servosystem, rad/\( \text{mm/sec} \)
- \( n \) = rotational speed of the spindle, rev/min
- \( N \) = nonlinear time-varying element of the force feedback control system, mm/rev
- \( N' \) = modified nonlinear time-varying element of the force feedback control system, mm/rev
- \( p \) = order of the feed of cutting force
- \( p_2, p_3, p_4 \) = coefficients of the characteristic equation
- \( q \) = imaginary element of the complex variable
- \( s \) = complex variable of the Laplace transform
- \( T \) = period of revolution of the spindle \( T = 60/n \), sec/rev
- \( U_i \) = input signal of the servosystem
- \( V_f \) = feed rate of the cutting tool, mm/sec
- \( w \) = imaginary element of the complex variable
- \( \xi \) = damping ratio of the servosystem
- \( \omega_n \) = natural frequency of the servosystem, rad/\( \text{sec} \)

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f^{p-1} \text{ plant, it is necessary to discuss the I/O relation, i.e., (feed)/(cutting force) relation, and I/O characteristic of the cutting process under control. Figure 2 illustrates the statistical assessment after practical cutting experiments when the cutting tool P10 (0, 5, 11, 18, 15, 0.8) cuts the S45C carbon steel workpiece with the spindle speed at 390 rpm, where symbols +, *, and x denote the real cutting force datum with respect to the cutting depth 3mm, 2mm, and 1mm, respectively. According to the results of the least square error fitting, } \kappa_f \text{ and } p \text{ can be obtained as 1773 N/mm}^2 \text{ and 0.83, respectively.}

To maintain the cutting force, the feed rate would be changed with respect to the current cutting depth. According to Fig. 2, the gain of the cutting process is of a high nonlinear time-varying value for a large cutting depth but a low value for a small cutting depth. The steady state of the cutting force is along a horizontal dotted line as shown in Fig. 2. The operation range } a^{p-1} \text{ may be obtained from the following:}

\[ F_c = (a^{p-1})\kappa_f f \]

where } a^{p-1} \in [0, 4.4] \text{ mm rev} \text{ (10)} \text{ i.e., } 0 \text{ mm rev} \leq a^{p-1} \leq 4.4 \text{ mm rev}. 4.4 \text{ mm rev} \text{ is obtained by } a = 3 \text{ mm and } f = 0.094 \text{ mm rev when } F_c = 750 N. 0 \text{ mm rev} \text{ is selected for convenient and conservative analysis. According to the definition of } b \text{ and } d \text{ in the section on dynamic model, it is obvious that } b = 4.4 \text{ mm rev and } d = 0 \text{ mm rev. The main design philosophy is to synthesize a robust PI controller according to the middle cutting duty } a^{p-1} = 2.2 \text{ mm rev} ((b + d)/2 = 2.2 \text{ mm rev}) \text{ such that the controller can tolerate the variation of cutting gain } a^{p-1} \text{ within the sector [0, 4.4] mm rev during the constant force control process. Because the ratio of the maximum transient cutting force to the steady state is proportional to the ratio of the changed cutting depth relative to the previous unchanged one, the changing ratio of the cutting depth should be limited to prevent the tool breakage during the control process. In this paper, triple times is selected to verify the system stability when the cutting force is maintained at 750N.}

4 Design Procedure

The system block diagram shown in Fig. 1 can be rearranged into an equivalent shown in Fig. 3, where } C(s) \text{ denotes the PI controller, and } P(s) \text{ denotes the combination of the linear servosystem and dynamics of the feed, i.e.,}

\[ P(s) = \frac{K_n T}{(s^2 + 2\xi \omega_n s + \omega_n^2)(T/2s + 1)} \]

\[ (11) \]

\[ Fig. 3 \text{ Simplified block diagram of force feedback control system} \]

\[ Fig. 4 \text{ Input/output relation of the nonlinear time-varying cutting process (a) original input/output relation (b) equivalent element of (a)} \]

\[ N \text{ denotes the gain of the nonlinear time-varying element, i.e.,} \]

\[ N = a^{p-1} \]

\[ (12) \]

which is illustrated in Fig. 4(a) with an operational range inside the sector } [d, b] \text{ centered at } (b + d)/2, \text{ i.e., } d \leq N \leq b. \text{ In the nonlinear time-varying system, the small gain theorem gives a sufficient condition to guarantee the stability under which a "bounded input" guarantees a "bounded output".}

Small Gain Theorem: For the feedback system in Fig. 5, if the linear elements } P'(s) \text{ and } F(s) \text{ are asymptotically stable and the nonlinear time-varying element } N' \text{ satisfies the inequality } |N'(f)| \leq k |f| \text{ where } k \text{ is a positive finite constant, then the closed loop system is bounded input and bounded output (BIBO) stable, in case}

\[ |kP'(j\omega)F(j\omega)| < 1 \quad \forall \omega \in [0, \infty) \]

\[ (13) \]

