4.2

(a)

$$\begin{aligned} x_a(t) &= Ae^{-at}u(t), \qquad a > 0 \\ X_a(F) &= \int_0^\infty Ae^{-at}e^{-j2\pi Ft}dt \\ &= \left.\frac{A}{-a-j2\pi F}e^{-(a+j2\pi F)t}\right]_0^\infty \\ &= \frac{A}{a+j2\pi F} \\ |X_a(F)| &= \frac{A}{\sqrt{a^2+(2\pi F)^2}} \\ \angle X_a(F) &= -tan^{-1}(\frac{2\pi F}{a}) \end{aligned}$$

Refer to fig 4.2-1

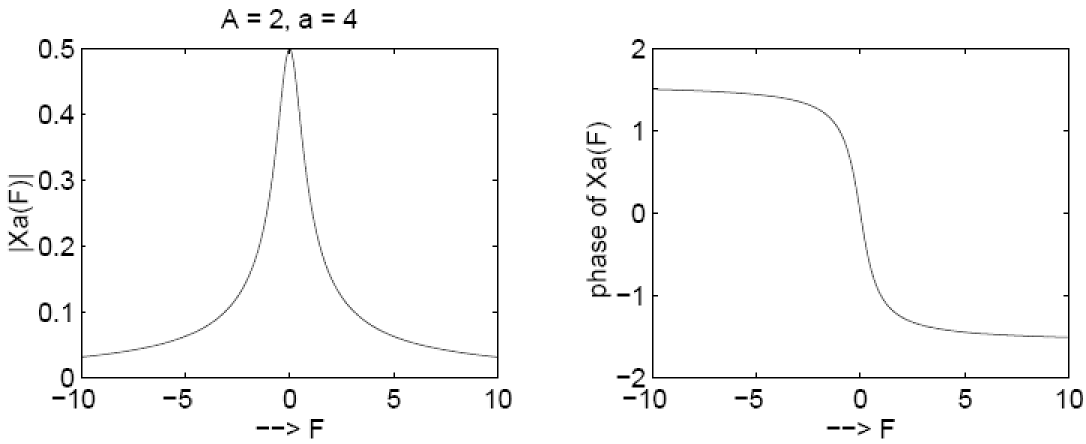

Figure 4.2-1:

(b)

$$\begin{aligned} X_a(F) &= \int_0^\infty Ae^{at}e^{-j2\pi Ft}dt + \int_0^\infty Ae^{-at}e^{-j2\pi Ft}dt \\ &= \frac{A}{a-j2\pi F} + \frac{A}{a+j2\pi F} \\ &= \frac{2aA}{a^2+(2\pi F)^2} \\ |X_a(F)| &= \frac{2aA}{a^2+(2\pi F)^2} \\ \angle X_a(F) &= 0 \end{aligned}$$

Refer to fig 4.2-2

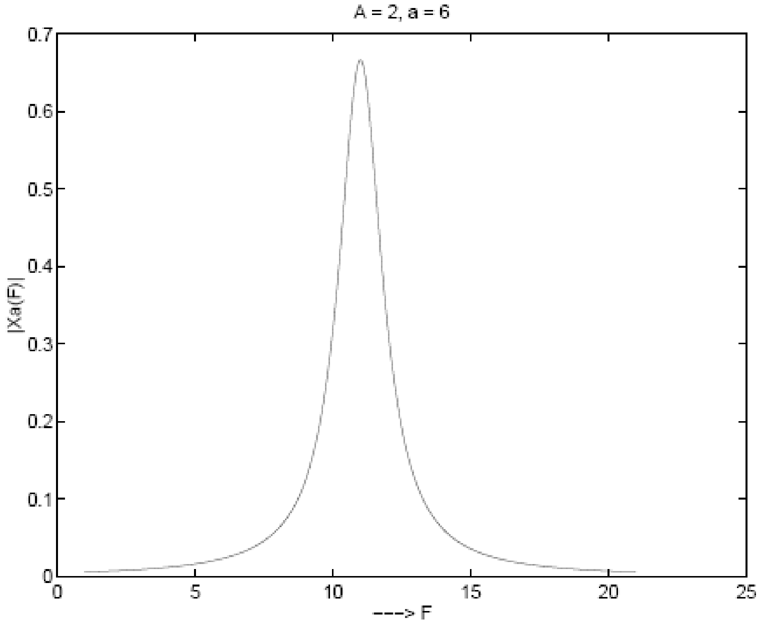

Figure 4.2-2:

4.4

$$x(n) = 2 + 2cos\pi n/4 + cos\pi n/2 + \frac{1}{2}cos3\pi n/4, \Rightarrow N = 8$$

(a)

$$\begin{aligned} c_k &= \frac{1}{8}\sum_{n=0}^{7} x(n)e^{-j\pi kn/4} \\ x(n) &= \left\{\frac{11}{2}, 2+\frac{3}{4}\sqrt{2}, 1, 2-\frac{3}{4}\sqrt{2}, \frac{1}{2}, 2-\frac{3}{4}\sqrt{2}, 1, 2+\frac{3}{4}\sqrt{2}\right\} \\ \text{Hence, } c_0 &= 2, c_1 = c_7 = 1, c_2 = c_6 = \frac{1}{2}, c_3 = c_5 = \frac{1}{4}, c_4 = 0 \end{aligned}$$

(b)

$$\begin{aligned} P &= \sum_{i=0}^{7} |c(i)|^2 \\ &= 4 + 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} \\ &= \frac{53}{8} \end{aligned}$$

4.7

(a)

$$\begin{aligned}
x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)e^{jwn}dw \\
&= \frac{1}{2\pi}\int_{-\pi}^{-w_0} e^{jwn}dw + \frac{1}{2\pi}\int_{w_0}^{\pi} e^{jwn}dw \\
x(0) &= \frac{1}{2\pi}(\pi - w_0) + \frac{1}{2\pi}(\pi - w_0) \\
&= \frac{\pi - w_0}{\pi} \\
\text{For } n \neq 0, \int_{-\pi}^{-w_0} e^{jwn}dw &= \frac{1}{jn}e^{jwn}|_{-\pi}^{-w_0} \\
&= \frac{1}{jn}(e^{-jw_0n} - e^{-j\pi n}) \\
\int_{w_0}^{\pi} e^{jwn}dw &= \frac{1}{jn}e^{jwn}|_{w_0}^{\pi} \\
&= \frac{1}{jn}(e^{j\pi n} - e^{jw_0n}) \\
\text{Hence, } x(n) &= -\frac{sinnw_0}{n\pi}, n \neq 0
\end{aligned}$$

(b)

$$\begin{aligned}
X(w) &= cos^2(w) \\
&= (\frac{1}{2}e^{jw} + \frac{1}{2}e^{-jw})^2 \\
&= \frac{1}{4}(e^{j2w} + 2 + e^{-j2w}) \\
x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)e^{jwn}dw \\
&= \frac{1}{8\pi}\left[2\pi\delta(n+2) + 4\pi\delta(n) + 2\pi\delta(n-2)\right] \\
&= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]
\end{aligned}$$

(c)

$$\begin{aligned} x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)e^{jwn}dw \\ &= \frac{1}{2\pi}\int_{w_0-\frac{\delta w}{2}}^{w_0+\frac{\delta w}{2}} e^{jwn}dw \\ &= \frac{2}{\pi}\delta w\left(\frac{sin(n\delta w/2)}{n\delta w/2}\right)e^{jnw_0} \end{aligned}$$

(d)

$$\begin{aligned} x(n) &= \frac{1}{2\pi}Re\left\{\int_0^{\pi/8} 2e^{jwn}dw + \int_{\pi/8}^{3\pi/8} e^{jwn}dw + \int_{6\pi/8}^{7\pi/8} e^{jwn}dw + \int_{7\pi/8}^{\pi} e^{jwn}dw\right\} \\ &= \frac{1}{\pi}\left[\int_0^{\pi/8} 2coswndw + \int_{\pi/8}^{3\pi/8} coswndw + \int_{6\pi/8}^{7\pi/8} coswndw + \int_{7\pi/8}^{\pi} 2coswndw\right] \\ &= \frac{1}{n\pi}\left[sin\frac{7\pi n}{8} + sin\frac{6\pi n}{8} - sin\frac{3\pi n}{8} - sin\frac{\pi n}{8}\right] \end{aligned}$$

4.10

(a) $X(0) = \sum_n x(n) = -1$
(b) $\angle X(w) = \pi$ for all w
(c) $x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)dw$ Hence, $\int_{-\pi}^{\pi} X(w)dw = 2\pi x(0) = -6\pi$
(d)

$$X(\pi) = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\pi} = \sum_n (-1)^n x(n) = -3 - 4 - 2 = -9$$

(e) $\int_{-\pi}^{\pi} |X(w)|^2 dw = 2\pi \sum_n |x(n)|^2 = (2\pi)(19) = 38\pi$

4.12

(a)

$$\sum_n x^*(n)e^{-jwn} = (\sum_n x(n)e^{-j(-w)n})^* = X^*(-w)$$

(b)

$$\sum_n x^*(-n)e^{-jwn} = \sum_{n=-\infty}^{\infty} x^*(n)e^{jwn} = X^*(w)$$

(c)

$$\begin{aligned}\sum_n y(n)e^{-jwn} &= \sum_n x(n)e^{-jwn} - \sum_n x(n-1)e^{-jwn} \\ Y(w) &= X(w) + X(w)e^{-jw} \\ &= (1-e^{-jw})X(w)\end{aligned}$$

(d)

$$\begin{aligned}y(n) &= \sum_{k=-\infty}^{n} x(k) \\ &= y(n) - y(n-1) \\ &= x(n) \\ \text{Hence, } X(w) &= (1-e^{-jw})Y(w) \\ \Rightarrow Y(w) &= \frac{X(w)}{1-e^{-jw}}\end{aligned}$$

(e)

$$\begin{aligned}Y(w) &= \sum_n x(2n)e^{-jwn} \\ &= \sum_n x(n)e^{-j\frac{w}{2}n} \\ &= X(\frac{w}{2})\end{aligned}$$

(f)

$$\begin{aligned}Y(w) &= \sum_n x(\frac{n}{2})e^{-jwn} \\ &= \sum_n x(n)e^{-j2wn} \\ &= X(2w)\end{aligned}$$

4.14

$$
\begin{aligned}
c_k^y &= \frac{1}{N}\sum_{n=0}^{N-1} y(n)e^{-j2\pi kn/N} \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right]e^{-j2\pi kn/N} \\
&= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)e^{-j2\pi k(m+lN)/N} \\
\text{But } \sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)e^{-jw(m+lN)} &= X(w) \\
\text{Therefore, } c_k^y &= \frac{1}{N}X(\frac{2\pi k}{N})
\end{aligned}
$$

4.16

(a) $Y_1(w) = \sum_n y_1(n)e^{-jwn} = \sum_{n,n \text{ even}} x(n)e^{-jwn}$ The fourier transform $Y_1(w)$ can easily be obtained by combining the results of (b) and (c).

(b)

$$\begin{aligned} y_2(n) &= x(2n) \\ Y_2(w) &= \sum_n y_2(n)e^{-jwn} \\ &= \sum_n x(2n)e^{-jwn} \\ &= \sum_m x(m)e^{-jwm/2} \\ &= X(\frac{w}{2}) \end{aligned}$$

Refer to fig 4.23-1.

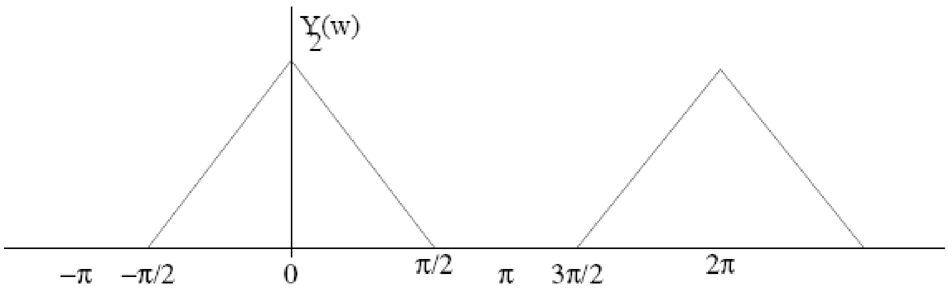

Figure 4.23-1:

(c)

$$y_3(n) = \begin{cases} x(n/2), & n \text{ even} \\ 0, & \text{otherwise} \end{cases}$$

$$Y_3(w) = \sum_n y_3(n)e^{-jwn}$$

$$
\begin{aligned}
&= \sum_{n \text{ even}} x(n/2)e^{-jwn} \\
&= \sum_{m} x(m)e^{-j2wm} \\
&= X(2w)
\end{aligned}
$$

We now return to part(a). Note that $y_1(n)$ may be expressed as

$$
y_1(n) = \begin{cases} y_2(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}
$$

Hence, $Y_1(w) = Y_2(2w)$. Refer to fig 4.23-2.

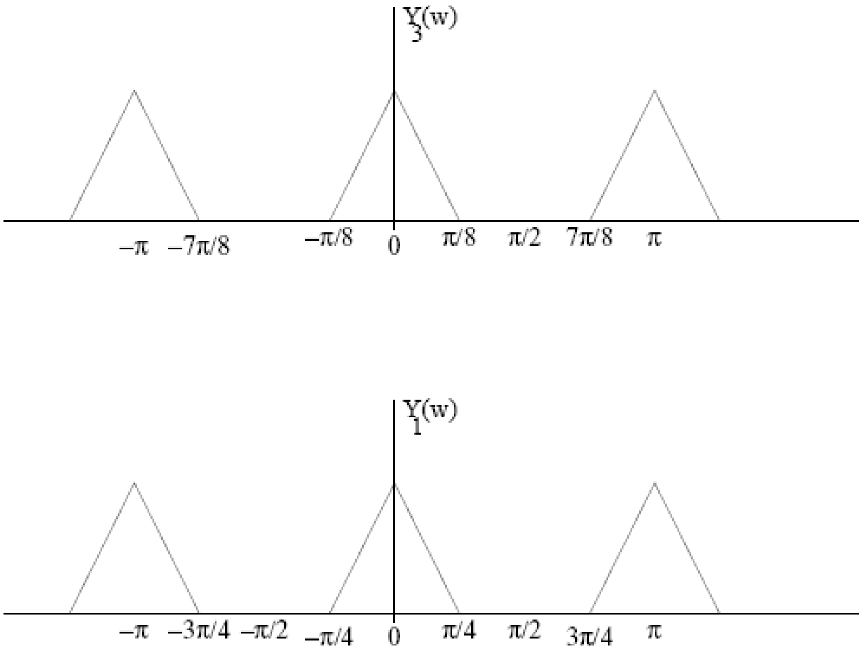

Figure 4.23-2: