Homework Problem Set #5

(Due by 2008/05/12)

This problem set covers the content of Lessons 9−10 or EK 12−13.

1) (10%) For $-1<p<1$, prove that:

$$\sum_{n=0}^{\infty} p^n (\cos nx) = \frac{1-p\cos x}{1-2p\cos x+p^2}, \quad \sum_{n=0}^{\infty} p^n (\sin nx) = \frac{p\sin x}{1-2p\cos x+p^2}.$$

2) (5%) Problem **13.2.29**. Are the two roots a complex conjugate pair as in polynomial equations of real coefficients?

3) (10%) Problem **13.3.26(e–f)**.

4) (10%) Problem **13.4.11**.

5) Two curves C_1: $z_1(t)=1+it$, C_2: $z_2(t)=t+it$ in the z-plane are mapped by $f(z)=e^z$ onto C_1^*, C_2^* in the w-plane.

5a) (5%) Plot C_1, C_2, and evaluate the included angle θ between them at $z=1+i$.

5b) (5%) Derive the mathematical forms of C_1^*, C_2^*, and plot them in the w-plane.

5c) (10%) Evaluate the included angle ϕ between C_1^*, C_2^* at $w=e^{1+i}$.Whether $\phi=\theta$?

6) (10%) Problem **13.7.30(a)**.

7) Evaluate $\int_C z^* dz$ (z^* means complex conjugate), if contour C is:

7a) (5%) A unit circle centered at $z=0$ in counterclockwise sense;

7b) (10%) A wedge shown in Fig. 1.

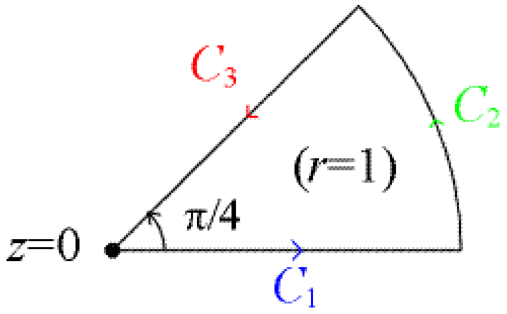

Fig. 1

8a) (5%) Evaluate $I=\int_{-i}^{+i} z^{-1}dz$ by eq. (10.3) if the integral path C is a unit circular arc in counterclockwise sense (Fig. 2).

8b) (5%) Can you still apply antiderivative approach [eq. (10.5)] to evaluate I? Justify your answer.

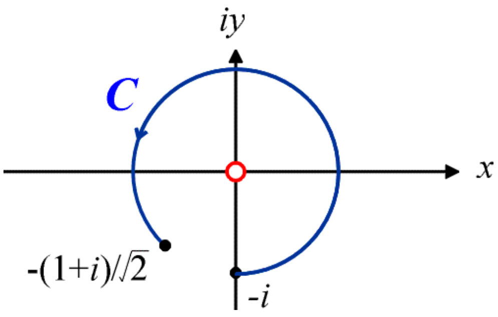

Fig. 2

9a) (5%) Evaluate $I=\oint_C \frac{e^z}{z(z+1)}dz$, where C: $|z-1|=3$.

9b) (5%) Evaluate $I=\oint_C \frac{5z^2-3z+2}{(z-1)^3}dz$, where C: $|z-1|=3$.

(Hint: Using Cauchy's integral formulas.)