In order to employ the small gain theorem to treat our problem, the force feedback control system of Fig. 3 must be rearranged into an equivalent system shown in Fig. 5 by the method of Fig. 4, i.e.,

\[ F_c' = Nf = \left(\frac{b + d}{2} + N'\right)f = \frac{b + d}{2}f + y \]

\[ (14) \]

and } k = (b - d)/2. \text{ } P'(s) \text{ in Fig. 5 is expressed as:}

\[ P'(s) = \frac{P(s)C(s)}{1 + \left(\frac{b + d}{2}\right)P(s)C(s)F} \]

\[ (15) \]

Actually, } P'(s) \text{ can be considered as the nominal force feedback control system with the center gain } (b + d)/2. \text{ Suppose we choose } C(s) \text{ such that the nominal force feedback control system } P'(s) \text{ is asymptotically
stable, then, by the small gain theorem [22, 23], it has been shown that the equivalent system in Fig. 5 is stable if

$$P'(jw)F \frac{b-d}{2} < 1 \quad \forall w \in [0, \infty)$$

(16)

where $(b-d)/2$ denotes the boundary gain of $N'$, i.e., $-\{(b-d)/2\} \leq N' \leq (b-d)/2$. $b$ and $d$ should satisfy $0 \leq d \leq b \leq \infty$. When $b = d$, the system is a linear one whose stability can be obtained by the conventional stability criterion. When the system is perturbed $(b \neq d)$, the stability can be analyzed by equation (16). Because the nominal gain $(b+d)/2$ is always located at the center within the positive boundary $b$ and $d$, the limitation of $(b-d)/2$ must be $(b-d)/2 \leq (b+d)/2$. The stable region in gain space decreases when the boundary of the perturbed region $(b-d)/2$ increases.

The physical meaning of equation (16) is that while the force feedback control system is stabilized around the nominal center $(b+d)/2$, and the variance within the sector $[-(b-d)/2, (b-d)/2]$ is also attenuated around the loop, i.e., if the loop gain of the variation is less than 1, the force feedback control system with the nonlinear time-varying element is BIBO stable.

From the above analysis, our design work becomes how to choose two gain parameters $g$ and $h$ such that the nominal force feedback control system $P'(s)$ is asymptotically stable while the robust stability condition in equation (16) is also satisfied simultaneously.

Let us define the sensitivity function of the nominal force feedback control system as:

$$S(jw) \triangleq \frac{1}{1 + \frac{b+d}{2} P(jw)C(jw)F}$$

(17)

then the robust stability condition in equation (16) is reduced to

$$\frac{b-d}{2} (1 - S(jw)) < 1 \quad \forall w \in [0, \infty)$$

(18)

or equivalent to

$$\left(\frac{b-d}{b+d}\right)^2 (1 - S(jw))(1 - S(jw)) < 1 \quad \forall w \in [0, \infty)$$

(19)

where the symbol * denotes the conjugate of the complex term.

According to the above analysis, a design procedure is proposed as follows:

(a) According to the Routh-Hurwitz criterion [20], the region of $g$ and $h$ which guarantees that the poles of $P'(s)$ or $S(s)$ are all on the left side of the complex plane is determined first (see region I in Fig. 6).

(b) Inside the region of $g$-$h$ gain space determined in step (a), we find the sub-region which satisfies the requirement of robust stability set up in equation (16) or equation (19) (see region III in Fig. 6).

(c) Within the $g$-$h$ regions qualified by procedure (b), we obtain the better pair of $g$ and $h$ with the aid of computer simulation, which enables the force feedback control systems to have a desired output response.

5 Simulation and Experimental Results

In order to prevent the tool breakage owing to the transient variation of the cutting force, the reference cutting force $F_c = 3000$ is chosen, i.e., the cutting force sensed by dynamometer with $K_e = 4.0$ N⁻¹ tends to maintain at 750 N when levels of the square waveform cutting depth are changed between 1 mm and 3 mm as shown in Fig. 7. According to the previous description, $b = 4.4$ mm⁻¹ and $d = 0$ mm⁻¹ can be obtained.

$$n = 390 \text{ rpm}, K_f = 1773 \text{N/mm}^2 \text{and } \nu = 0.83 \text{ are the same as what was described before}. \quad K_f = 3136 \text{rad}^2 \text{mm}/\text{sec}^2, \xi = 0.39 \text{ and } \omega_n = 56 \text{ rad/sec} \text{ are chosen for the servosystem.}$$

To implement and verify the robustness of the force feedback control system in the practical turning process, a fast-processing 16-bits TCS-6000 micro-computer is connected with a motor driver and a dynamometer. The computer, employed as a controller, drives the cutting tool while the dynamometer serves to detect the cutting force.

