Homework Problem Set #4

(Due by 2008/04/14)

This problem set covers the content of Lessons 7 or part of EK 12.5 and EK 12.10. The total score is **50** points.

1) (10%) Problem **12.5.34**.

2) (10%) In performing separation of variables to solve Laplace's equation in polar coordinates, we claim that $\frac{r^2R''+rR'}{R}=-\frac{\ddot{\Theta}}{\Theta}=\lambda$ ($\lambda=k^2$ in P7-5 of lecture notes). Can you justify why the eigenvalue λ cannot be negative?

3) (10%) Problem **12.10.9**. Besides, what is the solution if the region of interest is either $r<R$ (interior problem) or $r>R$ (exterior problem)?

4) (10%) Problem **12.10.6**. Besides, what is the solution if the region of interest is either $r<R$ (interior problem) or $r>R$ (exterior problem)? What have you found by comparing the difference between problem 2 and 3?

5) (10%) Solve the problem:

PDE: $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$, where ROI = $\{1<r<2,\ 0<\theta<\frac{\pi}{2}\}$.

BCs: $u(1,\theta)=\sin(2\theta)$, $u(2,\theta)=0$, $u(r,0)=0$, $u\left(r,\frac{\pi}{2}\right)=0$.

Hint: Modify the eigenfunction of $\Theta(\theta)$.