Abstract: A new figure of merit (Poincaré angular error gradient) is proposed to optimize the noise immunity of polarimeter. The typical polarization state analyzer is modified to intuitively realize the optimized scheme.

I. INTRODUCTION

Measurement of the state of polarization (SOP) of electromagnetic waves is useful in remote sensing [1], astronomy [2], and high-speed fiber telecommunications [3]. It is known that SOP can be characterized by a 4-D gradient vector \( \nabla \theta_e \). Since the power measurement error can be described by the angle \( \theta_e \) between the measured Stokes vector \( \mathbf{S}' = (S'_1, S'_2, S'_3) \) and the actual one \( \mathbf{S} = (S_1, S_2, S_3) \) (assuming \( S_0 = 1 \)) [7]. The dependence of \( \theta_e \) on the perturbations \( \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \) can be quantitatively characterized by a 4-D gradient vector

\[
\nabla \theta_e = \begin{bmatrix} \partial \theta_e / \partial \epsilon_1, \partial \theta_e / \partial \epsilon_2, \partial \theta_e / \partial \epsilon_3, \partial \theta_e / \partial \epsilon_4 \end{bmatrix}.
\]

Since \( \nabla \theta_e \) depends on the test SOP, the chosen polarization components, or the bias error vector \( \epsilon \), our goal is to find a set of four polarization components \( \{J_{1,2,3,4}\} \) corresponding to the lowest ensemble average \( E \) of \( \| \nabla \theta_e (\epsilon) \|_2 \)

\[
E = \sum_{n=1}^{N} \| \nabla \theta_e \|_2 \times w_n,
\]

where \( w_n \) is the relative solid angle of the \( n \)th possible SOP on the Poincaré sphere.

II. THEORY AND EXPERIMENT

Consider an incident light whose SOP can be represented by a Jones vector \( \mathbf{J} = [a_x, a_y e^{i \phi}]^T \), a Stokes vector \( \mathbf{S} = [S_0, S_1, S_2, S_3]^T \), or \( \mathbf{s} = [s_0, s_x, s_y, s_z]^T \), which are related by

\[
S_0 = (a_x)^2 + (a_y)^2, \quad S_1 = (a_x)^2 - (a_y)^2, \quad S_2 = 2 a_x a_y \cos \phi, \quad S_3 = 2 a_x a_y \sin \phi.
\]

The power measurement vector, \( \mathbf{p} = [p_1, p_2, p_3] \), is related to the test SOP via \( \mathbf{p} = \mathbf{W} \times \mathbf{s} \), where \( \mathbf{W} \) is a 4x4 matrix determined by the PSA. As a result, the vector \( \mathbf{s} \) (thus SOP) can be calculated by \( \mathbf{s} = \mathbf{W}^{-1} \times \mathbf{p} \) as long as \( \mathbf{W}^{-1} \) exists. In the presence of perturbation, the four measured powers are modeled by \( p^\epsilon = p + \mathbf{e} \) (the four elements of \( \mathbf{e} \), \( \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \) are normalized to the total power \( S_0 )\). The SOP measurement error can be described by the angle \( \theta_e \) between the measured SOP of the test SOP into the polarization such that a linear polarizer can sample the corresponding powers [7].

PSA using single rotatable retarder still works, but can only sample a subset of polarization components [5]. As a result, it takes longer to calculate the SOP (matrix inversion is required) and the noise immunity optimization is subject to a tighter boundary condition. Optimization of the orientation angles and retardance of the single retarder in an RRFP polarimeter against random noise was carried out by the Monte Carlo algorithm to minimize the ensemble average of \( \| \nabla \theta_e \|_2 \). To measure arbitrary polarization components, we replace one QWP in the 2RRFP by a half waveplate (HWP). Our experiments confirmed the feasibility of \( \nabla \theta_e \) and the improvement of the new 2RRFP configuration.

Figure 1(a) shows the distribution of gradient magnitude \( \| \nabla \theta_e \|_2 \) due to the standard 2RRFP-based PSA using \( \{J_1 = [1,0]^T, J_2 = [0,1]^T, J_3 = 2^{0.5} [1,1]^T, J_4 = 2^{-0.5} [1,j]^T\} \), corresponding to an ensemble average of \( E_{\text{m}} = 182.070^\circ \). This means a signal-to-noise ratio (SNR) of \( 10^2 (\epsilon_1 \sim 10^{-2}) \) may incur a Poincaré angular error of \( \theta_e \sim 1.82^\circ \). By using...
Monte Carlo algorithm, we got a new set of four polarization components 
\[
J_1 = \begin{bmatrix} 0.89 \\ 0.45 \end{bmatrix}, J_2 = \begin{bmatrix} 0.20 \\ 0.98 \end{bmatrix}, J_3 = \begin{bmatrix} 0.79 \\ 0.62 \end{bmatrix}, J_4 = \begin{bmatrix} 0.81 \\ 0.59 \end{bmatrix}
\]
leading to a reduced ensemble average \(E=142.894^\circ\) (EWV=10.3). The corresponding gradient magnitude distribution [Fig. 1(b)] is lower than that of the standard 2RRFP except for a nominal fraction (~2%) of all possible SOPs [Fig. 1(c), blue area]. Surprisingly, the four EWV-optimized polarization components proposed in [5] correspond to a slightly lower ensemble average \(E=140.45^\circ\) (EWV=10.0) under our definition [Fig. 1(d)]. No better result can be found by using the EWV-optimized solution as the initial condition to our Monte Carlo codes. This implies a complicated potential surface of \(E\) that is prone to trapping the optimization process by local minima.

The experimental setup is shown in Fig. 2. The light source is a distributed feedback laser operated at 1550 nm. The linear polarizer (LP) and the first QWP (QWP1) are utilized to generate test SOPs, while the remaining components are responsible of SOP measurement. A half waveplate (HWP) replaces the second QWP in a standard 2RRFP PSA such that the four “optimized” polarization components \(J_{1,2,3,4}\) can be measured by proper orientation angles \(\theta_i\) of QWP2 and HWP (Table 1).

![Fig. 2. Experimental setup. LP: Linear polarizer. Q(H)WP: λ/4(λ/2) waveplate. PBS: Polarization beam splitter. PD: Photodetector.](image)

<table>
<thead>
<tr>
<th>(\theta_i) (QWP)</th>
<th>(\theta_i) (HWP)</th>
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<tbody>
<tr>
<td>26.9°</td>
<td>-11.5°</td>
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<tr>
<td>-25.4°</td>
<td>-34.2°</td>
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<tr>
<td>15.0°</td>
<td>38.2°</td>
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<tr>
<td>150.4°</td>
<td>150.4°</td>
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<tr>
<td>170.7°</td>
<td>170.7°</td>
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We experimentally suppressed the impact of deterministic perturbations by using the calibration procedure [4,7], such that \(\mathbf{g}\) can be modeled by a vector of zero-mean random variables. The Poincaré angular error predicted by an inner product \(\theta_i = (V \theta_j) \mathbf{g}\), however, is not zero-mean (for \(\theta_i > 0\)). For simplicity, we estimate the mean of Poincaré angular error by using a constant vector \(\kappa \mathbf{g}\) in place of \(\mathbf{g}\), where \(\kappa\) is a constant to be determined and \(\sigma_{1,2,3,4}\) are the (normalized) standard deviations of the four measured powers \(p_{1,2,3,4}\). In Fig. 3(a), 48 data points corresponding to 9 different SOPs [including 0°, 45°, 90°-linear polarizations, right-hand circular (RHC) polarization, and 5 different elliptical polarizations] and different noise powers \(\sigma\) (ranging from 4×10^{-4} to 4.5×10^{-2} of the total power) are presented. The correlation coefficient between the predicted and experimentally measured angular errors, i.e. \(\theta_i / \kappa\) and \(\theta_i\), is 0.935. The fit line in Fig. 3(a) indicates a quasi-linear relation \(\theta_i = 0.353 \times (\theta_i / \kappa)^{0.957}\), suggesting a scaling constant of \(\kappa = 0.353\). Fig. 3(b) shows that the optimized 2RRFP configuration gives lower angular error compare with the standard 2RRFP counterpart when measuring 9 different SOPs under different noise powers (20 data points).

![Fig. 3. Relation between the Poincaré angular errors obtained by (a) prediction and experiment, (b) experiments using optimized and standard 2RRFP configurations, respectively.](image)

### III. CONCLUSIONS

A new figure of merit, Poincaré angular error gradient, is proposed to optimize the noise immunity of polarimeter. The standard 2RRFP polarization state analyzer is modified by replacing one QWP by a HWP to realize the optimized scheme. Experiments confirmed that Poincaré angular error gradient can quantitatively predict the SOP measurement error, and the optimized scheme does outperform the standard one.

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### REFERENCES