To assure the asymptotical stability of $P'(s)$ by the Routh-Hurwitz criterion, the characteristic equation of the nominal force feedback control system can be written as:

$$P_s^4 + P_s^3 + P_s^2 + (2\omega_n^2 + K_e)P_s + K_e = 0$$

(20)

where

$$P_s = T$$

$$P_1 = 2 + 2T\xi\omega_n$$

$$P_2 = 4\xi^2\omega_n + T\omega_n^2$$

In order to let equation (20) be a Hurwitz polynomial, the boundary value of $h$ and $g$ can be obtained as follows:

$$P_s^2P_3 - P_s(2\omega_n^2 + K_e) > 0$$

(21)

i.e.,

$$\frac{P_s^2P_3 - 2\omega_n^2}{P_sK_e} > h \geq 0$$

(22)

Subject to equation (22), the boundary of $g$ is:

$$\frac{P_s^2\omega_n^2 + K_e}{P_sK_e} > g > 0$$

(23)
According to the above data and the inequality equation (22), \( h \) can be located to be in the region of \([0, 17 \times 10^{-4}]\), i.e., \( 0 \leq h \leq 17 \times 10^{-4} \). The region of \( g \) can be obtained by equation (23) when \( h \) is fixed at a certain value satisfying equation (22). The \( g-h \) gain space region I satisfying equations (22) and (23) is below the solid line in Fig. 6, within which the gain parameters \( g \) and \( h \) may assure the asymptotical stability of nominal system \( P'(s) \) with the nominal gain \( (b+d)/2 = 2.2 \text{mm}\cdot \text{rev} \).

Region III below the dotted line in Fig. 6 satisfies the robust stability criterion equation (16) or equation (19). It can ensure the stability of the nominal system \( P'(s) \) against the gain \( a \) variation within \([0, 4.4] \text{mm}\cdot \text{rev} \) (0mm < \( a \leq 3 \text{mm} \)).

When cutting depth is maintained at \( a = 3 \text{mm} \) (\( b-d = 4.4 \text{mm}\cdot \text{rev} \)), region II below the single dot-dash line in Fig. 6 can be obtained by Routh-Hurwitz criterion too. As shown in Fig. 8(a) \( (g = 0.9 \times 10^{-2}, h = 1.0 \times 10^{-4}) \), the cutting force responses of the PI controllers are capable of stability when the cutting depth is maintained at 3mm. However, as shown in Fig. 8(b), when the cutting depth changes from 1mm to 3mm, the responses of the system are oscillatory. Although the controller is designed by constant \( 4.4 \text{mm}\cdot \text{rev} \) (\( a = 3 \text{mm} \)), from the results in Fig. 8, the controller
Gain parameters near the boundary of Region III to be verified are shown as follows:
\[ g = 0.3 \times 10^{-2}, \quad h = 1.0 \times 10^{-4} \] shown in Fig. 9
\[ g = 0.8 \times 10^{-2}, \quad h = 5.0 \times 10^{-4} \] shown in Fig. 10

It is obvious that all of the responses are stable. The controller gains satisfying equation (16) or equation (19) show stabler system responses than they do in other regions in gain space.

The performance of the force feedback control system can be verified with the aid of the computer simulation to obtain the better pair of \( g \) and \( h \) in region III. Two gain parameters in region III with the same \( g \) portion are chosen:

Case 1: \[ g = 0.3 \times 10^{-2}, \quad h = 1.0 \times 10^{-4} \] shown in Fig. 11
Case 2: \[ g = 0.3 \times 10^{-2}, \quad h = 3.0 \times 10^{-4} \] shown in Fig. 12

Figures 11 and 12 show that the simulated responses are all stable. It is obvious that the overshoots in Fig. 12 are smaller than those in Fig. 11, i.e., the performance of the force response and feed response in Case 2 is better than the one in Case 1. Consequently, the gain parameters of Case 2 can be selected for implementation in the practical force feedback control system. Figure 13 illustrates the practical responses of Case 2, in which the responses are better than those in Fig. 9.

6 Conclusions

To maintain a constant turning force, a PI controller is designed to assure the stability of the force feedback control system. Some conclusions may be drawn as follows:

(a) Based on small gain theorem, this paper offers a good rationale for selecting an acceptable gain space within which the system responses are relatively well damped.

(b) The input/output relation between cutting parameters and main cutting force is analyzed in order to find out the input/output characteristics of a cutting process under control.

(c) The design procedure is to rearrange the force feedback control system into a nominal linearized one with a nonlinear time-varying perturbation. According to the Routh-Hurwitz criterion, the gain parameters, \( g \) and \( h \), can be obtained to stabilize the nominal linearized system. We test the loop gain.
for all frequencies to find the available $g$ and $h$ satisfying the robust stability criterion of equation (16) or equation (19), and then select a satisfactory gain pair by computer simulation.

(d) The experiments can highlight the stability of the PI controllers designed by using the maximum cutting depth and the small gain theorem. The experimental results show that the controller gains designed by using the latter show a stabler system response than other regions in gain space.

Further research will need to investigate the cutting efficiency of the force feedback control system, such as the improvement on tool life, metal removal rates, machining stability, etc. The cutting speed would be manipulated by these constraints. The robustness of the system with variable gain of $p^{-1}$ and $T$ will be discussed in the implementation of the above functions in the future.

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