6.2

(a) $F_s = 1/T \geq 2B \Rightarrow A = T, F_c = B.$

(b) $X_a(F) = 0$ for $|F| \geq 3B$. $F_s = 1/T \geq 6B \Rightarrow A = T, F_c = 3B.$

(c) $X_a(F) = 0$ for $|F| \geq 5B$. $F_s = 1/T \geq 10B \Rightarrow A = T, F_c = 5B.$

6.3

(a)

$$\begin{aligned} x(n) = x_a(nT) &= nTe^{-nT}u_a(nT) \\ &= nTa^n u_a(nT) \end{aligned}$$

where $a = e^{-T}$.
Define $x_1(n) = a^n u_a(n)$. The Fourier transform of $x_1(n)$ is

$$\begin{aligned} X_1(F) &= \sum_{n=0}^{\infty} a^n e^{-j2\pi F n} \\ &= \frac{1}{1 - ae^{-j2\pi F}} \end{aligned}$$

Using the differentiation in frequency domain property of the Fourier transform

$$\begin{aligned} X(F) &= Tj\frac{X_1(F)}{dF} \\ &= \frac{Tae^{-j2\pi F}}{(1 - ae^{-j2\pi F})^2} \\ &= \frac{T}{e^{(T+j2\pi F)} + e^{-(T+j2\pi F)} - 2} \end{aligned}$$

(b) The Fourier transform of $x_a(t)$ is

$$X_a(F) = \frac{1}{(1 + j2\pi F)^2}$$

Fig. 6.4-1(a) shows the original signal $x_a(t)$ and its spectrum $X_a(F)$. Sampled signal $x(n)$ and its spectrum $X(F)$ are shown for $F_s = 3$ Hz and $F_s = 1$ Hz in Fig. 6.4-1(b) and Fig. 6.4-1(c), respectively.

(c) Fig. 6.4-2 illustrates the reconstructed sugnal $\hat{x}_a(t)$ and its spectrum for $F_s = 3$ Hz and $F_s = 1$ Hz.

$$\hat{x}_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT)\frac{sin\,(\pi(t - nT)/T)}{\pi(t - nT)/T}$$

6.6

1.

$$g_{SH}(n) = \begin{cases} 1, & 0 \leq n \leq I \\ 0, & otherwise \end{cases}$$

2.

$$\begin{aligned} G_{SH}(w) &= \sum_{n=-\infty}^{\infty} g_{SH}(n) e^{-jwn} \\ &= \sum_{n=0}^{I} e^{-jwn} \\ &= e^{-jw(I-1)/2} \frac{\sin[wI/2]}{\sin(w/2)} \end{aligned}$$

3. The linear interpolator is defined as

$$g_{lin}[n] = \begin{cases} 1 - |n|/I, & |n| \leq I \\ 0, & otherwise \end{cases}$$

Taking the Fourier transform, we obtain

$$G_{lin}(w) = \frac{1}{I} \left[\frac{\sin(wI/2)}{\sin(w/2)} \right]^2$$

Fig. 6.8-1 shows magnitude and phase responses of the ideal interpolator (dashed-dotted line), the linear interpolator (dashed line), and the sample-and-hold interpolator (solid line).

6.7

Since $\frac{F_c+\frac{B}{2}}{B} = \frac{50+10}{20} = 3$ is an integer, then $F_s = 2B = 40Hz$

6.9

$$
\begin{aligned}
s_a(t) &= x_a(t) + \alpha x_a(t-\tau), \qquad |\alpha| < 1 \\
s_a(n) &= x_a(n) + \alpha x_a(n - \frac{\tau}{T}) \\
\frac{S_a(w)}{X_a(w)} &= 1 + \alpha e^{-j\frac{\tau w}{T}}
\end{aligned}
$$

If $\frac{\tau}{T}$ is an integer, then we may select

$$
H(z) = \frac{1}{1 - \alpha z^{-2}} \text{ where } \frac{\tau}{T} = L
$$

6.10

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x^2(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(w)|^2 dw \\
X(w) &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(\frac{w-2\pi k}{T}\right) \\
&= \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(\frac{w}{T}\right), \qquad |w| \le \pi \\
\sum_{n=-\infty}^{\infty} x^2(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{T^2}|X_a(\frac{w}{T})|^2 dw \\
&= \frac{1}{2\pi T^2}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} |X_a(\lambda)|^2 T d\lambda \\
&= \frac{1}{2\pi T}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} |X_a(\lambda)|^2 d\lambda \\
\text{Also, } E_a &= \int_{-\infty}^{\infty} x_a^2(t) dt \\
&= \int_{-\infty}^{\infty} |X_a(f)|^2 df \\
&= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |X_a(f)|^2 df \\
\text{Therefore, } \sum_{n=-\infty}^{\infty} x^2(n) &= \frac{E_a}{T}
\end{aligned}$$

