2.4

First, we prove that

$$\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) = 0$$

$$\begin{aligned}\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) &- \sum_{m=-\infty}^{\infty} x_e(-m)x_o(-m)\\ &= -\sum_{m=-\infty}^{\infty} x_e(m)x_o(m)\\ &= -\sum_{n=-\infty}^{\infty} x_e(n)x_o(n)\\ &= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n)\\ &= 0\end{aligned}$$

Then,

$$\begin{aligned}\sum_{n=-\infty}^{\infty} x^2(n) &= \sum_{n=-\infty}^{\infty} [x_e(n)+x_o(n)]^2\\ &= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n)\\ &= E_e + E_o\end{aligned}$$

2.5

(a) Static, nonlinear, time invariant, causal, stable.
(b) Dynamic, linear, time invariant, noncausal and unstable. The latter is easily proved. For the bounded input $x(k) = u(k)$, the output becomes

$$y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

since $y(n) \to \infty$ as $n \to \infty$, the system is unstable.
(c) Static, linear, timevariant, causal, stable.
(d) Dynamic, linear, time invariant, noncausal, stable.
(e) Static, nonlinear, time invariant, causal, stable.
(f) Static, nonlinear, time invariant, causal, stable.
(g) Static, nonlinear, time invariant, causal, stable.
(h) Static, linear, time invariant, causal, stable.
(i) Dynamic, linear, time variant, noncausal, unstable. Note that the bounded input $x(n) = u(n)$ produces an unbounded output.
(j) Dynamic, linear, time variant, noncausal, stable.
(k) Static, nonlinear, time invariant, causal, stable.
(l) Dynamic, linear, time invariant, noncausal, stable.
(m) Static, nonlinear, time invariant, causal, stable.
(n) Static, linear, time invariant, causal, stable.

2.6

(a) True. If

$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$
$$v_2(n) = \mathcal{T}_1[x_2(n)],$$

then

$$\alpha_1 x_1(n) + \alpha_2 x_2(n)$$

yields

$$\alpha_1 v_1(n) + \alpha_2 v_2(n)$$

by the linearity property of $\mathcal{T}_1$. Similarly, if

$$y_1(n) = \mathcal{T}_2[v_1(n)] \text{ and}$$
$$y_2(n) = \mathcal{T}_2[v_2(n)],$$

then

$$\beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$$

by the linearity property of $\mathcal{T}_2$. Since

$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$

$$v_2(n) = \mathcal{T}_2[x_2(n)],$$

it follows that

$$A_1 x_1(n) + A_2 x_2(n)$$

yields the output

$$A_1 \mathcal{T}[x_1(n)] + A_2 \mathcal{T}[x_2(n)],$$

where $\mathcal{T} = \mathcal{T}_1 \mathcal{T}_2$. Hence $\mathcal{T}$ is linear.
(b) True. For $\mathcal{T}_1$, if

$$x(n) \rightarrow v(n) \text{ and}$$

$$x(n-k) \rightarrow v(n-k),$$

For $\mathcal{T}_2$, if

$$v(n) \rightarrow y(n)$$

$$\text{and} v(n-k) \rightarrow y(n-k).$$

Hence, For $\mathcal{T}_1 \mathcal{T}_2$, if

$$x(n) \rightarrow y(n) \text{ and}$$

$$x(n-k) \rightarrow y(n-k)$$

Therefore, $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ is time invariant.
(c) True. $\mathcal{T}_1$ is causal $\Rightarrow v(n)$ depends only on x(k) for $k \leq n$. $\mathcal{T}_2$ is causal $\Rightarrow y(n)$ depends only on $v(k)$ for $k \leq n$. Therefore, $y(n)$ depends only on $x(k)$ for $k \leq n$. Hence, $\mathcal{T}$ is causal.
(d) True. Combine (a) and (b).
(e) True. This follows from $h_1(n) * h_2(n) - h_2(n) * h_1(n)$
(f) False. For example, consider

$$\mathcal{T}_1 : y(n) = nx(n) \text{ and}$$

$$\mathcal{T}_2 : y(n) = nx(n+1).$$

Then,

$$\begin{aligned}
\mathcal{T}_2[\mathcal{T}_1[\delta(n)]] &= \mathcal{T}_2(0) = 0. \\
\mathcal{T}_1[\mathcal{T}_2[\delta(n)]] &= \mathcal{T}_1[\delta(n+1)] \\
&= -\delta(n+1) \\
&\neq 0.
\end{aligned}$$

(g) False. For example, consider

$$\mathcal{T}_1 : y(n) = x(n) + b \text{ and}$$

$$\mathcal{T}_2 : y(n) = x(n) - b, \text{ where } b \neq 0.$$

Then,

$$\mathcal{T}[x(n)] = \mathcal{T}_2[\mathcal{T}_1[x(n)]] = \mathcal{T}_2[x(n) + b] = x(n).$$

Hence $\mathcal{T}$ is linear.
(h) True.

$$\mathcal{T}_1 \text{ is stable } \Rightarrow v(n) \text{ is bounded if } x(n) \text{ is bounded.}$$

$$\mathcal{T}_2 \text{ is stable } \Rightarrow y(n) \text{ is bounded if } v(n) \text{ is bounded .}$$

Hence, y(n) is bounded if x(n) is bounded $\Rightarrow \mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ is stable.
(i) Inverse of (c). $\mathcal{T}_1$ and for $\mathcal{T}_2$ are noncausal $\Rightarrow \mathcal{T}$ is noncausal. Example:

$$\begin{aligned}
\mathcal{T}_1 : y(n) &= x(n+1) \text{ and} \\
\mathcal{T}_2 : y(n) &= x(n-2) \\
\Rightarrow \mathcal{T} : y(n) &= x(n-1),
\end{aligned}$$

which is causal. Hence, the inverse of (c) is false.
Inverse of (h): $\mathcal{T}_1$ and/or $\mathcal{T}_2$ is unstable, implies $\mathcal{T}$ is unstable. Example:

$$\mathcal{T}_1 : y(n) = e^{x(n)}, \text{ stable and } \mathcal{T}_2 : y(n) = ln[x(n)], \text{ which is unstable.}$$

But $\mathcal{T} : y(n) = x(n),$ which is stable. Hence, the inverse of (h) is false.

2.8

since

$$x_1(n) + x_2(n) = \delta(n)$$

and the system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \left\{0, \underset{\uparrow}{3}, -1, 2, 1\right\}.$$

If the system were time invariant, the response to $x_3(n)$ would be

$$\left\{\underset{\uparrow}{3}, 2, 1, 3, 1\right\}.$$

But this is not the case.

2.11

(a)

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ \sum_n y(n) &= \sum_n \sum_k h(k)x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k) \\ &= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right) \end{aligned}$$

(b) (1)

$$y(n) = h(n) * x(n) = \{1, 3, 7, 7, 7, 6, 4\}$$

$$\sum_n y(n) = 35, \qquad \sum_k h(k) = 5, \qquad \sum_k x(k) = 7$$

(2)

$$y(n) = \{1, 4, 2, -4, 1\}$$

$$\sum_n y(n) = 4, \qquad \sum_k h(k) = 2, \qquad \sum_k x(k) = 2$$

(3)

$$y(n) = \left\{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\right\}$$

$$\sum_n y(n) = -5, \qquad \sum_n h(n) = 2.5, \qquad \sum_n x(n) = -2$$

(4)

$$y(n) = \{1, 2, 3, 4, 5\}$$

$$\sum_n y(n) = 15, \qquad \sum_n h(n) = 1, \qquad \sum_n x(n) = 15$$

(5)

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \qquad \sum_n h(n) = 4, \qquad \sum_n x(n) = 2$$

(6)

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \qquad \sum_n h(n) = 2, \qquad \sum_n x(n) = 4$$

(7)

$$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$$

$$\sum_n y(n) = -2, \quad \sum_n h(n) = -1, \quad \sum_n x(n) = 2$$

(8)

$$y(n) = u(n) - u(n-1) + 2u(n-2)$$

$$\sum_n y(n) = \infty, \quad \sum_n h(n) = \infty, \quad \sum_n x(n) = 4$$

(9)

$$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$$

$$\sum_n y(n) = 0, \quad \sum_n h(n) = 0, \quad \sum_n x(n) = 4$$

(10)

$$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$$

$$\sum_n y(n) = 36, \quad \sum_n h(n) = 6, \quad \sum_n x(n) = 6$$

(11)

$$y(n) = [2(\frac{1}{2})^n - (\frac{1}{4})^n]u(n)$$

$$\sum_n y(n) = \frac{8}{3}, \quad \sum_n h(n) = \frac{4}{3}, \quad \sum_n x(n) = 2$$

2.13

$$\begin{aligned} y(n) &= \sum_{k=0}^{4} h(k)x(n-k), \\ x(n) &= \left\{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots, \alpha^5\right\} \\ h(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\} \end{aligned}$$

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{4} x(n-k), -3 \le n \le 9 \\
&= 0, \text{ otherwise.}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(-3) &= \alpha^{-3}, \\
y(-2) &= x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}, \\
y(-1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \\
y(0) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 \\
y(1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \\
y(2) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2} \\
y(3) &= \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}, \\
y(4) &= \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1 \\
y(5) &= \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(6) &= \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5} \\
y(7) &= \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(8) &= \alpha^{4} + \alpha^{5}, \\
y(9) &= \alpha^{5}
\end{aligned}
$$

2.17

(a)

$$
\begin{aligned}
\delta(n) &= \gamma(n) - a\gamma(n-1) \text{ and,} \\
\delta(n-k) &= \gamma(n-k) - a\gamma(n-k-1). \text{ Then,} \\
x(n) &= \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \\
&= \sum_{k=-\infty}^{\infty} x(k)[\gamma(n-k) - a\gamma(n-k-1)]
\end{aligned}
$$

$$
\begin{aligned}
x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1) \\
x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k) \\
&= \sum_{k=-\infty}^{\infty} [x(k) - ax(k-1)]\gamma(n-k) \\
\text{Thus, } c_k &= x(k) - ax(k-1)
\end{aligned}
$$

(b)

$$
\begin{aligned}
y(n) &= \mathcal{T}[x(n)] \\
&= \mathcal{T}[\sum_{k=-\infty}^{\infty} c_k\gamma(n-k)] \\
&= \sum_{k=-\infty}^{\infty} c_k\mathcal{T}[\gamma(n-k)] \\
&= \sum_{k=-\infty}^{\infty} c_k g(n-k)
\end{aligned}
$$

(c)

$$
\begin{aligned}
h(n) &= \mathcal{T}[\delta(n)] \\
&= T[\gamma(n) - a\gamma(n-1)] \\
&= g(n) - ag(n-1)
\end{aligned}
$$

2.20

$$\begin{aligned}
h(n) &= h_1(n) * h_2(n) \\
&= \sum_{k=-\infty}^{\infty} a^k [u(k) - u(k-N)][u(n-k) - u(n-k-M)] \\
&= \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k-M) \\
&\quad - \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k-M) \\
&= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k \right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-M} a^k \right) \\
&= 0
\end{aligned}$$

2.22

(c)

$$\begin{aligned}
x(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\right\} \\
h(n) &= \left\{2, \underset{\uparrow}{2}, 2, 2\right\} \\
N_1 &= -2, \\
N_2 &= 4, \\
M_1 &= -1, \\
M_2 &= 2,
\end{aligned}$$

Partial overlap from left: $n = -3 \quad n = -1 \quad L_1 = -3$

Full overlap: $n = 0 \quad n = 3$

Partial overlap from right:$n = 4 \quad n = 6 \quad L_2 = 6$

(a) $L_1 = N_1 + M_1$ and $L_2 = N_2 + M_2$
(b) Partial overlap from left:

$$\text{low } N_1 + M_1 \qquad \text{high } N_1 + M_2 - 1$$

$$\text{Full overlap: low } N_1 - M_2 \qquad \text{high } N_2 + M_1$$

Partial overlap from right:

$$\text{low } N_2 + M_1 + 1 \qquad \text{high } N_2 + M_2$$

2.24

First, we determine

$$\begin{aligned} s(n) &= u(n) * h(n) \\ s(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\ &= \sum_{k=0}^{n} h(n-k) \\ &= \sum_{k=0}^{\infty} a^{n-k} \\ &= \frac{a^{n+1}-1}{a-1}, n \geq 0 \end{aligned}$$

For $x(n) = u(n+5) - u(n-10)$, we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10)$$

From figure P2.33,

$$\begin{aligned} y(n) &= x(n) * h(n) - x(n) * h(n-2) \\ \text{Hence, } y(n) &= \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10) \\ &\quad - \frac{a^{n+4}-1}{a-1}u(n+3) + \frac{a^{n-11}-1}{a-1}u(n-12) \end{aligned}$$

2.27

We may use the result in problem 2.36 with $a = \frac{1}{2}$. Thus,

$$y(n) = 2\left[1 - (\frac{1}{2})^{n+1}\right]u(n) - 2\left[1 - (\frac{1}{2})^{n-9}\right]u(n-10)$$

2.31

(a) Refer to fig 2.46-1
(b) Refer to fig 2.46-2

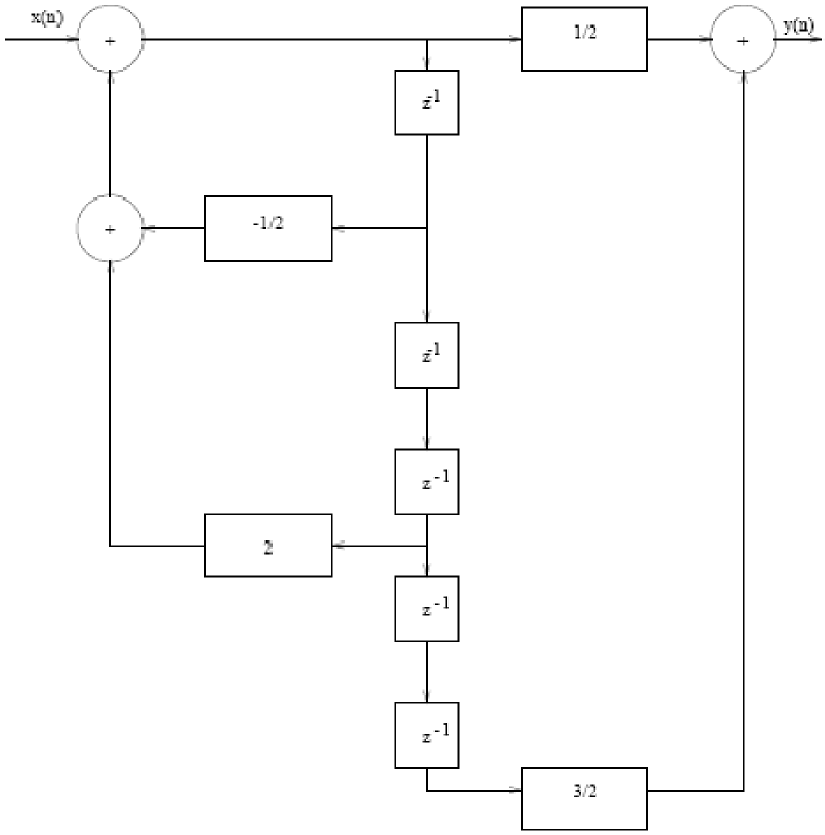


Figure 2.46-1:

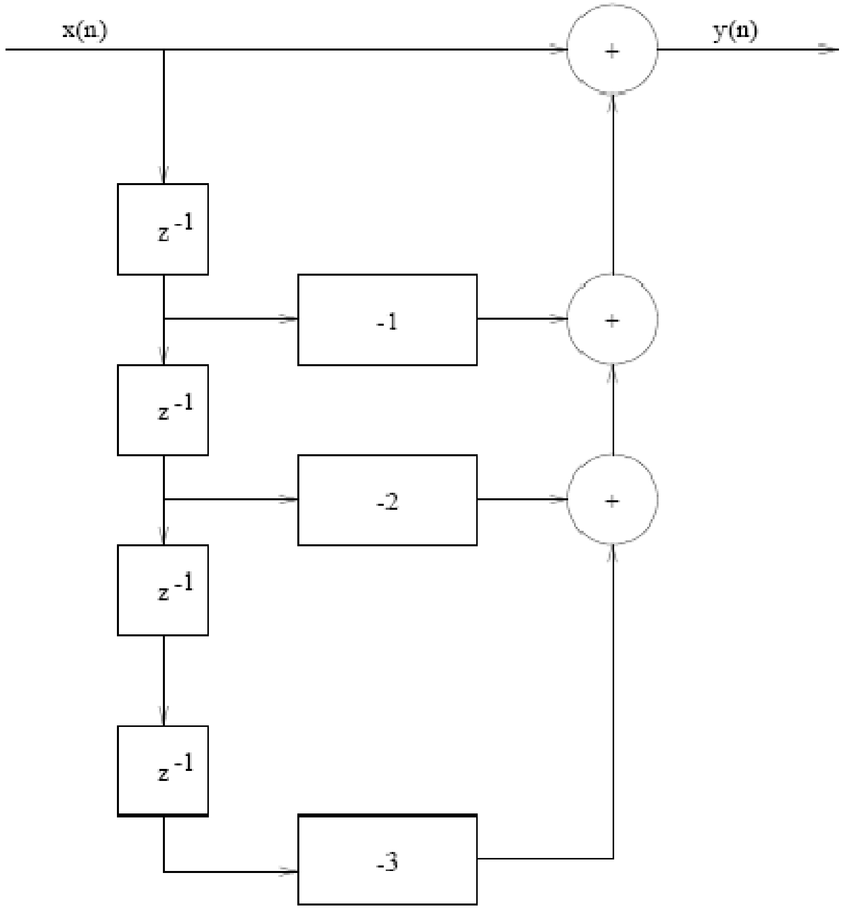


Figure 2.46-2:

2.34

$y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$
(a)For $x(n) = \delta(n)$, we have

$$\begin{aligned} y(0) &= 1, \\ y(1) &= 2.9, \\ y(2) &= 5.61, \\ y(3) &= 5.049, \\ y(4) &= 4.544, \\ y(5) &= 4.090, \ldots \end{aligned}$$

(b)

$$\begin{aligned}
s(0) &= y(0) = 1, \\
s(1) &= y(0) + y(1) = 3.91 \\
s(2) &= y(0) + y(1) + y(2) = 9.51 \\
s(3) &= y(0) + y(1) + y(2) + y(3) = 14.56 \\
s(4) &= \sum_{0}^{4} y(n) = 19.10 \\
s(5) &= \sum_{0}^{5} y(n) = 23.19
\end{aligned}$$

(c)

$$\begin{aligned}
h(n) &= (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2) \\
&= \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)
\end{aligned}$$

2.36

(a)

$$\begin{aligned}
y(n) &= \frac{1}{2} y(n-1) + x(n) + x(n-1) \\
\text{For } y(n) - \frac{1}{2} y(n-1) &= \delta(n), \text{ the solution is} \\
h(n) &= (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)
\end{aligned}$$

(b) $h_1(n) * [\delta(n) + \delta(n-1)] = (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$.

2.40

$$\begin{aligned}
x(n) &= x(n) * \delta(n) \\
&= x(n) * [u(n) - u(n-1)] \\
&= [x(n) - x(n-1)] * u(n) \\
&= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)
\end{aligned}$$

2.43

(a)

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n-l)$$

$$
\begin{aligned}
&= \sum_{n=-\infty}^{\infty} [s(n) + \gamma_1 s(n-k_1) + \gamma_2 s(n-k_2)] * \\
&\quad [s(n-l) + \gamma_1 s(n-l-k_1) + \gamma_2 s(n-l-k_2)] \\
&= (1+\gamma_1^2+\gamma_2^2)\gamma_{ss}(l) + \gamma_1 [\gamma_{ss}(l+k_1) + \gamma_{ss}(l-k_1)] \\
&\quad + \gamma_2 [\gamma_{ss}(l+k_2) + \gamma_{ss}(l-k_2)] \\
&\quad + \gamma_1\gamma_2 [\gamma_{ss}(l+k_1-k_2) + \gamma_{ss}(l+k_2-k_1)]
\end{aligned}
$$

(b) $\gamma_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. Suppose that $k_1 < k_2$. Then, we can determine γ_1 and k_1. The problem is to determine γ_2 and k_2 from the other peaks.
(c) If $\gamma_2 = 0$, the peaks occur at $l = 0$ and $l = \pm k_1$. Then, it is easy to obtain γ_1 and k_1.

2.44

(a) The shift at which the crosscorrelation is maximum is the amount of delay D.
(b) variance = 0.01. Refer to fig 2.65 1.
(b) Delay D = 20. Refer to fig 2.65-1.

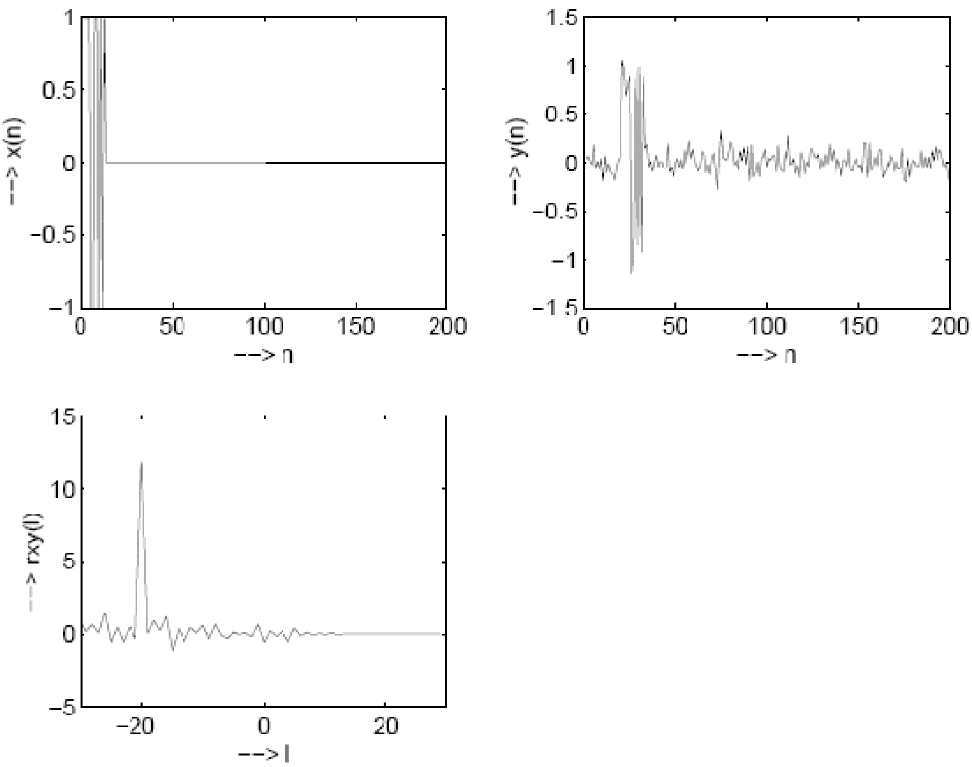


Figure 2.65-1: variance = 0.01

(c) variance = 0.1. Delay D = 20. Refer to fig 2.65-2.

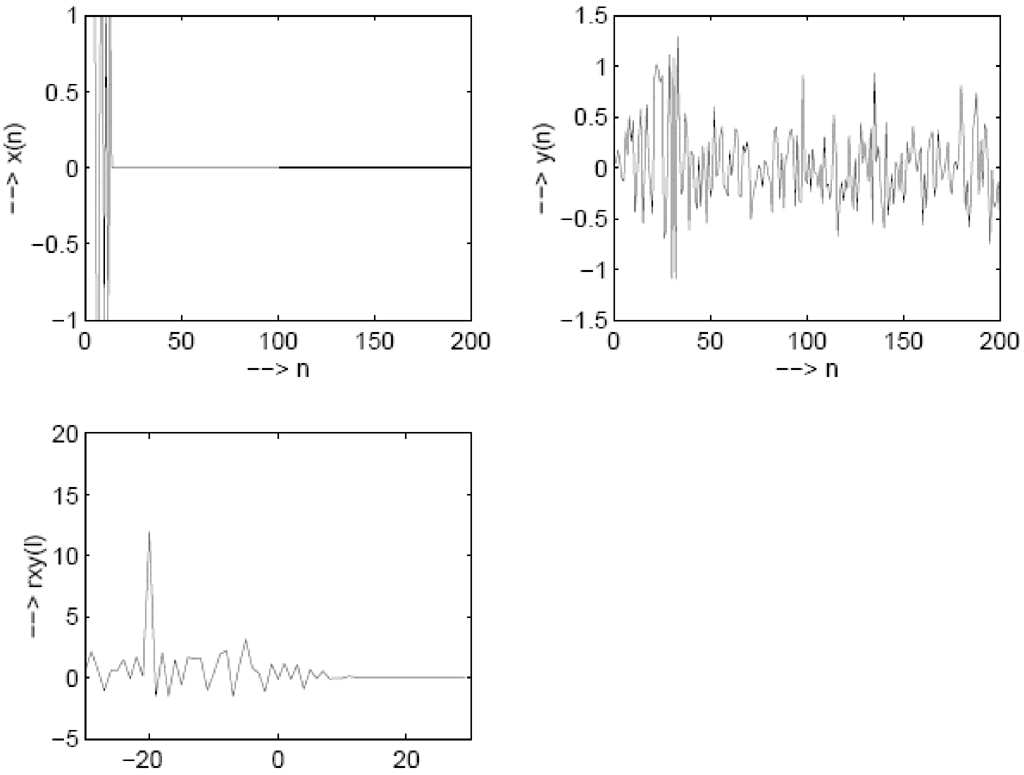


Figure 2.65-2: variance = 0.1

(d) Variance = 1. delay D = 20. Refer to fig 2.65-3.

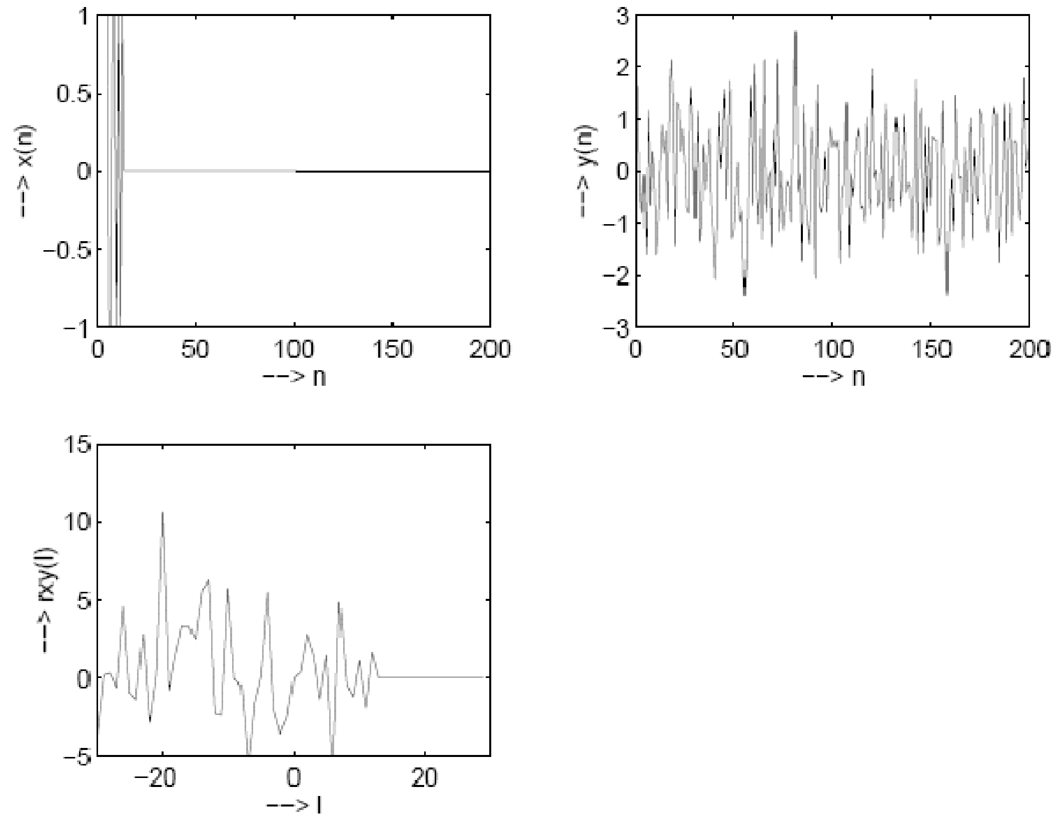
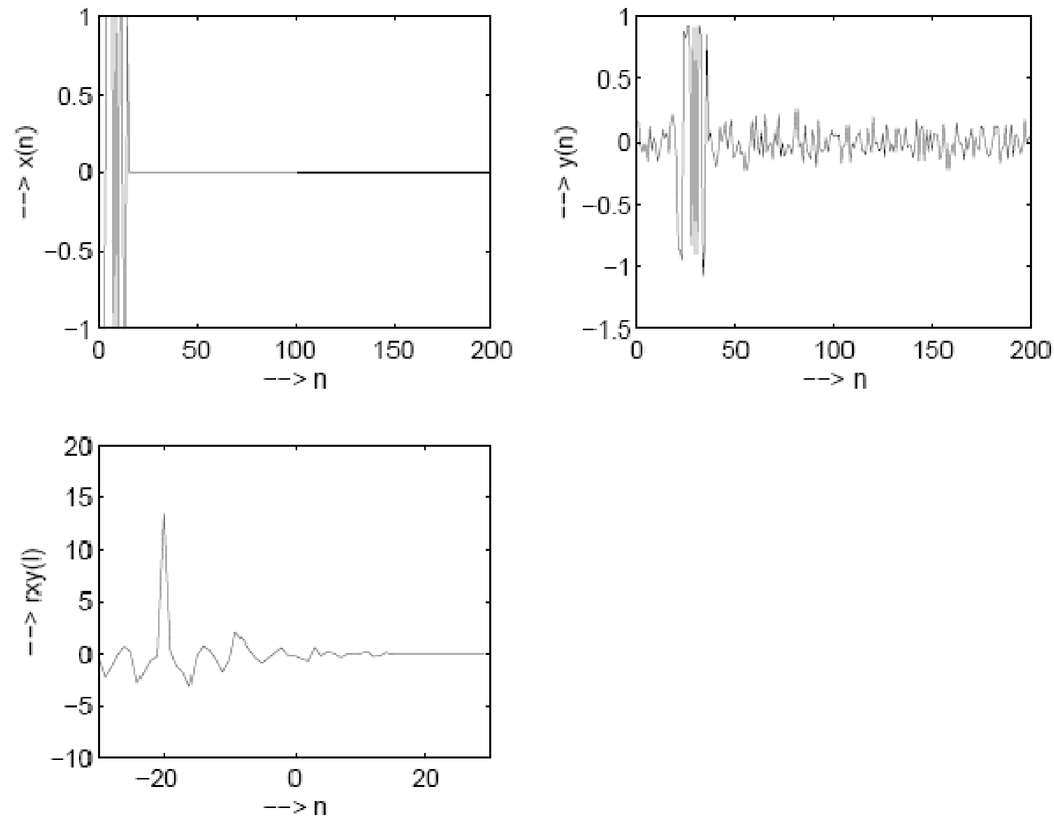


Figure 2.65-3: variance = 1

(e) $x(n) = \{-1, -1, -1, +1, +1, +1, +1, -1, +1, -1, +1, +1, -1, -1, +1\}$. Refer to fig 2.65-4.



(f) Refer to fig 2.65-5.

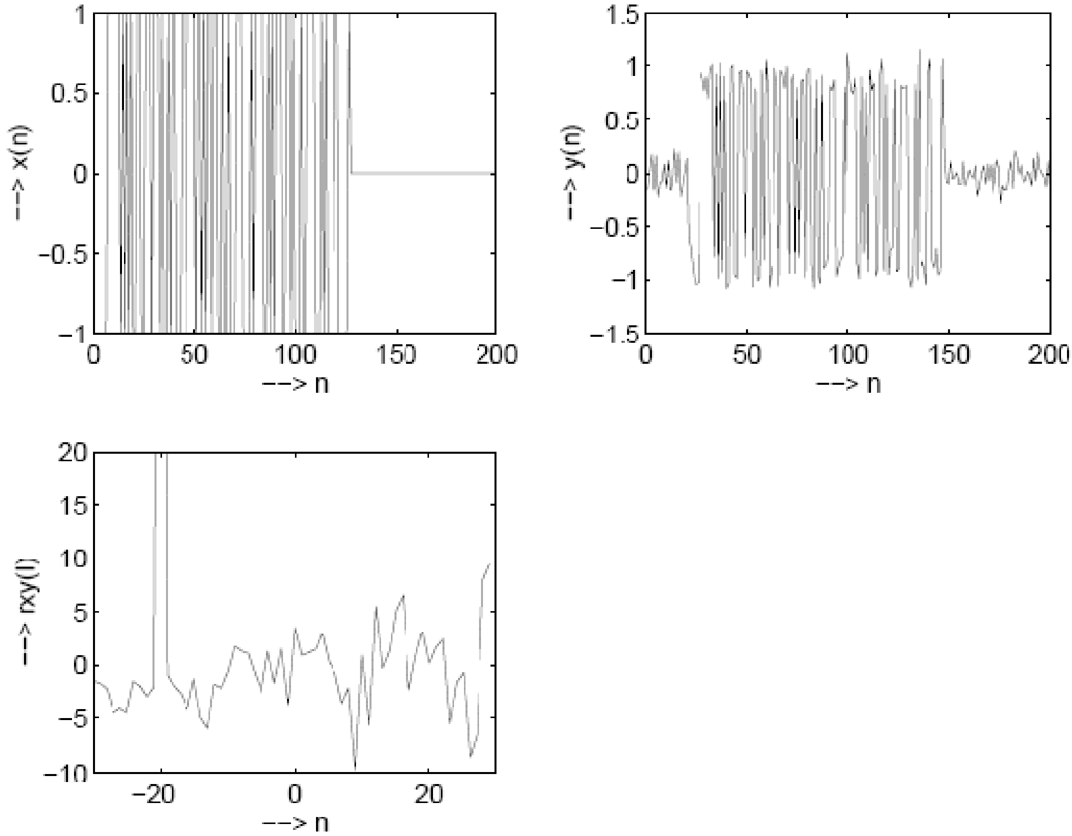


Figure 2.65-5: