5.2

(a)

$$\begin{aligned} W_R(w) &= \sum_{n=0}^{M} w_R(n)e^{-jwn} \\ &= \sum_{n=0}^{M} e^{-jwn} \\ &= \frac{1-e^{-j(M+1)w}}{1-e^{-jw}} \\ &= e^{-jMw/2}\frac{sin(\frac{M+1}{2})w}{sin\frac{w}{2}} \end{aligned}$$

(b) Let $w_T(n) = h_R(n) * h_R(n-1)$,

$$h_R(n) = \begin{cases} 1, & 0 \le n \le \frac{M}{2} - 1 \\ 0, & \text{otherwise} \end{cases}$$

Hence,

$$\begin{aligned} W_T(w) &= H_R^2(w)e^{-jw} \\ &= \left(\frac{sin\frac{M}{4}w}{sin\frac{w}{2}}\right)^2 e^{-jwM/2} \end{aligned}$$

(c)

$$\begin{aligned}
\text{Let } c(n) &= \frac{1}{2}(1 + cos\frac{2\pi n}{M}) \\
\text{Then, } C(w) &= \pi\left[\delta(w) + \frac{1}{2}\delta(w - \frac{2\pi}{M}) + \frac{1}{2}\delta(w + \frac{2\pi}{M})\right] \qquad -\pi \le w \le \pi \\
W_c(w) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} c(\Theta)W_R(w-\Theta)d\Theta \\
&= \frac{1}{2}W_R(w) + \frac{1}{2}W_R(w - \frac{2\pi}{M}) + \frac{1}{2}W_R(w + \frac{2\pi}{M})
\end{aligned}$$

Refer to fig 5.2-1

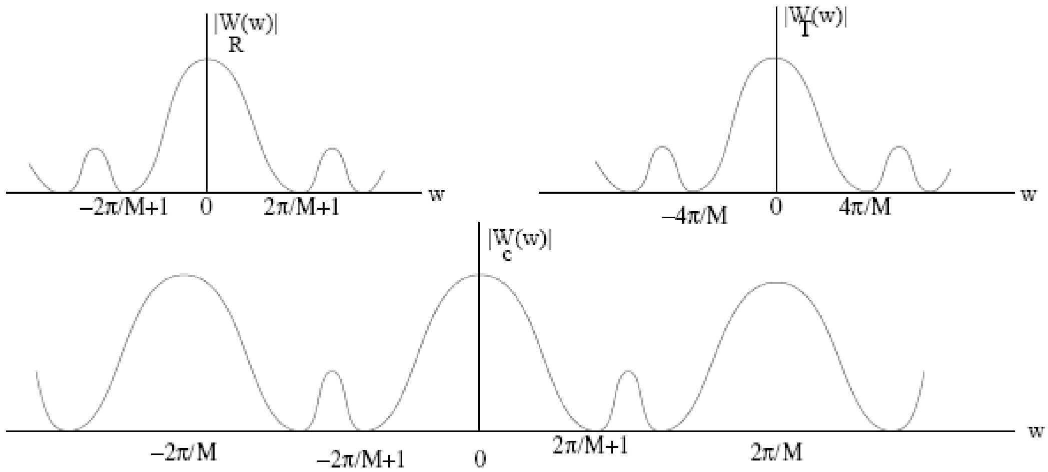

Figure 5.2-1:

5.5

(a)

$$\begin{aligned} y(n) &= x(n) + x(n-4) \\ Y(w) &= (1 + e^{-j4w})X(w) \\ H(w) &= (2cos2w)e^{-j2w} \end{aligned}$$

Refer to fig 5.7-1.

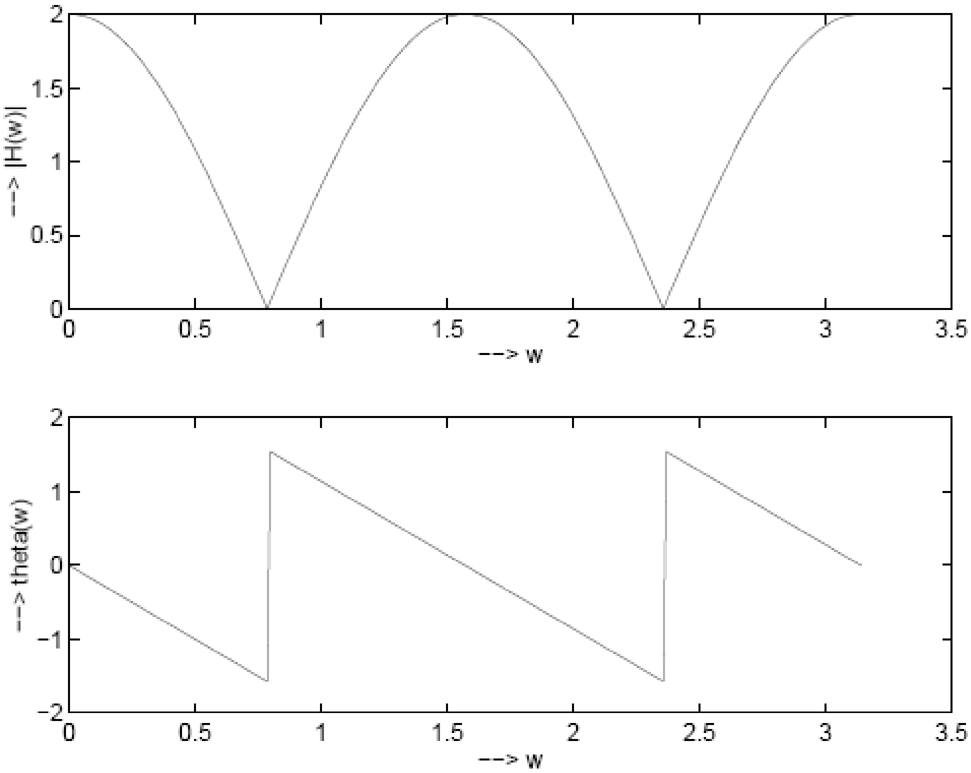

Figure 5.7-1:

(b)

$$\begin{aligned} y(n) &= cos\frac{\pi}{2}n + cos\frac{\pi}{4}n + cos\frac{\pi}{2}(n-4) + cos\frac{\pi}{4}(n-4) \\ \text{But } cos\frac{\pi}{2}(n-4) &= cos\frac{\pi}{2}ncos2\pi + sin\frac{\pi}{2}nsin2\pi \\ &= cos\frac{\pi}{2}n \\ \text{and } cos\frac{\pi}{4}(n-4) &= cos\frac{\pi}{4}ncos\pi - sin\frac{\pi}{4}nsin\pi \\ &= -cos\frac{\pi}{4}n \\ \text{Therefore, } y(n) &= 2cos\frac{\pi}{2}n \end{aligned}$$

(c) Note that $H(\frac{\pi}{2}) = 2$ and $H(\frac{\pi}{4}) = 0$. Therefore, the filter does not pass the signal $cos(\frac{\pi}{4}n)$.

5.7

(a)

$$\begin{aligned} y(n) &= \frac{1}{2}[x(n) + x(n-1)] \\ Y(w) &= \frac{1}{2}(1+e^{-jw})X(w) \\ H(w) &= \frac{1}{2}(1+e^{-jw}) \\ &= cos(\frac{w}{2})e^{-j\frac{w}{2}} \end{aligned}$$

Refer to fig 5.10-1.

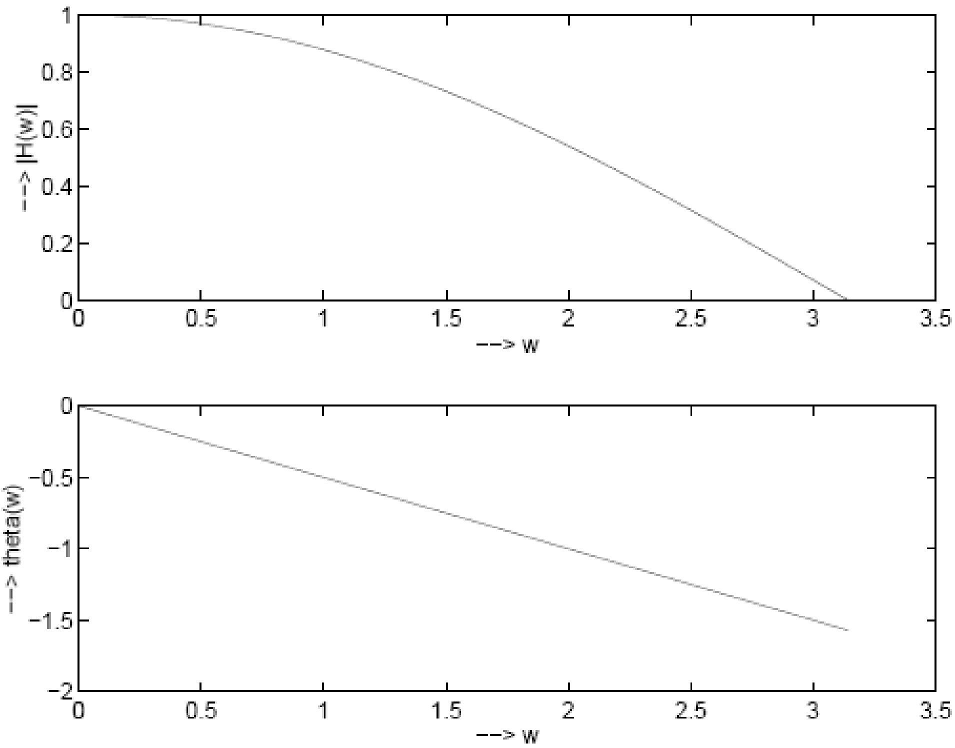

Figure 5.10-1:

(b)

$$\begin{aligned} y(n) &= -\frac{1}{2}\left[x(n) - x(n-1)\right] \\ Y(w) &= -\frac{1}{2}(1 - e^{-jw})X(w) \\ |H(w)| &= sin\frac{w}{2} \\ \Theta(w) &= e^{j(\frac{\pi}{2} - \frac{w}{2})} \end{aligned}$$

Refer to fig 5.10-2.

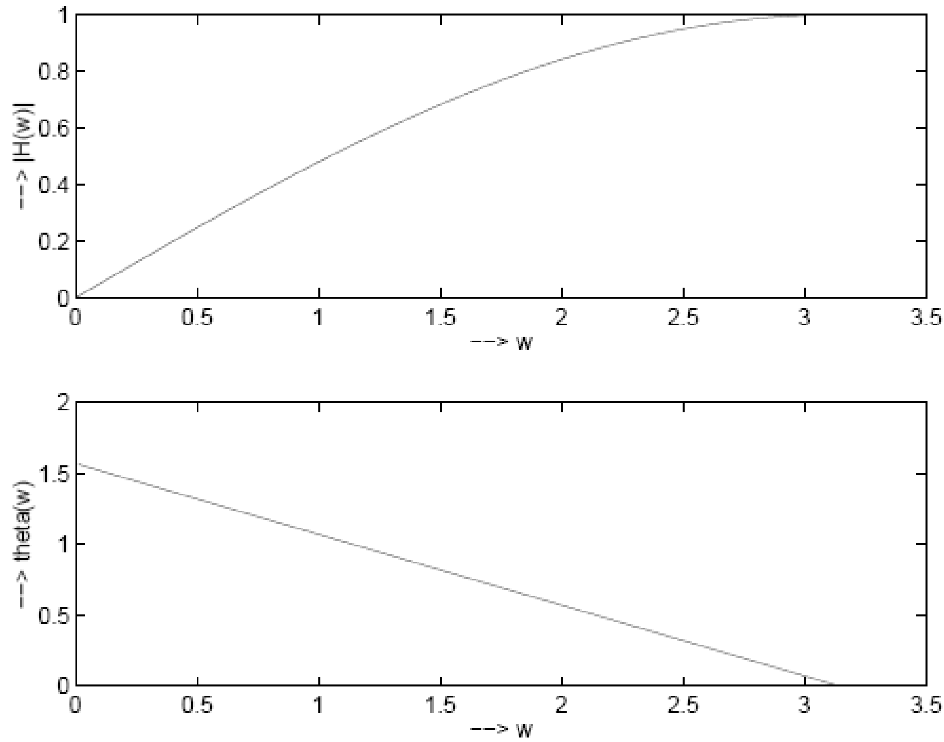

Figure 5.10-2:

(c)

$$
\begin{aligned}
y(n) &= \frac{1}{8}\left[x(n) + 3x(n-1) + 3x(n-2) + x(n-3)\right] \\
Y(w) &= \frac{1}{8}(1+e^{-jw})^3 X(w) \\
H(w) &= \frac{1}{8}(1+e^{-jw})^3 \\
&= cos^3(\frac{w}{2})e^{-j\frac{3w}{2}}
\end{aligned}
$$

Refer to fig 5.10-3.

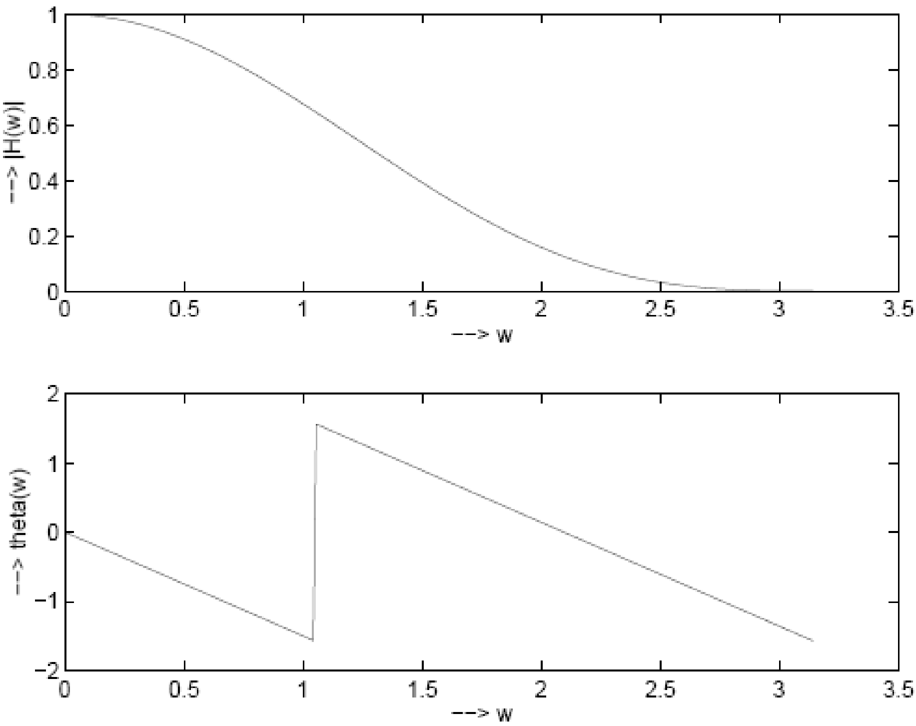

Figure 5.10-3:

5.10

(a) Refer to fig 5.14-1

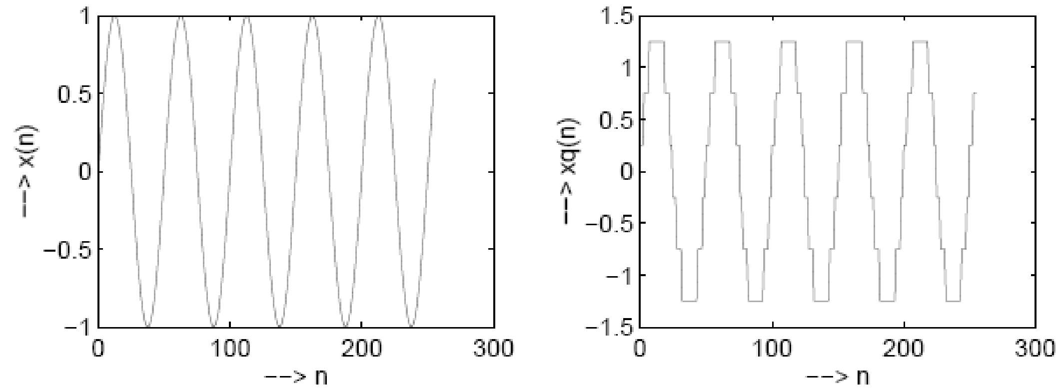

Figure 5.14-1:

(b) $f_0 = \frac{1}{50}$

bits	4	6	8	16
THD	$9.4616e-04$	$5.3431e-05$	$3.5650e-06$	$4.2848e-11$

(c) $f_0 = \frac{1}{100}$

bits	4	6	8	16
THD	$9.1993e-04$	$5.5965e-05$	$3.0308e-06$	$4.5383e-11$

(d) As the number of bits are increased, THD is reduced considerably.

5.12

(a)

$$\begin{aligned} y(n) &= x(n) - 2cosw_0 x(n-1) + x(n-2) \\ h(n) &= \delta(n) - 2cosw_0 \delta(n-1) + \delta(n-2) \end{aligned}$$

(b)

$$\begin{aligned} H(w) &= 1 - 2cosw_0 e^{-jw} + e^{-j2w} \\ &= (1 - e^{-jw_0} e^{-jw})(1 - e^{jw_0} e^{jw}) \\ &= -4e^{-jw} sin\frac{w+w_0}{2} sin\frac{w-w_0}{2} \\ &= -2e^{-jw}(cosw - cosw_0) \\ |H(w)| &= 2|cosw - cosw_0| \\ \Rightarrow |H(w)| &= 0 \text{ at } w = \pm w_0 \end{aligned}$$

Refer to fig 5.17-1.

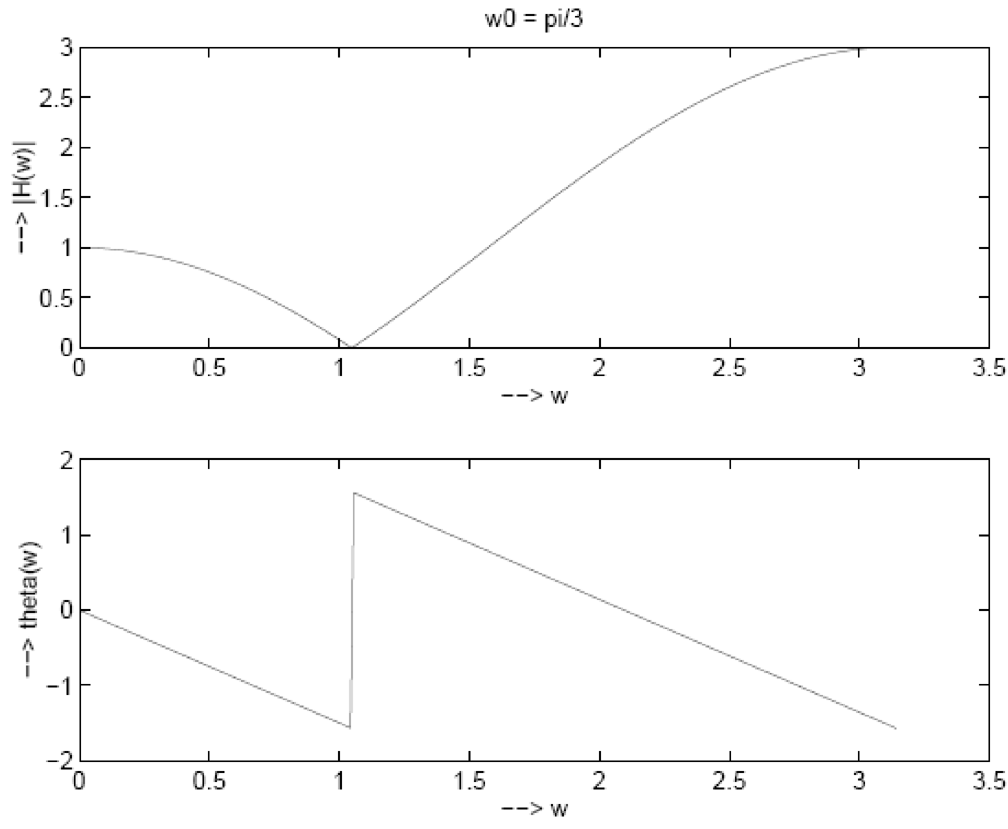

Figure 5.17-1:

(c)

$$\begin{aligned} \text{when } w_0 = \pi/2, H(w) &= 1 - e^{j2w} \\ \text{at } w = \pi/3, H(\pi/3) &= 1 - e^{j2\pi/3} = 1e^{j\pi/3} \\ y(n) &= |H(\pi/3)| 3cos(\frac{\pi}{3}n + 30^o - 60^o) \\ &= 3cos(\frac{\pi}{3}n - 30^o) \end{aligned}$$

5.16

(a)

$$\begin{aligned} h(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)e^{jwn}dw \\ &= \frac{1}{2\pi}\left[\int_{-\frac{3\pi}{8}}^{\frac{3\pi}{8}} e^{jwn}dw - \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} e^{-jwn}dw\right] \\ &= \frac{1}{\pi n}\left[sin\frac{3\pi}{8}n - sin\frac{\pi}{8}n\right] \\ &= \frac{2}{\pi n}sin\frac{\pi}{8}ncos\frac{\pi}{4}n \end{aligned}$$

(b) Let

$$h_1(n) = \frac{2sin\frac{\pi}{8}n}{n\pi}$$

Then,

$$H_1(w) = \begin{cases} 2, & |w| \le \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |w| < \pi \end{cases}$$

and

$$h(n) = h_1(n)cos\frac{\pi}{4}n$$

5.19

(a) $H(z) = b_0 \frac{1+bz^{-1}}{1+az^{-1}}$. Refer to fig 5.28-1.

(b) For $a = 0.5, b = -0.6$, $H(z) = b_0 \frac{1-0.6z^{-1}}{1+0.5z^{-1}}$. Since the pole is inside the unit circle and the filter is causal, it is also stable. Refer to fig 5.28-2.

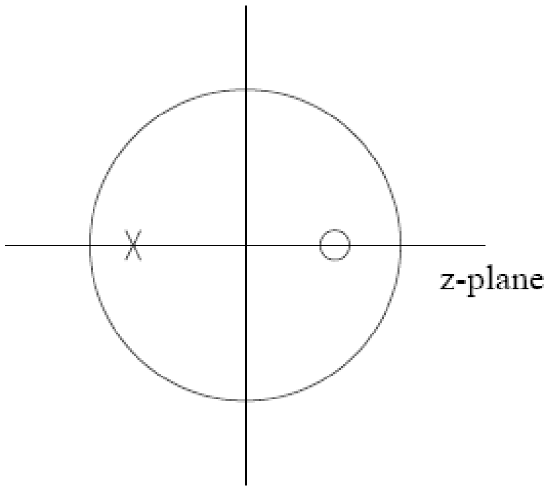

Figure 5.28-2:

(c)

$$
\begin{aligned}
H(z) &= b_0 \frac{1+0.5z^{-1}}{1-0.5z^{-1}} \\
\Rightarrow |H(w)|^2 &= b_0^2 \frac{\frac{5}{4}+cosw}{\frac{5}{4}-cosw}
\end{aligned}
$$

The maximum occurs at $w=0$. Hence,

$$
\begin{aligned}
H(w)|_{w=0} &= b_0^2 \frac{\frac{9}{4}}{\frac{1}{4}} \\
&= 9b_0^2 = 1 \\
\Rightarrow b_0 &= \pm\frac{1}{3}
\end{aligned}
$$

(d) Refer to fig 5.28-3.

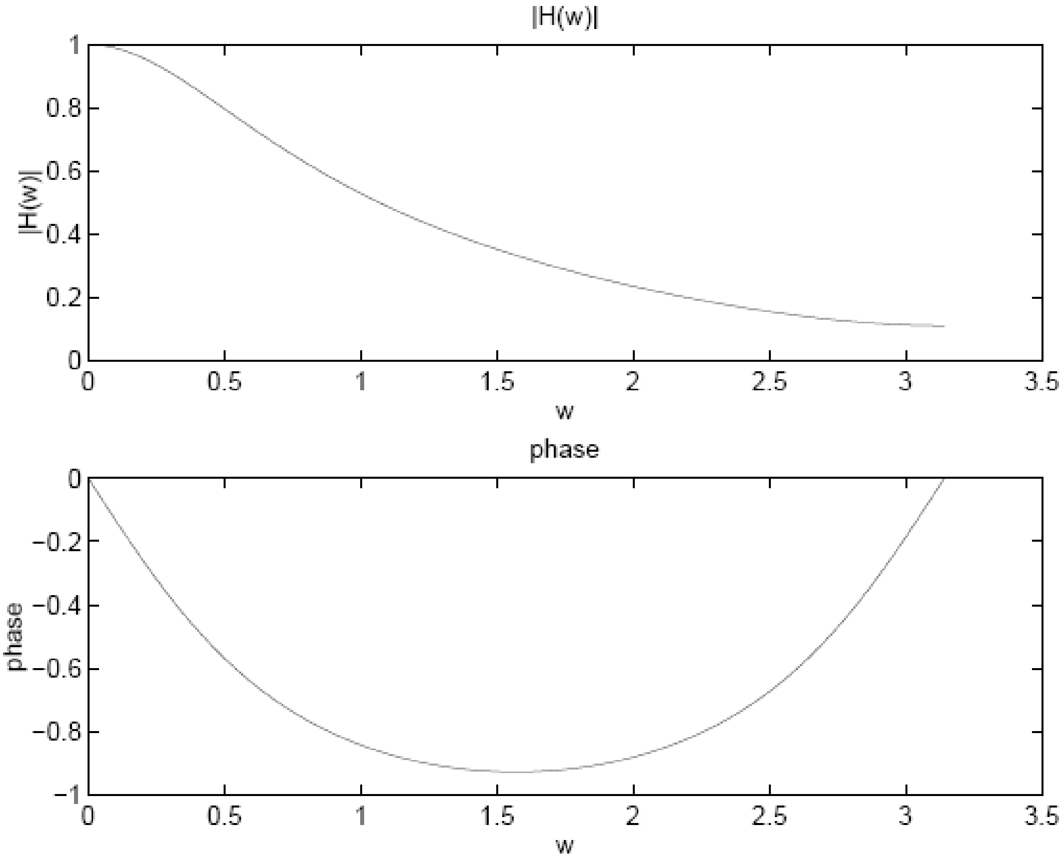

Figure 5.28-3:

(e) Refer to fig 5.28-4.

obviously, this is a highpass filter. By selecting $b = -1$, the frequency response of the highpass filter is improved.

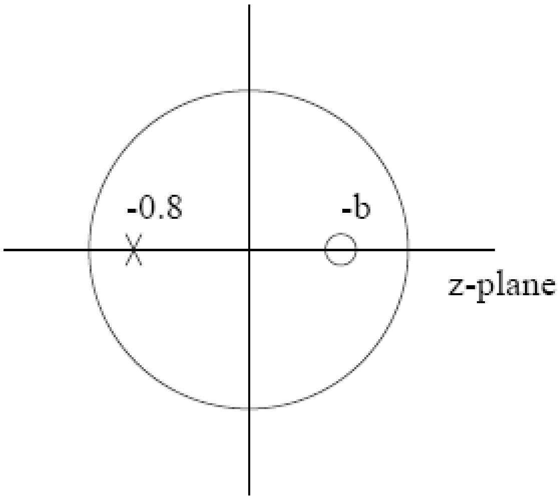

Figure 5.28-4:

5.22

$$\begin{aligned} y(n) &= b_0x(n) + b_1x(n-1) + b_2x(n-2) \\ H(w) &= b_0 + b_1e^{-jw} + b_2e^{-j2w} \end{aligned}$$

(a)

$$\begin{aligned} H(\frac{2\pi}{3}) &= b_0 + b_1e^{-j\frac{2\pi}{3}} + b_2e^{-j\frac{4\pi}{3}} = 0 \\ H(0) &= b_0 + b_1 + b_2 = 1 \\ \text{For linear phase, } b_0 &= \pm b_2. \\ \text{select } b_0 &= b_2 \text{ (otherwise } b_1 = 0). \end{aligned}$$

These conditions yield

$$\begin{aligned} b_0 = b_1 = b_2 &= \frac{1}{3} \\ \text{Hence, } H(w) &= \frac{1}{3}e^{-jw}(1 + 2cosw) \end{aligned}$$

(b)

$$H(w) = \frac{1}{3}(1 + 2cosw)$$

$$\Theta(w) = \begin{cases} -w, & \text{for } 1 + 2cosw > 0 \\ -w + \pi, & \text{for } 1 + 2cosw < 0 \end{cases}$$

5.26

$$\begin{aligned} H_z(w) &= (1-re^{j\theta}e^{-jw})(1-re^{-j\theta}e^{-jw}) \\ &= (1-re^{-j(w-\theta)})(1-re^{-j(w+\theta)}) \\ &= A(w)B(w) \end{aligned}$$

(a)

$$\begin{aligned} |H_z(w)| &= |A(w)b(w)| \\ &= |A(w)||B(w)| \\ |H_z(w)|_{\text{dB}} &= 20\log_{10}|H_z(w)| \\ &= 10\log_{10}[1-2rcos(w-\theta)+r^2] + 10\log_{10}[1-2rcos(w+\theta)+r^2] \end{aligned}$$

(b)

$$\begin{aligned} \angle H_z(w) &= \angle A(w) + \angle B(w) \\ &= tan^{-1}\frac{rsin(w-\theta)}{1-rcos(w-\theta)} + tan^{-1}\frac{rsin(w+\theta)}{1-rcos(w+\theta)} \end{aligned}$$

(c)

$$\begin{aligned} \tau_g^z(w) &= -\frac{d\Theta_z(w)}{dw} \\ &= \tau_A^z(w) + \tau_g^B(w) \\ &= \frac{r^2 - rcos(w-\theta)}{1+r^2-2rcos(w-\theta)} + \frac{r^2 - rcos(w+\theta)}{1+r^2-2rcos(w+\theta)} \end{aligned}$$

(d)

$$\begin{aligned} H_p(w) &= \frac{1}{H_z(w)} \\ \text{Therefore, } |H_p(w)| &= \frac{1}{|H_z(w)|} \\ |H_p(w)|_{\text{dB}} &= -|H_z(w)|_{\text{dB}} \\ \text{on the same lines of prob4.62} & \\ \Theta_p(w) &= -\Theta_z(w) \text{ and} \\ \tau_g^p(w) &= -\tau_g^z(w) \end{aligned}$$

(e) Refer to fig 5.38-1.

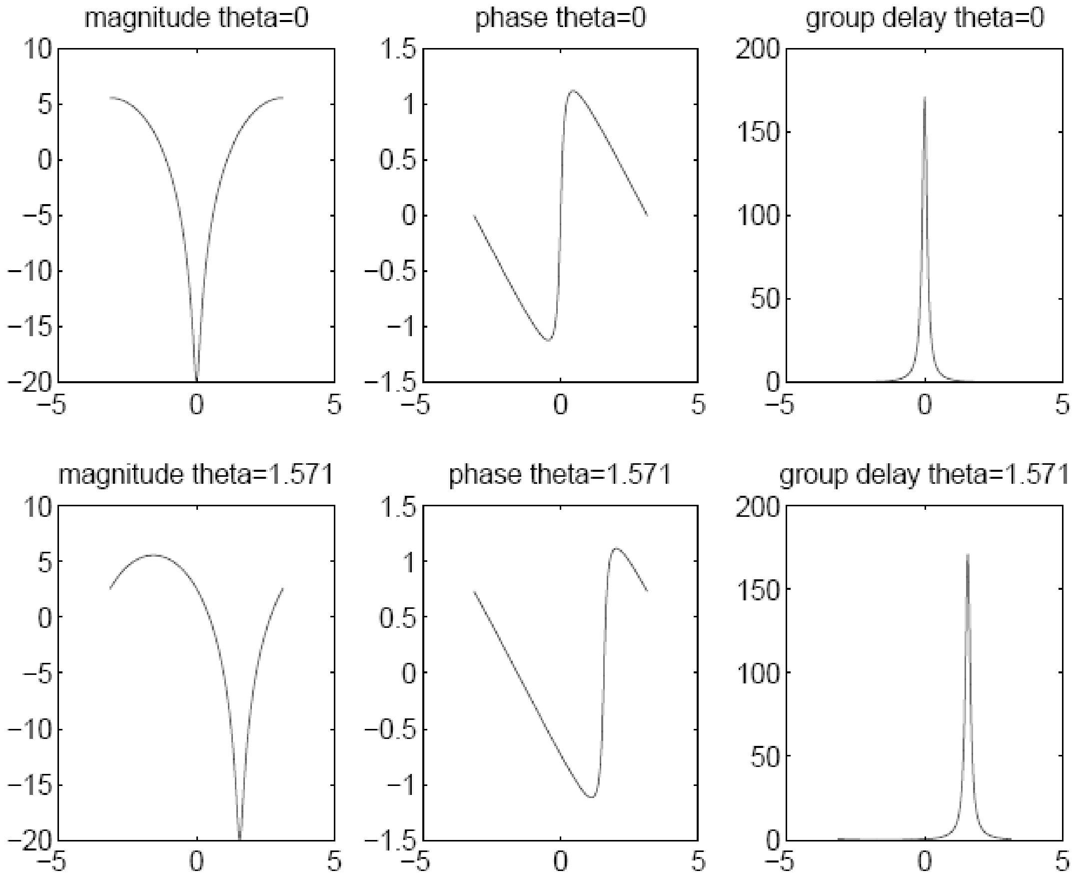

Figure 5.38-1:

5.29

For the sampling frequency $F_s = 500$samples/sec., the rejected frequency should be $w_1 = 2\pi(\frac{60}{100}) = \frac{6}{25}\pi$. The filter should have unity gain at $w_2 = 2\pi(\frac{200}{500}) = \frac{4}{5}\pi$. Hence,

$$\begin{aligned}
H(\frac{6}{25}\pi) &= 0 \\
\text{and } H(\frac{4}{5}\pi) &= 1 \\
H(w) &= G(1 - e^{j\frac{6\pi}{25}}e^{-jw})(1 - e^{-j\frac{6\pi}{25}}e^{-jw}) \\
&= Ge^{-jw}[2cosw - 2cos\frac{6\pi}{25}] \\
H(\frac{4}{5}\pi) &= 2G|[cos(\frac{4}{5}\pi) - cos(\frac{6}{25}\pi)]| = 1 \\
\text{Hence, } G &= \frac{\frac{1}{2}}{cos\frac{6}{25}\pi - cos\frac{4}{5}\pi}
\end{aligned}$$

5.33

(a) Replace z by z^8. We need 8 zeros at the frequencies $w = 0, \pm\frac{\pi}{4}, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \pi$ Hence,

$$
\begin{aligned}
H(z) &= \frac{1 - z^{-8}}{1 - az^{-8}} \\
&= \frac{Y(z)}{X(z)} \\
\text{Hence, } y(n) &= ay(n-8) + x(n) - x(n-8)
\end{aligned}
$$

(b) Zeros at $1, e^{\pm j\frac{\pi}{4}}, e^{\pm j\frac{\pi}{2}}, e^{\pm j\frac{3\pi}{4}}, -1$

Poles at $a^{\frac{1}{8}}, a^{\frac{1}{8}}e^{\pm j\frac{\pi}{4}}, a^{\frac{1}{8}}e^{\pm j\frac{\pi}{2}}, a^{\frac{1}{8}}e^{\pm j\frac{3\pi}{4}}, -1$. Refer to fig 5.49-1.

(c)

$$
|H(w)| = \frac{2|cos4w|}{\sqrt{1 - 2acos8w + a^2}}
$$

$$
\angle H(w) = \begin{cases} -tan^{-1}\frac{asin8w}{1-acos8w}, & cos4w \geq 0 \\ \pi - tan^{-1}\frac{asin8w}{1-acos8w}, & cos4w < 0 \end{cases}
$$

Refer to fig 5.49-2.

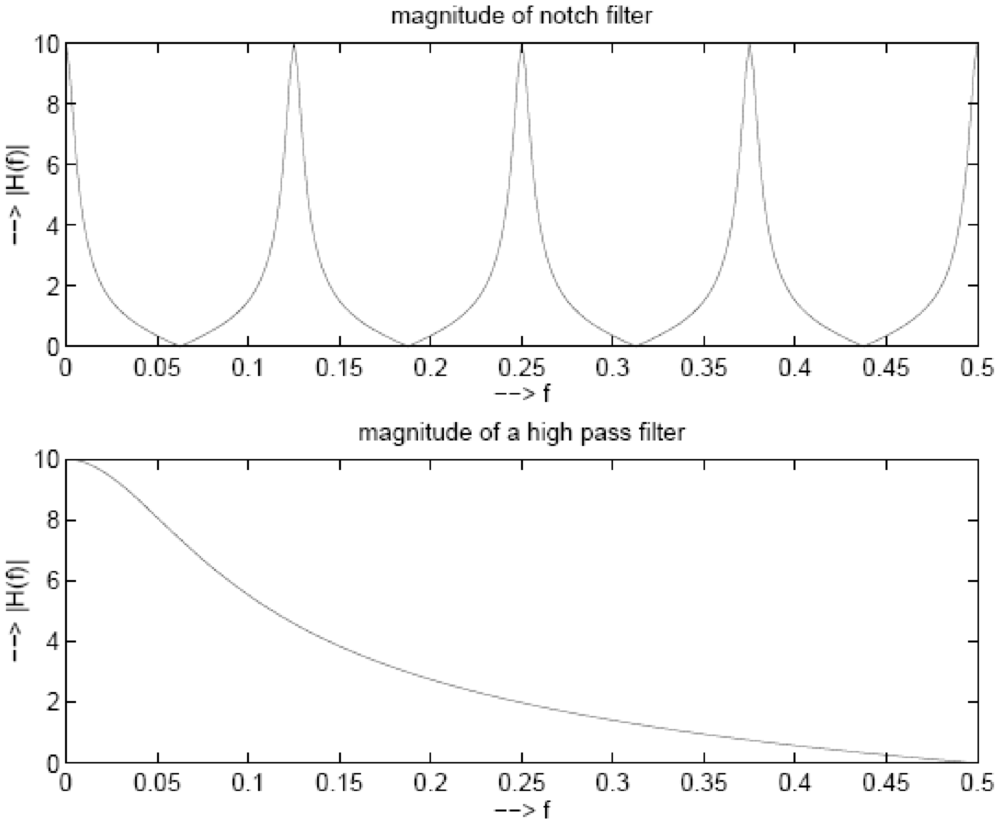

Figure 5.49-2:

5.36

$$
\begin{aligned}
h(n) &= \{h(0), h(1), h(2), h(3)\} \text{ where } h(0) = -h(3), h(1) = -h(2) \\
\text{Hence, } H_r(w) &= 2(h(0) sin\frac{3w}{2} + h(1) sin\frac{w}{2}) \\
H_r(\frac{\pi}{4}) &= 2h(0) sin\frac{3\pi}{8} + 2h(1) sin\frac{\pi}{8}) = \frac{1}{2} \\
H_r(\frac{3\pi}{4}) &= 2h(0) sin\frac{9\pi}{8} + 2h(1) sin\frac{3\pi}{8}) = 1 \\
1.85h(0) + 0.765h(1) &= \frac{1}{2} \\
-0.765h(0) + 1.85h(1) &= 1 \\
h(1) &= 0.56, h(0) = 0.04
\end{aligned}
$$

5.39

(a) Since $X(w)$ and $Y(w)$ are periodic, it is observed that $Y(w) = X(w - \pi)$. Therefore, $y(n) = e^{j\pi n}x(n) = (-1)^n x(n)$
(b) $x(n) = (-1)^n y(n)$.

5.43

$$\begin{aligned} Y(z) &= X(z) + bz^{-2}X(z) + z^{-4}X(z) \\ H(z) &= \frac{Y(z)}{X(z)} \\ &= 1 + bz^{-2} + z^{-4} \\ \text{For } b = 1, H(w) &= 1 + e^{j2w} + e^{-j4w} \\ &= (1 + 2cosw)e^{-jw} \\ |H(w)| &= |1 + 2cosw| \end{aligned}$$

$$\angle H(w) = \begin{cases} -w, & 1 + 2cosw \geq 0 \\ \pi - w, & 1 + 2cosw < 0 \end{cases}$$

Refer to fig 5.64-1.

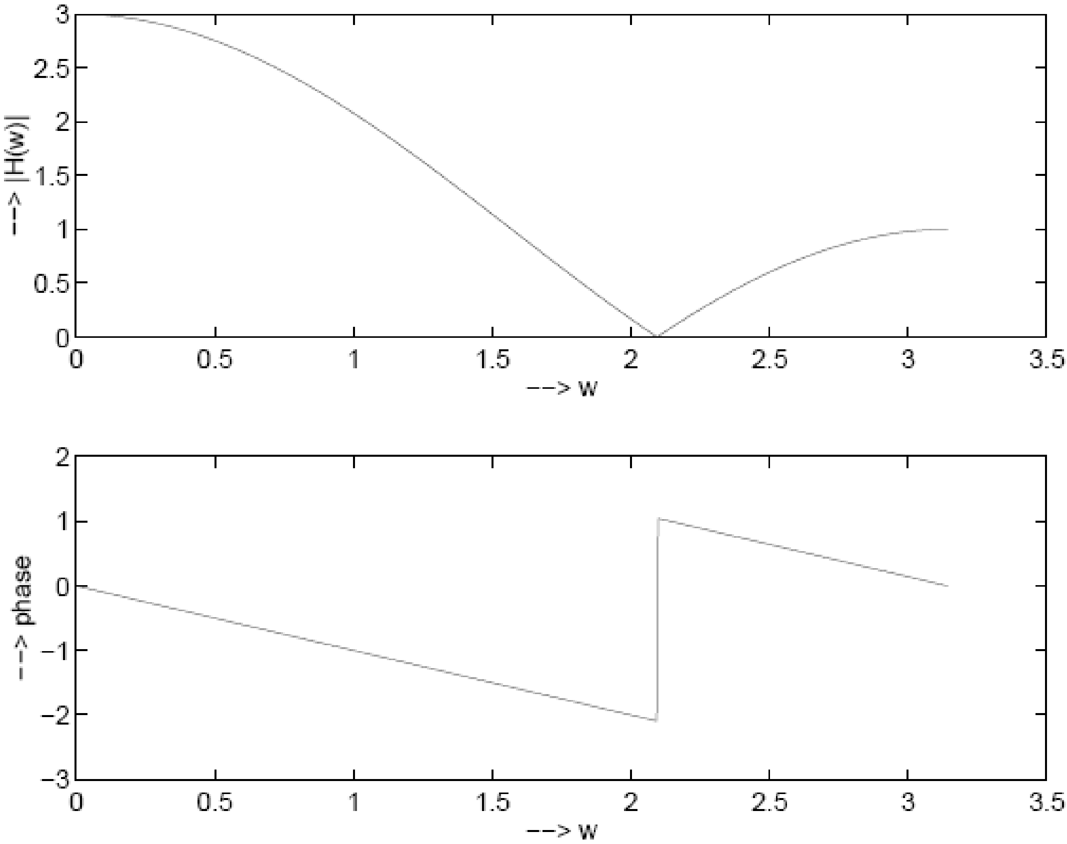

Figure 5.64-1:

$$\begin{aligned} b = -1, H(w) &= 1 - e^{-jw} + e^{-j2w} \\ &= (2cosw - 1)e^{-jw} \\ |H(w)| &= |2cosw - 1| \end{aligned}$$

$$\angle H(w) = \begin{cases} -w, & -1 + 2cosw \geq 0 \\ \pi - w, & -1 + 2cosw < 0 \end{cases}$$

Refer to fig 5.64-2.

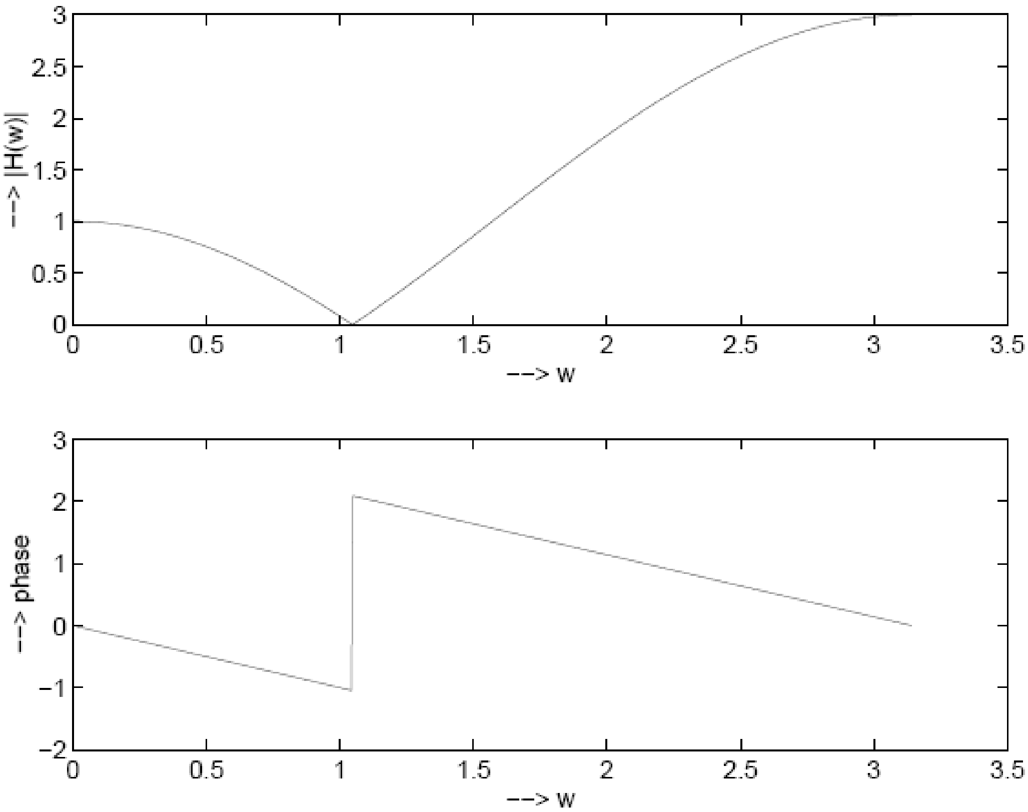

Figure 5.64-2:

5.46

$$\begin{aligned}
y(n) &= \frac{1}{2}y(n-1)+x(n)\\
x(n) &= (\frac{1}{4})^n u(n)\\
H(z) &= \frac{Y(z)}{X(z)}\\
&= \frac{1}{1-\frac{1}{2}z^{-1}}\\
X(z) &= \frac{1}{1-\frac{1}{4}z^{-1}}\\
Y(z) &= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}\\
R_{xx}(z) &= X(z)X(z^{-1})\\
&= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{4}z)}\\
&= \frac{-4z^{-1}}{(1-\frac{1}{4}z^{-1})(1-4z^{-1})}\\
&= \frac{16}{15}\frac{1}{1-\frac{1}{4}z^{-1}}-\frac{16}{15}\frac{1}{1-4z^{-1}}\\
\text{Hence, } r_{xx}(n) &= \frac{16}{15}(\frac{1}{4})^n u(n)+\frac{16}{15}(4)^n u(-n-1)\\
R_{hh}(z) &= H(z)H(z^{-1})\\
&= \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{2}z)}\\
&= \frac{-2z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}\\
&= \frac{4}{3}\frac{1}{1-\frac{1}{2}z^{-1}}-\frac{4}{3}\frac{1}{1-2z^{-1}}\\
\text{Hence, } r_{hh}(n) &= \frac{4}{3}(\frac{1}{2})^n u(n)+\frac{4}{3}(2)^n u(-n-1)\\
R_{xy}(z) &= X(z)Y(z^{-1})\\
&= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{4}z)(1-\frac{1}{2}z)}\\
&= -\frac{16}{17}\frac{1}{1-2z^{-1}}+\frac{16}{15}\frac{1}{1-4z^{-1}}+\frac{128}{105}\frac{1}{1-\frac{1}{4}z^{-1}}\\
\text{Hence, } r_{xy}(n) &= \frac{16}{17}(2)^n u(-n-1)-\frac{16}{15}(4)^n u(-n-1)+\frac{128}{105}(\frac{1}{4})^n u(n)
\end{aligned}$$

$$\begin{aligned}
R_{yy}(z) &= Y(z)Y(z^{-1})\\
&= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z)(1-\frac{1}{2}z)}\\
&= -\frac{64}{21}\frac{1}{1-2z^{-1}}+\frac{128}{105}\frac{1}{1-4z^{-1}}+\frac{64}{21}\frac{1}{1-\frac{1}{2}z^{-1}}-\frac{128}{105}\frac{1}{1-\frac{1}{4}z^{-1}}\\
\text{Hence, } r_{yy}(n) &= \frac{64}{21}(2)^n u(-n-1)-\frac{128}{105}(4)^n u(-n-1)+\frac{64}{21}(\frac{1}{2})^n u(n)-\frac{128}{105}(\frac{1}{4})^n u(n
\end{aligned}$$

5.48

$h(n) = b_0\delta(n) + b_1\delta(n-D) + b_2\delta(n-2D)$ (a) If the input to the system is $x(n)$, the output is $y(n) = b_0x(n) + b_1x(n-D) + b_2x(n-2D)$. Hence, the output consists of $x(n)$, which is the input signal, and the delayed signals $x(n-D)$ and $x(n-2D)$. The latter may be thought of as echoes of $x(n)$.

(b)

$$\begin{aligned} H(w) &= b_0 + b_1e^{-jwD} + b_2e^{-j2wD} \\ &= b_0 + b_1coswD + b_2cos2wD - j(b_1sinwD + b_2sin2wD) \\ |H(w)| &= \sqrt{b_0{}^2 + b_1{}^2 + b_2{}^2 + 2b_1(b_0+b_2)coswD + 2b_0b_2cos2wD} \\ \Theta(w) &= -tan^{-1}\frac{b_1sinwD + b_2sin2wD}{b_0 + b_1coswD + b_2cos2wD} \end{aligned}$$

(c) If $|b_0 + b_2| << |b_1|$, then the dominant term is b_1e^{-jwD} and

$$|H(w)| = \sqrt{b_0{}^2 + b_1{}^2 + b_2{}^2 + 2b_1(b_0+b_2)coswD}$$

and $|H(w)|$ has maxima and minima at $w = \pm\frac{k}{D}\pi, k = 0, 1, 2, \ldots$

(d) The phase $\Theta(w)$ is approximately linear with a slope of $-D$. Refer to fig 5.71-1.

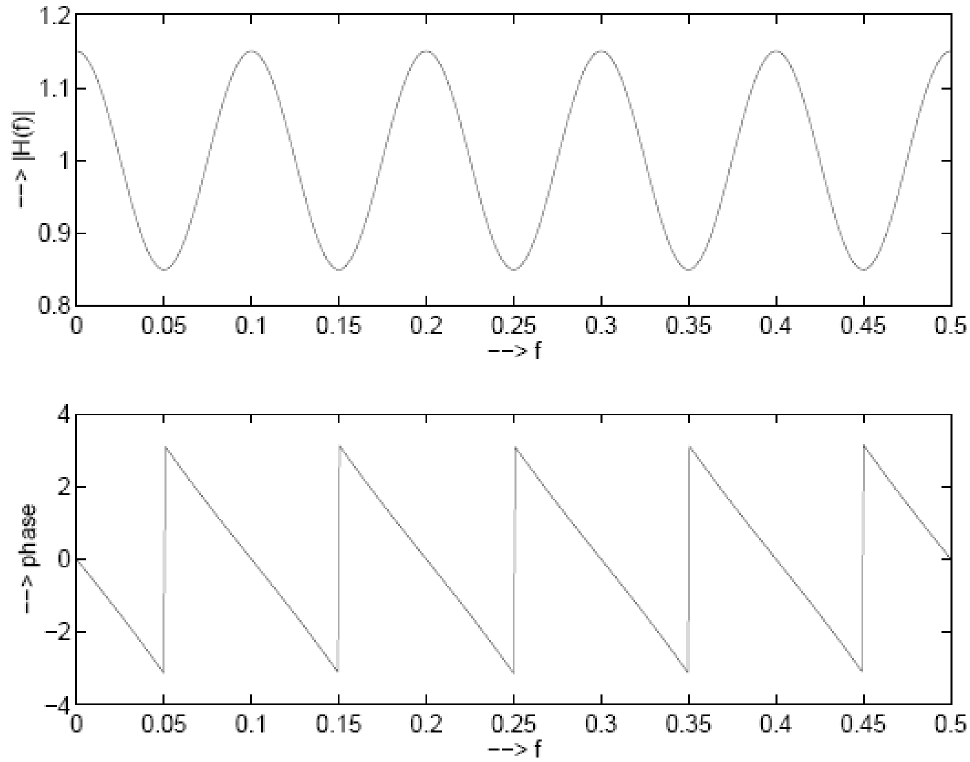

Figure 5.71-1:

5.52

(a)

$$\begin{aligned} |H(w)|^2 &= \frac{\frac{5}{4} - cosw}{\frac{10}{9} - \frac{2}{3}cosw} \\ &= H(z)H(z^{-1})|_{z=e^{-jw}} \\ \text{Hence, } H(z)H(z^{-1}) &= \frac{\frac{5}{4} - \frac{1}{2}(z + z^{-1})}{\frac{10}{9} - \frac{1}{3}(z + z^{-1})} \\ &= \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{3}z^{-1}} \end{aligned}$$

(b)

$$\begin{aligned} |H(w)|^2 &= \frac{2(1 - a^2)}{1 + a^2 - 2acosw} \\ H(z)H(z^{-1}) &= \frac{2(1 - a^2)}{1 + a^2 - a(z + z^{-1})} \\ H(z)H(z^{-1}) &= \frac{2(1 + a)(1 - a)}{(1 - az^{-1})(1 - az)} \\ \text{Hence, } H(z) &= \frac{\sqrt{2(1 - a^2)}}{1 - az^{-1}} \\ \text{or } H(z) &= \frac{\sqrt{2(1 - a^2)}}{1 - az} \end{aligned}$$

5.54

(a) The impulse response is given in pr10fig 5.80-1.

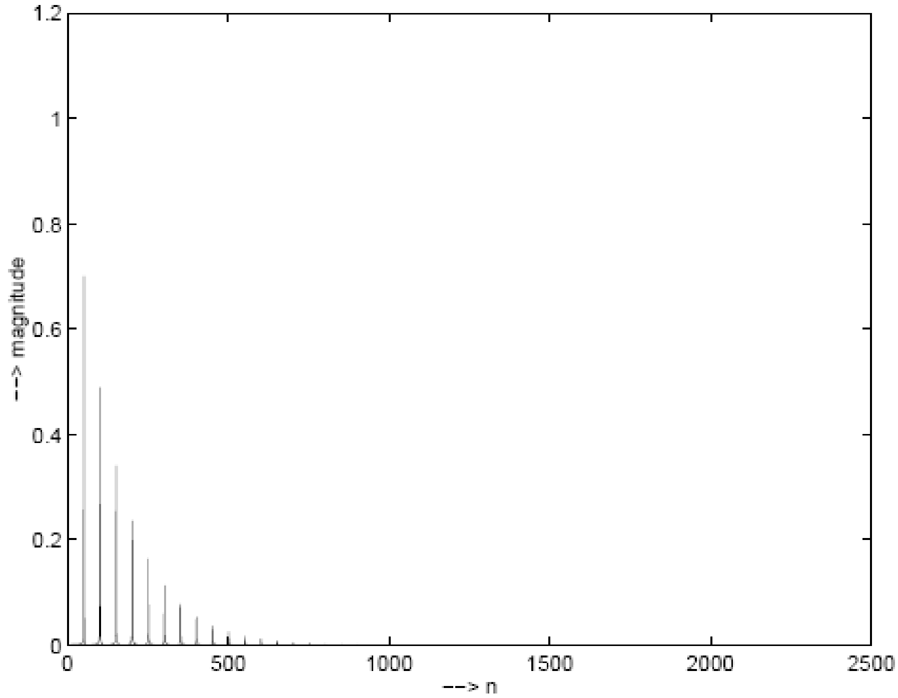

Figure 5.80-1:

(b) Reverberator 1: refer to fig 5.80-2.

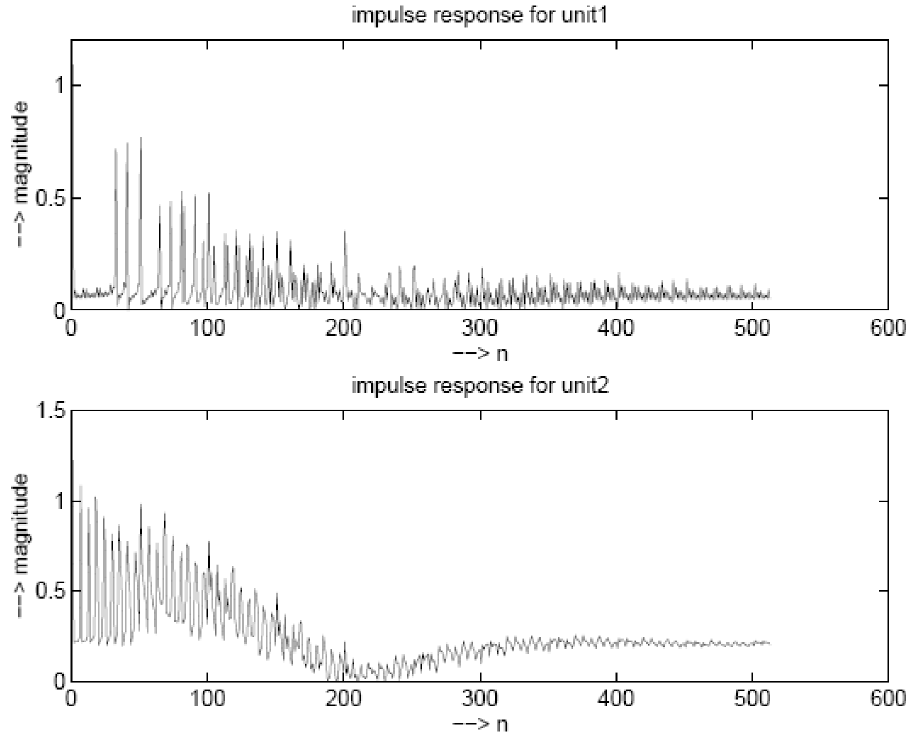

Figure 5.80-2:

Reverberator 2: refer to fig 5.80-2.

(c) Unit 2 is a better reverberator.
(d) For prime number of D_1, D_2, D_3, the reverberations of the signal in the different sections do not overlap which results in the impulse response of the unit being more dense.
(e) Refer to fig 5.80-3.
(f) Refer to fig 5.80-4 for the delays being prime numbers.