6.11

(a)

$$\begin{aligned}
d(n) &= x(n) - ax(n-1) \\
E[d(n)] &= E[x(n)] - aE[x(n-1)] = 0 \\
E[d^2(n)] \equiv \sigma_d^2 &= E\left\{[x(n) - ax(n-1)]^2\right\} \\
&= \sigma_x^2 + a^2\sigma_x^2 - 2aE[x(n)x(n-1)] \\
&= \sigma_x^2 + a^2\sigma_x^2 - 2a\gamma_x(1) \\
&= \sigma_x^2(1 + a^2 - 2a\rho_x(1)) \\
\text{where } \rho_x(1) &= \frac{\gamma_x(1)}{\sigma_x^2} \\
&\equiv \frac{\gamma_x(1)}{\gamma_x(0)}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{d}{da}\left[\sigma_x^2(1 + a^2 - 2a\rho_x(1))\right] &= 2a - 2\rho_x(1) = 0 \\
a &= \rho_x(1)
\end{aligned}$$

For this value of α we have

$$\begin{aligned}
\sigma_d^2 &= \sigma_x^2[1 + \rho_x^2(1) - 2\rho_x^2(1)] \\
&= \sigma_x^2[1 - \rho_x^2(1)]
\end{aligned}$$

(c) $\sigma_d^2 < \sigma_x^2$ is always true if $|\rho_x(1)| > 0$. Note also that $|\rho_x(1)| \le 1$.

(d)

$$\begin{aligned}
d(n) &= x(n) - a_1x(n-1) - a_2x(n-2) \\
E[d^2(n)] &= E\left\{[x(n) - a_1x(n-1) - a_2x(n-2)]^2\right\} \\
\sigma_d^2 &= \sigma_x^2(1 + a_1^2 + a_2^2 + 2a_1(a_2 - 1)\rho_x(1) - 2a_2\rho_x(2)) \\
\frac{d}{da_1}\sigma_d^2 &= 0 \\
\Rightarrow a_1 &= \frac{\rho_x(1)[1 - \rho_x(2)]}{1 - \rho_x^2(1)} \\
\frac{d}{da_2}\sigma_d^2 &= 0 \\
\Rightarrow a_2 &= \frac{\rho_x(2) - \rho_x^2(1)}{1 - \rho_x^2(1)} \\
\text{Then, } \sigma_{d\,min}^2 &= \frac{1 - 3\rho_x^2(1) - \rho_x^2(2) + 2\rho_x^2(1)\rho_x(2) + 2\rho_x^4(1) + \rho_x^2(1)\rho_x^2(2) - 2\rho_x^4(1)\rho_x(2)}{[1 - \rho_x^2(1)]^2}
\end{aligned}$$

6.13

(a)

$$\begin{aligned}
S_e(F) &= \frac{\sigma_e^2}{F_s} \\
|H_n(F)| &= 2|sin\frac{\pi F}{F_s}| \\
\sigma_n^2 &= \int_{-B}^{B} |H_n(F)|^2 S_e(F) dF \\
&= 2\int_0^B 4sin^2(\frac{\pi F}{F_s})\frac{\sigma_e^2}{F_s} dF \\
&= \frac{4\sigma_e^2}{F_s}\int_0^B (1 - cos\frac{2\pi F}{F_s})dF \\
&= \frac{4\sigma_e^2}{F_s}[B - \frac{F_s}{2\pi}sin\frac{2\pi B}{F_s}] \\
&= \frac{2\sigma_e^2}{\pi}[\frac{2\pi B}{F_s} - sin\frac{2\pi B}{F_s}]
\end{aligned}$$

(b)

$$\begin{aligned}
\text{For } \frac{2\pi B}{F_s} &<< 1, \\
sin\frac{2\pi B}{F_s} &\approx \frac{2\pi B}{F_s} - \frac{1}{6}(\frac{2\pi B}{F_s})^3 \\
\text{Therefore, } \sigma_n^2 &= \frac{2\sigma_e^2}{\pi}[\frac{2\pi B}{F_s} - \frac{2\pi B}{F_s} - \frac{1}{6}(\frac{2\pi B}{F_s})^3] \\
&= \frac{1}{3}\pi^2\sigma_e^2(\frac{2B}{F_s})^3
\end{aligned}$$

6.14

(a)

$$\begin{aligned}
\{[X(z) - D_q(z)]\frac{1}{1-z^{-1}} - D_q(z)\}\frac{z^{-1}}{1-z^{-1}} &= D_q(z) - E(z) \\
D_q(z) &= z^{-1}X(z) + (1-z^{-1})^2E(z) \\
\text{Therefore, } H_s(z) &= z^{-1} \\
\text{and } H_n(z) &= (1-z^{-1})^2
\end{aligned}$$

(b)

$$\begin{aligned}
|H_n(F)| &= 4sin^2(\frac{\pi F}{F_s}) \\
&= 2(1 - cos(\frac{2\pi F}{F_s}))
\end{aligned}$$

(c)

$$\begin{aligned}
\sigma_n^2 &= \int_{-B}^{B} |H_n(F)|^2 \frac{\sigma_e^2}{F_s} dF \\
&\approx 2\int_0^B [4(\frac{\pi F}{F_s})^2]^2 \frac{\sigma_e^2}{F_s} dF \\
&= \frac{32\pi^4\sigma_e^2}{F_s^5}\int_0^B F^4 dF \\
&= \frac{1}{5}\pi^4\sigma_e^2(\frac{2B}{F_s})^5
\end{aligned}$$

6.16

(a) Refer to fig 6.23-1.

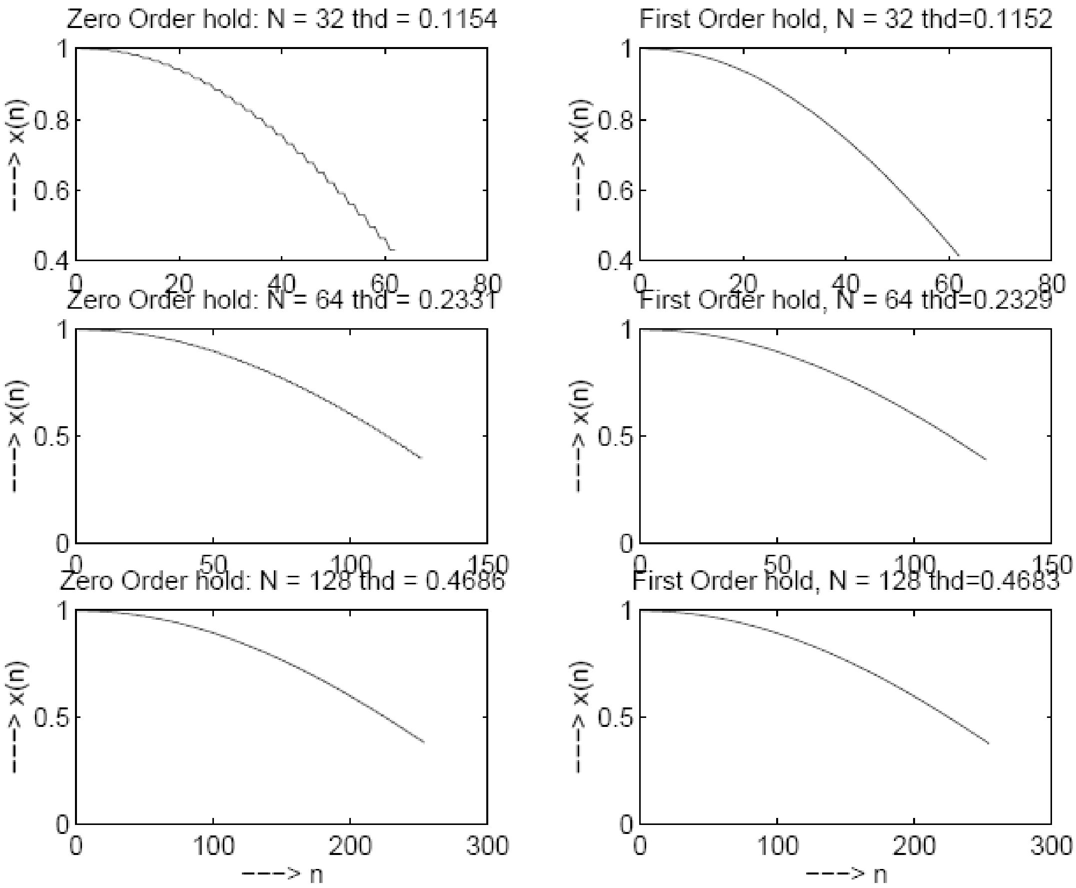

Figure 6.23-1:

(b) Refer to fig 6.23-2.

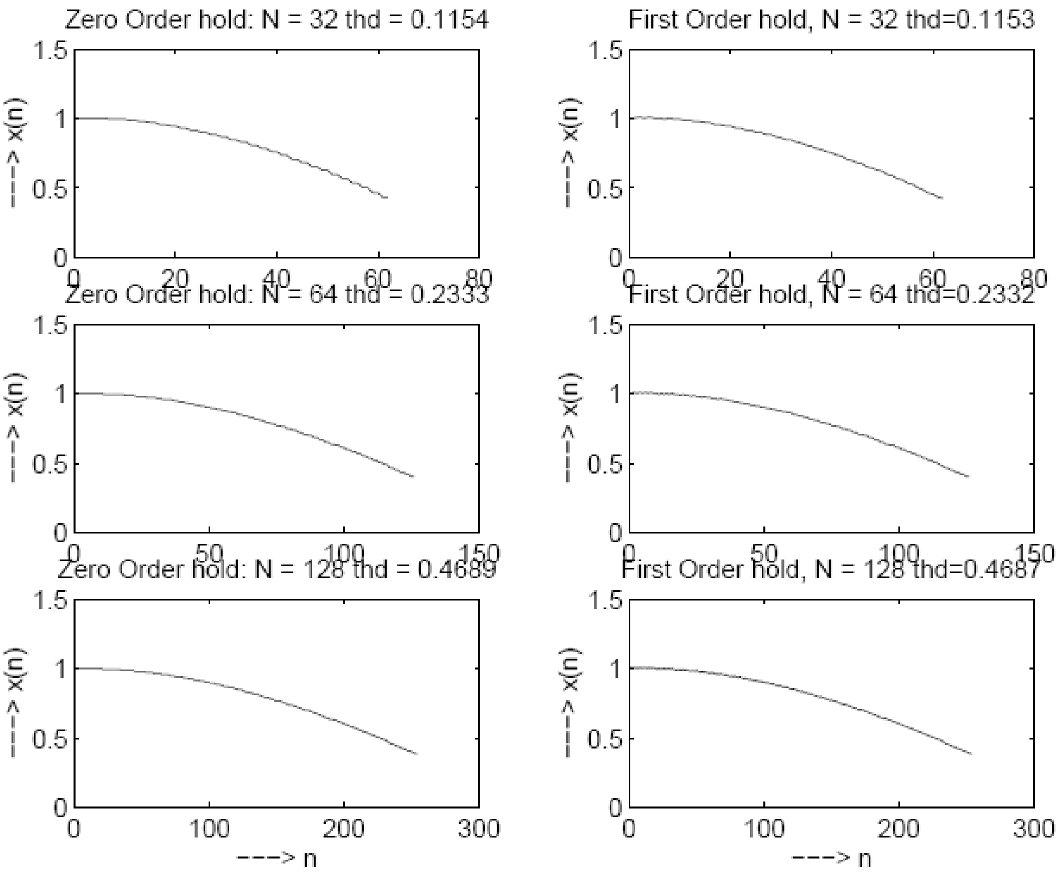

Figure 6.23-2:

(c) Refer to fig 6.23-3. The first order hold interpolator performs better than the zero order interpolator because the frequency response of the first order hold is more closer to the ideal interpolator than that of the zero order hold case.

