Homework Problem Set #2

(Due by 2008/03/24)

This problem set covers the content of Lessons 3−4 or EK 12.5-12.6.

1) (10%) Problem **12.5.20**.

2) (15%) Solve the initial-value problem:

PDE: $u_t = \alpha^2 u_{xx}$, $\{-\infty<x<\infty,\ t>0\}$

IC: $u(x,0)=e^{-(x/L)^2}$

3a) (20%) Solve the initial-boundary-value problem:

PDE: $u_t = u_{xx}$, $\{0<x<1,\ t>0\}$

BCs: $u(0,t)=1$, $u_x(1,t) + hu(1,t)=1$

IC: $u(x,0)=1-x$

(Hint: transform the BCs into homogeneous ones)

3b) (10%) Please give an intuitive interpretation for your results when $h=10^{-3}$ and $h=10^{3}$, respectively.

4) Consider the initial-boundary-value problem:

PDE: $u_t = u_{xx}$, $\{0<x<1,\ t>0\}$

BCs: $u(0,t)=g_1(t)$, $u_x(1,t) + hu(1,t)= g_2(t)$

IC: $u(x,0)=\phi(x)$

Since BCs are time-varying, so is the steady-state solution: $S(x,t)\equiv u(x,t\rightarrow\infty)$. To let $S(x,t)$ satisfy BCs of $u(x,t)$, $S(x,t)=A(t)(1-x)+B(t)x$.

4a) (5%) Find $A(t)$, $B(t)$.

4b) (10%) The total solution $u(x,t) \equiv S(x,t) + U(x,t)$. Find the PDE, BCs, IC for the transient solution $U(x,t)$.

4c) (5%) Can $U(x,t)$ be solved by separation of variables? Why?

5) (15%) Problem **12.5.24**.

6) (10%) Problem **12.5.27**.