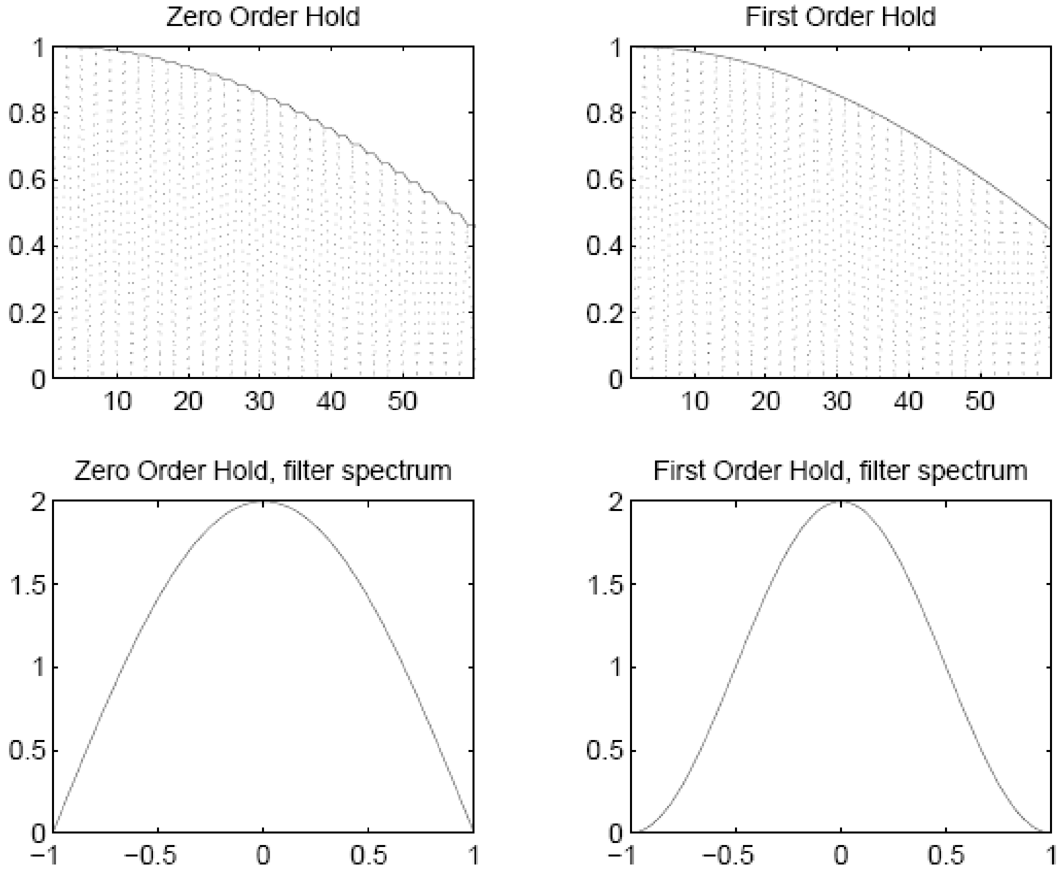

Figure 6.23-3:

(d) Refer to fig 6.23-4.

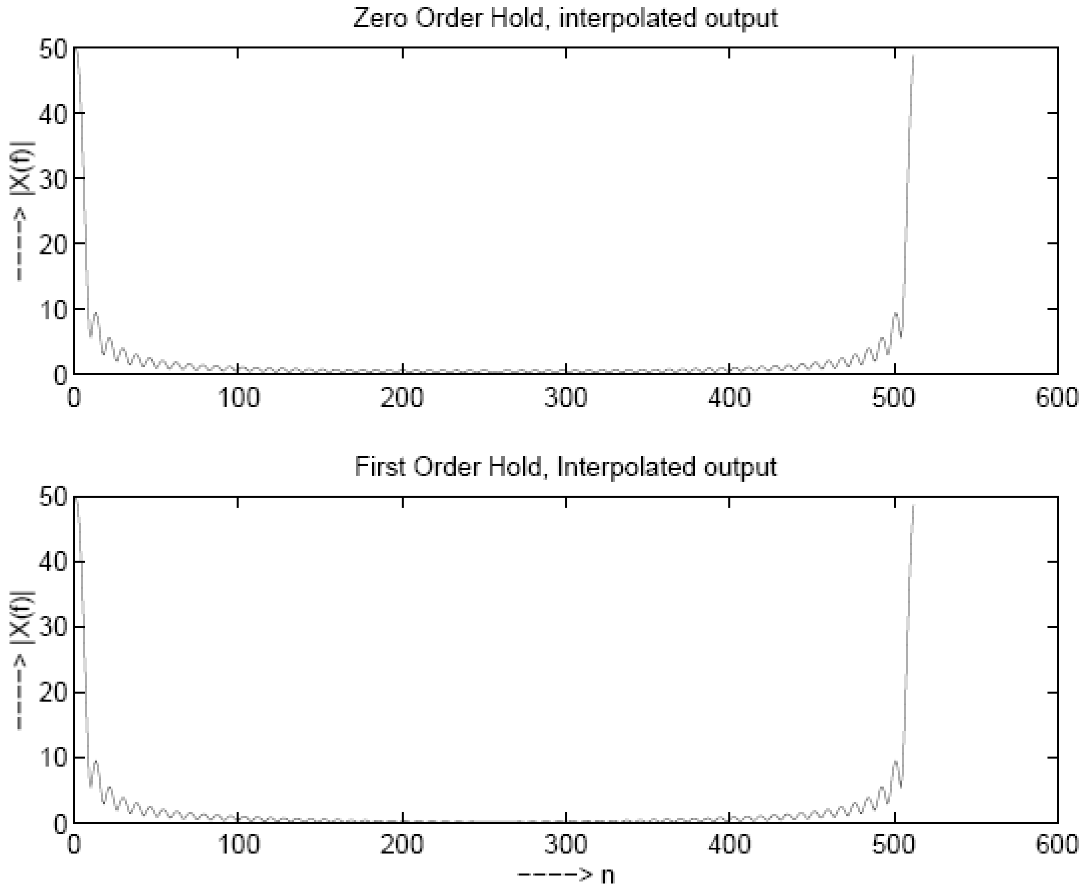

Figure 6.23-4:

(e) Refer to fig 6.23-5. Higher order interpolators with more memory or cubic spline interpolators would be a better choice.

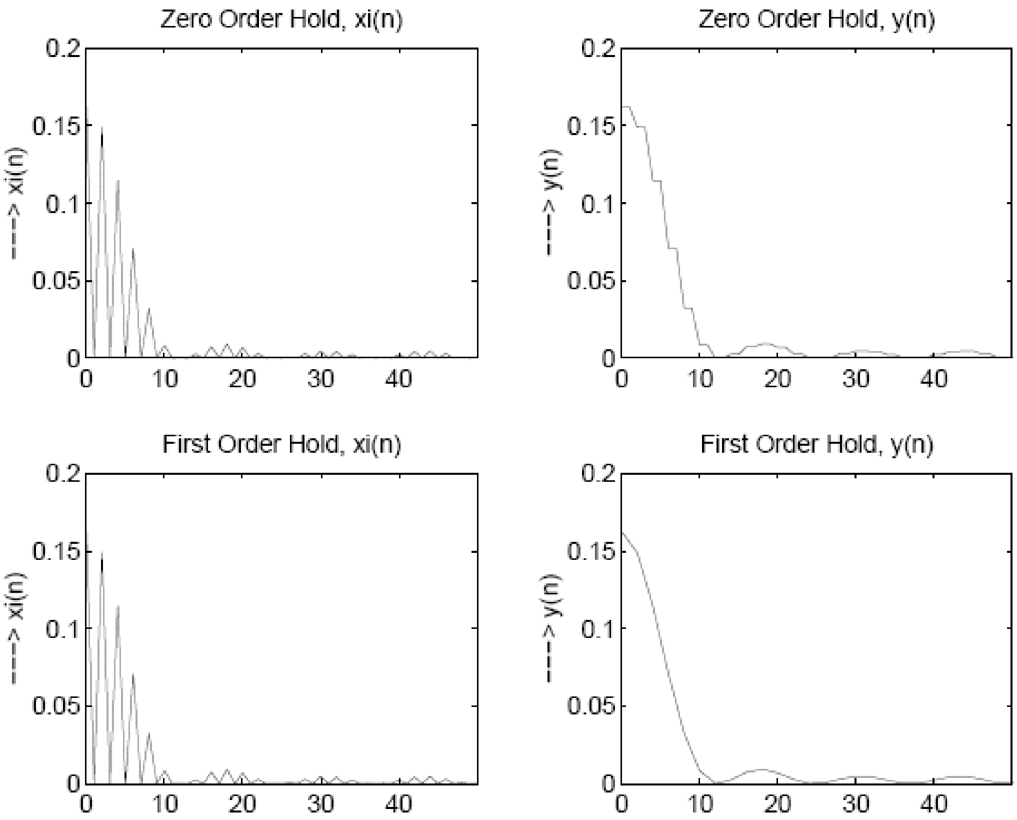

Figure 6.23-5: