A Time Varying Filtering Theory for Constrained Traffic Regulation and Dynamic Service Guarantees

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Abstract—**By extending the filtering theory under the $(\min, +)$-algebra to the time varying setting, we solve the problem of constrained traffic regulation and develop a calculus for dynamic service guarantees. For a constrained traffic regulation problem with maximum tolerable delay d and maximum buffer size q, the optimal regulator that generates the output traffic conforming to a subadditive envelope f and minimizes the number of discarded packets is a concatenation of the g-clipper with $g(t) = \min[f(t+d), f(t)+q]$ and the maximal f-regulator. The g-clipper is a *bufferless* device which optimally *drops* packets as necessary in order that its output be conformant to an envelope g. The maximal f-regulator is a *buffered* device that *delays* packets as necessary in order that its output be conformant to an envelope f. The f-regulator is a linear time invariant filter with impulse response f, under the $(\min, +)$-algebra.**

To provide dynamic service guarantees in a network, we develop the concept of a dynamic server as a basic network element. Dynamic servers can be joined by concatenation, "filter bank summation," and feedback to form a composite dynamic server. We also show that dynamic service guarantees for multiple input streams sharing a work conserving link can be achieved by a dynamic SCED (Service Curve Earliest Deadline) scheduling algorithm, if an appropriate admission control is enforced.

Keywords—**constrained traffic regulation, traffic clipping, service guarantees, min-plus algebra, scheduling, admission control**

I. Introduction

Future high speed digital networks aim to provide integrated services, including voice, video, fax, and data. To control interaction among traffic generated by different sources, traffic regulation seems inevitable. In [12], Cruz proposed the following deterministic traffic characterization. A traffic stream, described by a non-decreasing sequence $A \equiv \{A(t), t = 0, 1, 2, \}$ (with $A(0) = 0$), conforms to a function f, called an *envelope*, if

$$A(t) - A(s) \le f(t-s), \quad \forall s \le t.$$

Without loss of generality, an envelope f can be assumed to be subadditive [8], i.e., $f(s) + f(t-s) \ge f(t)$ for all $s \le t$. Using this characterization, a calculus is developed in [12], [13] to compute deterministic performance measures, such as bounds on delay and bounds on queue length. Traffic regulation addresses the problem of modifying a traffic stream so that it conforms to a subadditive envelope f. The problem of traffic regulation was treated systematically in [10]. There it is shown that the optimal traffic regulator that generates an output B conforming to a subadditive envelope f for an input A is a linear time invariant filter with the impulse response f under the $(\min, +)$-algebra, i.e.,

$$B(t) = \min_{0 \le s \le t}[A(s) + f(t-s)].$$

We call such a filter the maximal f-regulator. This characterization was also observed in [1][2][20].

As the buffer in the maximal f-regulator is assumed to be infinite, packets from the input might be queued at the regulator. For a real-time service, the delay of a queued packet at the regulator might exceed its maximum tolerable delay and such a packet should be discarded (i.e. clipped). The problem of traffic regulation with such a delay constraint is called the constrained traffic regulation problem in [17]. Its objective is to find a regulator that not only generates traffic conforming to an envelope, but also minimizes the number of discarded packets. In addition to the delay constraint, Konstantopoulos and Anantharam [17] also considered the buffer constraint for the regulator. For $f(t) = \rho t + \sigma$, they derived optimal traffic regulators that satisfied either the delay constraint or the buffer constraint.

Cruz and Taneja [16] considered the zero delay case of the constrained traffic regulation problem. This is also the case without any buffer. By extending the time invariant filtering theory under the $(\min, +)$-algebra to the time varying setting, it is shown there that the departure process of the optimal zero-delay regulator, that generates a departure process conformant to f, is the subadditive closure [10] of the arrival process convolved with f. Such a bufferless regulator is called the f-clipper in [16].

Le Boudec and Thiran [7] used the nonlinear filtering theory under the $(\min, +)$-algebra to find the representations of the optimal regulators for the constrained traffic regulation problems that have either the delay constraint or the buffer constraint. Their representations are in the form of the closure of nonlinear operators, and seem difficult to implement. Though they did not address the implementation issue of the optimal regulators, they were able to use the representations to obtain comparison results for various suboptimal traffic regulation schemes. For these suboptimal schemes, their performance bounds, in terms of the cumulative losses, are much easier to obtain and could be used as bounds for the performance of the optimal regulators.

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Motivated by all these works, one of the main objectives of this paper is to provide an optimal and implementable solution for the general constrained traffic regulation problem with both the delay constraint and the buffer constraint. As in [16], our approach is based on the time varying filtering theory under the $(\min, +)$-algebra. By extending the subadditive closure in [10] to the time varying setting, we show that the f-clipper with input A and output B can be implemented by the recursive equation as follows:

$$B(t) = \min\Big[B(t-1)+A(t)-A(t-1), \min_{0\le s<t}[B(s)+f(t-s)]\Big].$$

The computation complexity of the f-clipper is almost the same as that of the maximal f-regulator. The recursive equation also implies that the f-clipper is greedy. Packets are discarded only when needed.

For the constrained traffic regulation problem with maximum tolerable delay d and maximum buffer size q, the optimal traffic regulator is shown to be a concatenation of the g-clipper with $g(t) = \min[f(t)+q, f(t+d)]$ and the maximal f-regulator. The solution is intuitive as the output from the g-clipper conforms to the envelope g that yields bounded delay d and bounded queue length q at the maximal f-regulator. For example, when $f(t) = \min_{1\le i\le K}[\rho_i t + \sigma_i]$, the corresponding g-clipper can be implemented by K parallel bufferless $(\sigma_i + \min[q, \rho_i d], \rho_i)$-leaky buckets. A packet is discarded if it cannot be admitted to one of these K leaky buckets. The output from the g-clipper is then fed into K parallel (σ_i, ρ_i)-leaky buckets.

The time varying filtering theory can also be used for dynamic service guarantees. By extending the concept of the service curve in [14], [1], [6] to a bivariate function $F(\cdot,\cdot)$, we define a dynamic F-server for an input A if its output B satisfies

$$B(t) \ge \min_{0\le s\le t}[A(s) + F(s,t)], \quad \forall t.$$

Analogous to the time invariant filtering theory in [10], [1], [6], a dynamic F-server can be viewed as a linear filter with the time varying impulse response F. It can be combined by concatenation, "filter bank summation," and feedback to form a composite dynamic server. We illustrate the use of the dynamic server by considering a work conserving link with a time varying capacity and a dynamic window flow control problem. We also show that dynamic service guarantees for multiple input streams sharing a work conserving link can be achieved by a dynamic SCED (Service Curve Earliest Deadline) scheduling algorithm, if an appropriate admission control is enforced. As the SCED algorithm in [19], the dynamic SCED algorithm is an EDF (Earliest Deadline First) policy that schedules packets according to their deadlines.

The remainder of the paper is organized as follows. In the next section, we introduce the time varying filtering theory under the $(\min, +)$-algebra. The development is parallel to the time invariant filtering theory in [10], [1], [6]. The reader is also referred to [3], [4], which contains results overlapping with this paper. In Section III, we solve the problem of constrained traffic regulation by introducing dynamic maximal regulators and dynamic maximal clippers. In Section IV, we develop the concept of dynamic servers and their associated calculus. We show in Section V that the dynamic SCED algorithm can be used to achieve dynamic service guarantees. We conclude the paper in Section VI by discussing possible extensions and applications.

II. Time varying filtering theory under the min-plus algebra

In the section, we introduce the time varying filtering theory under the $(\min, +)$-algebra. The development is parallel to the time invariant filtering theory in [10], [1], [6]. To extend the $(\min, +)$-algebra to the time varying setting, we consider the family of bivariate functions.

$$\tilde{\mathcal{F}} = \{F(\cdot,\cdot) : F(s,t) \ge 0, F(s,t) \le F(s,t+1), \text{ for all } 0 \le s \le t\}$$

Thus, for any $F \in \tilde{\mathcal{F}}$, $F(s,t)$ is nonnegative and nondecreasing in t. For any two bivariate functions F and G in $\tilde{\mathcal{F}}$, we say $F = G$ (resp. $F \le G$) if $F(s,t) = G(s,t)$ (resp. $F(s,t) \le G(s,t)$) for all $0 \le s \le t$. We define the following two operations for functions in $\tilde{\mathcal{F}}$.

(i) (min) the pointwise minimum of two functions:

$$(F \oplus G)(s,t) = \min[F(s,t), G(s,t)].$$

(ii) (convolution) the convolution of two functions under the $(\min, +)$-algebra:

$$(F \star G)(s,t) = \min_{s\le\tau\le t}[F(s,\tau) + G(\tau,t)].$$

One can easily verify that $(\tilde{\mathcal{F}}, \oplus, \star)$ is a dioid (see e.g., [5]) with the zero function $\tilde{\epsilon}$ and the identity function $\tilde{\mathbf{e}}$, where $\tilde{\epsilon}(s,t) = \infty$ for all $s \le t$, and $\tilde{\mathbf{e}}(s,t) = 0$ if $s = t$ and ∞ otherwise. To be precise, we have the following properties.

1. (Associativity) $\forall F, G, H \in \tilde{\mathcal{F}}$,

$$(F \oplus G) \oplus H = F \oplus (G \oplus H),$$
$$(F \star G) \star H = F \star (G \star H).$$

2. (Commutativity) $\forall F, G \in \tilde{\mathcal{F}}$,

$$F \oplus G = G \oplus F.$$

3. (Distributivity) $\forall F, G, H \in \tilde{\mathcal{F}}$,

$$(F \oplus G) \star H = (F \star H) \oplus (G \star H),$$
$$H \star (F \oplus G) = (H \star F) \oplus (H \star G).$$

4. (Zero element) $\forall F \in \tilde{\mathcal{F}}$,

$$F \oplus \tilde{\epsilon} = F.$$

5. (Absorbing zero element) $\forall F \in \tilde{\mathcal{F}}$,

$$F \star \tilde{\epsilon} = \tilde{\epsilon} \star F = \tilde{\epsilon}.$$

6. (Identity element) $\forall F \in \tilde{\mathcal{F}}$,

$$F \star \tilde{\mathbf{e}} = \tilde{\mathbf{e}} \star F = F.$$

7. (Idempotency of addition) $\forall F \in \tilde{\mathcal{F}}$,

$$F \oplus F = F.$$

The key difference to the time invariant filtering theory is that we do not have the commutative property for $\star$ in $(\tilde{\mathcal{F}}, \oplus, \star)$, i.e., $F \star G \neq G \star H$ in general.

Let $\tilde{\mathcal{F}}_0 = \{F \in \tilde{\mathcal{F}} : F \oplus \tilde{\mathbf{e}} = F\}$. That is, a function $F \in \tilde{\mathcal{F}}_0$ if $F(t,t) = 0$ for all t. As in the time invariant case, we still have the following monotonicity.

8. (Monotonicity) $\forall F \leq \tilde{F}, G \leq \tilde{G}$,

$$F \oplus G \leq \tilde{F} \oplus \tilde{G} \leq \tilde{F},$$
$$F \star G \leq \tilde{F} \star \tilde{G}.$$

If F (resp. G) is in $\tilde{\mathcal{F}}_0$, then $F \star G \leq G$ (resp. $F \star G \leq F$). If both F and G are in $\tilde{\mathcal{F}}_0$, then $F \oplus G \geq F \star G$.

In addition to the algebraic properties, we present several important properties in Lemmas II.1-II.3 that will be used to prove results for constrained traffic regulation and service guarantees. The proofs for these results are identical to those in [11], [10] and thus omitted.

Lemma II.1: Consider two sequences of bivariate functions $\{F_m, m = 1, 2, \ldots\} \in \tilde{\mathcal{F}}$ and $\{G_m, m = 1, 2, \ldots\} \in \tilde{\mathcal{F}}$. If both sequences are non-increasing in m, then

$$(\lim_{m\to\infty} F_m) \star (\lim_{m\to\infty} G_m) = \lim_{m\to\infty} (F_m \star G_m).$$

An immediate corollary of Lemma II.1 is the distributivity holds for infinite "sums."

9. (Distributivity for infinite "sums") For any two sequences of functions F_m and G_m in $\tilde{\mathcal{F}}$,

$$\begin{aligned}(F_1 \oplus F_2 \oplus \ldots \oplus F_m \oplus \ldots) \star (G_1 \oplus G_2 \oplus \ldots \oplus G_m \oplus \ldots) \\ = (F_1 \star G_1) \oplus (F_1 \star G_2) \oplus (F_2 \star G_1) \oplus \ldots \oplus (F_m \star G_m) \\ \oplus \ldots \qquad (1)\end{aligned}$$

For any function $F \in \tilde{\mathcal{F}}$, define the unitary operator (called the closure operation in this paper)

$$F^* = \lim_{n\to\infty} (F \oplus \tilde{\mathbf{e}})^{(n)} = \lim_{n\to\infty} (\tilde{\mathbf{e}} \oplus F \oplus F^{(2)} \oplus \ldots \oplus F^{(n)}), \quad (2)$$

where $F^{(n)}$ is the self convolution of F for n times, i.e., $F^{(n)} = F^{(n-1)} \star F$, $n \geq 2$ and $F^{(1)} = F$.

Lemma II.2: Suppose that $F, G \in \tilde{\mathcal{F}}$.

(i) (Monotonicity) If $F \leq G$, then $F^* \leq G^*$.

(ii) (Closure properties) $F^* = F^* \oplus \tilde{\mathbf{e}} = F^* \star F^* = (F^*)^{(m)} = (F^*)^* \leq F \oplus \tilde{\mathbf{e}} \leq F$.

(iii) (Maximum solution) F^* is the maximum solution of the equation $H = (H \star F) \oplus \tilde{\mathbf{e}}$, i.e., for any H satisfying $H = (H \star F) \oplus \tilde{\mathbf{e}}$, $H \leq F^*$.

(iv) F^* can be computed recursively from the following equations:

$$F^*(s,s) = 0,$$
$$F^*(s,t) = \min_{s \leq \tau < t} [F^*(s,\tau) + F(\tau,t)].$$

(v) $(F \oplus G)^* = (F^* \oplus G^*)^* = (F^* \star G^*)^*$.

Lemma II.3: (Feedback)

(i) For the equation

$$B = (B \star F) \oplus A, \quad (3)$$

$B = A \star F^*$ is the maximum solution.

(ii) If $\inf_t F(t,t) > 0$, then $B = A \star F^*$ is the unique solution.

(iii) Under the condition in (ii), if

$$B \geq (B \star F) \oplus A,$$

then $B \geq A \star F^*$.

Remark II.4: As in [10], let $\mathcal{F} = \{f : f(0) \geq 0, f(s) \leq f(t), s \leq t\}$ be the set of nonnegative and non-decreasing functions. Also, let $\mathcal{F}_0$ be the subset of functions in $\mathcal{F}$ with $f(0) = 0$. Then one may define the convolution of a function $f \in \mathcal{F}$ and a bivariate function $G \in \tilde{\mathcal{F}}$ as follows:

$$(f \star G)(t) = \min_{0 \leq s \leq t} [f(s) + G(s,t)].$$

Under such a definition, $f \star G$ is in $\mathcal{F}$. One may view $f \star G$ as a special case of $F \star G$ for some $F \in \tilde{\mathcal{F}}$ with $F(0,t) = f(t)$ for all t and $F(s,t) = \infty$, for all t and $s > 0$. Thus, the results in Lemma II.3 still hold.

III. CONSTRAINED TRAFFIC REGULATION

Given a sequence $A \in \mathcal{F}_0$, it is defined in [12], [13] that A conforms to the (static) upper envelope $f \in \mathcal{F}_0$ if $A(t) - A(s) \leq f(t-s)$ for all $s \leq t$. It is also shown in [10], [1] that the optimal traffic regulator that generates output traffic conforming to a subadditive envelope f is a linear time invariant filter with the impulse response f under the $(\min, +)$-algebra. However, packets (or cells) may have to be queued and delayed at such a regulator. In practice, there might be constraints for the buffer size and the delay. In this regard, one might have to discard (i.e. clip) some packets from the input so that the buffer and delay constraints can be satisfied. The question is then how one discard packets *optimally* so that the number of clipped packets can be minimized. Such a problem is called constrained traffic regulation and was first considered in [17] for (σ, ρ)-leaky buckets. Our objective of this section is to provide a general, simple and optimal solution for the constrained traffic regulation problem.

We start from extending the definition of a static envelope to a dynamic envelope.

Definition III.1: A sequence $A \in \mathcal{F}_0$ is said to conform to the dynamic upper envelope $F \in \tilde{\mathcal{F}}_0$ if for all $s \leq t$ there holds $A(t) - A(s) \leq F(s,t)$.

As in [10], [11], this characterization has the following equivalent statements. The proof is omitted.

Lemma III.2: Suppose that $A \in \mathcal{F}_0$ and $F \in \tilde{\mathcal{F}}_0$. The following statements are equivalent.

(i) A conforms to the dynamic upper envelope F.

(ii) $A = A \star F$.

(iii) $A = A \star F^*$.

(iv) A conforms to the dynamic upper envelope F^*.

Given a dynamic upper envelope $F \in \tilde{\mathcal{F}}_0$, one can construct a regulator such that for any input $A \in \mathcal{F}_0$, the output from the regulator conforms to the dynamic upper envelope F. This is shown in the following theorem. Once again, the proof is omitted.

Theorem III.3: Suppose that $A \in \mathcal{F}_0$ and $F \in \tilde{\mathcal{F}}_0$. Let $B = A \star F^*$.
(i) (Traffic regulation) B conforms to the dynamic upper envelope F^* and thus B also conforms to the dynamic upper envelope F.
(ii) (Flow constraint) $B \le A$.
(iii) (Optimality) For any $\tilde{B} \in \mathcal{F}_0$ that satisfies (i) and (ii), one has $\tilde{B} \le B$.
(iv) (Conformity) A conforms to the dynamic upper envelope F if and only if $B = A$.

The construction $B = A \star F^*$ is called the *maximal dynamic F-regulator* (for the input A).

As in the time invariant case, the flow constraint $B \le A$ corresponds to one of the causal conditions in [17] as the number of departures cannot be larger than the number of arrivals. Theorem III.3(iii) shows that under the flow constraint and the constraint that the output traffic conforms to the dynamic upper envelope F, the maximal F-regulator is the best construction that one can implement.

The maximal dynamic F-regulator solves the traffic regulation problem with infinite buffer. In [16], the traffic regulation problem without buffer is solved. However, the solution in [16] cannot be easily implemented directly. In the following lemma, we present a recursive construction for the solution.

Theorem III.4: Suppose that $A \in \mathcal{F}_0$ and $F \in \tilde{\mathcal{F}}_0$. Let $B(t) = (\hat{A} \oplus F)^*(0,t)$, where $\hat{A}(s,t) = A(t) - A(s)$. Then the following statements hold.
(i) (Traffic regulation) B conforms to the dynamic upper envelope F.
(ii) (Clipping constraint) $B(t) - B(t-1) \le A(t) - A(t-1)$ for all t.
(iii) (Optimality) For any $\tilde{B} \in \mathcal{F}_0$ that satisfies (i) and (ii), one has $\tilde{B} \le B$.
(iv) B can be constructed by the following recursive equation:

$$B(t) = \min\Big[B(t-1)+A(t)-A(t-1), \min_{0 \le s < t}[B(s)+F(s,t)]\Big], \tag{4}$$

with $B(0) = 0$.
(v) (Conformity) A conforms to the dynamic upper envelope F if and only if $B = A$.
The construction in (4) is called the *maximal dynamic F-clipper* (for the input A) in the paper.

Proof. For any $s \le t$ we have

$$\begin{aligned} B(t) &= (\hat{A} \oplus F)^*(0,t) \\ &\le (\hat{A} \oplus F)^*(0,s) + (\hat{A} \oplus F)(s,t) \\ &\le B(s) + F(s,t)\ , \end{aligned}$$

and hence $B \le B \star F$, so that B is conformant to F, establishing (i).

To see (ii), note similarly that

$$\begin{aligned} B(t) &= (\hat{A} \oplus F)^*(0,t) \\ &\le (\hat{A} \oplus F)^*(0,t-1) + (\hat{A} \oplus F)(t-1,t) \\ &\le B(t-1) + \hat{A}(t-1,t)\ . \end{aligned}$$

Next, we establish (iii). Suppose that $\tilde{B} \in \mathcal{F}_0$ satisfies (i) and (ii). Since $\tilde{B}(0) = 0$,

$$\tilde{B} \le \mathbf{e}, \tag{5}$$

where $\mathbf{e}(0) = 0$ and $\mathbf{e}(s) = \infty$ for $s > 0$. As $\tilde{B}$ conforms to the dynamic envelope F,

$$\tilde{B} \le \tilde{B} \star F. \tag{6}$$

The inequality in the clipping constraint in (ii) is equivalent to $\tilde{B}(t) - \tilde{B}(s) \le A(t) - A(s)$ for all $s \le t$ and it can be rewritten as

$$\tilde{B} \le \tilde{B} \star \hat{A}, \tag{7}$$

with $\hat{A}(s,t) = A(t) - A(s)$. The constraints in (5)-(7) are equivalent to

$$\tilde{B} = \tilde{B} \oplus (\tilde{B} \star F) \oplus (\tilde{B} \star \hat{A}) \oplus \mathbf{e}. \tag{8}$$

Applying the distributivity and the fact that $\hat{A} \in \tilde{\mathcal{F}}_0$ yields

$$\begin{aligned} \tilde{B} &= (\tilde{B} \star (\tilde{\mathbf{e}} \oplus \hat{A} \oplus F)) \oplus \mathbf{e} \\ &= (\tilde{B} \star (\hat{A} \oplus F)) \oplus \mathbf{e} \end{aligned}$$

It then follows from Lemma II.3(i) that $\mathbf{e} \star (\hat{A} \oplus F)^*$ is the maximum solution of (8). Note that

$$(\mathbf{e} \star (\hat{A} \oplus F)^*)(t) = (\hat{A} \oplus F)^*(0,t) = B(t).$$

Thus, B is the maximum solution that satisfies (i) and (ii).

To see (iv), note from Lemma II.2(iv) that B can be constructed recursively as follows:

$$\begin{aligned} B(t) &= \min_{0 \le s < t}\Big[B(s) + \min[A(t) - A(s), F(s,t)]\Big] \\ &= \min\Big[\min_{0 \le s < t}[B(s) + A(t) - A(s)], \min_{0 \le s < t}[B(s) + F(s,t)]\Big], \end{aligned} \tag{9}$$

with $B(0) = 0$. Since B satisfies the clipping constraint,

$$\begin{aligned} B(s) + A(t) - A(s) &= B(s) + A(t-1) - A(s) \\ &\qquad + A(t) - A(t-1) \\ &\ge B(s) + B(t-1) - B(s) + A(t) - A(t-1) \\ &= B(t-1) + A(t) - A(t-1). \end{aligned}$$

This implies that

$$\min_{0 \le s < t}[B(s) + A(t) - A(s)] = B(t-1) + A(t) - A(t-1).$$

Thus,

$$B(t) = \min\Big[B(t-1)+A(t)-A(t-1), \min_{0 \le s < t}[B(s)+F(s,t)]\Big]. \tag{10}$$

To prove (v), note that if $B = A$, then it follows from (10) that $A = A \star F$. Thus, A conforms to the dynamic envelope F. On the other hand, if A conforms to the dynamic envelope F, then

$$A(t) - A(s) \le F(s,t).$$

This implies $\hat{A} \oplus F = \hat{A}$. As $\hat{A}$ is linear, $\hat{A}^{(2)}(s,t) = \hat{A}(s,t) = A(t) - A(s)$. We have $\hat{A}^* = \hat{A}$ and

$$B(t) = (\hat{A} \oplus F)^*(0,t) = A(t) - A(0) = A(t).$$

■

We note that the original representation in [16] is that $B(t) = (\hat{A} \star F^*)^*(0,t)$. This is equivalent to our result in Theorem III.4 from Lemma II.2(v) and $\hat{A}^* = \hat{A}$. As $(\hat{A} \oplus F)^* = (\hat{A} \oplus F^*)^*$ in Lemma II.2(v), one also has the following equivalent implementation

$$B(t)=\min\left[B(t-1)+A(t)-A(t-1),\ \min_{0\le s<t}[B(s)+F^*(s,t)]\right]. \quad (11)$$

Note that the key difference between Theorem III.3 and Theorem III.4 is the clipping constraint. The clipping constraint implies that in any given slot, the packets departing are a subset of the packets arriving in the same slot. Let $\ell(t)=A(t)-A(t-1)-(B(t)-B(t-1))$ be the number of packets clipped at time t. From (4), we have

$$\ell(t)=\max\left[0,B(t-1)+A(t)-A(t-1)-\min_{0\le s<t}[B(s)+F(s,t)]\right]. \quad (12)$$

Observe from (12) that packet loss occurs at time t only when at least one of the following inequalities is violated

$$(B(t-1)+A(t)-A(t-1))-B(s)\le F(s,t),\quad s=0,1\dots,t-1. \quad (13)$$

When this happens, one then discards packets to the extent so that the above inequalities are all satisfied. Note also that (4) implies that the maximal dynamic F-clipper can be implemented in real-time, since the value of $B(t)$ depends only on $B(s-1)$ and $A(s)$ for $s\le t$.

For the maximal dynamic F-clipper with the input A and the output B, let $L(t)=A(t)-B(t)$ be the cumulative losses at the clipper by time t. Let $S=\{t_0,t_1,t_2\dots t_m\}$ be a subset of $\{1,2,\dots,t\}$ (with $t_0=0<t_1<t_2<\dots<t_m=t$). Expanding (4) recursively yields

$$L(t)=\sup_S\sum_{i=1}^m[A(t_i)-A(t_{i-1})-F(t_{i-1},t_i)]^+. \quad (14)$$

This was previously shown in [16], Corollary 1. A similar result was also obtained in [7] for the continuous time setting.

Now we compare the output from the maximal dynamic $F_1\oplus F_2$-clipper and a concatenation of the maximal dynamic F_1-clipper and the maximal dynamic F_2-clipper. Let A be the input to both systems, B_1 be the output from the maximal dynamic F_1-clipper, B_2 be the output from the maximal dynamic F_2-clipper, and B be the output from the maximal dynamic $F_1\oplus F_2$-clipper. Also let $L(t)=A(t)-B(t)$ be the cumulative losses at the maximal dynamic $F_1\oplus F_2$-clipper by time t. Similar, let $L_1(t)=A(t)-B_1(t)$ and $L_2(t)=B_1(t)-B_2(t)$. From Theorem III.4, we have for all $s\le t$,

$$B_1(t)-B_1(s)\le A_1(t)-A_1(s),$$
$$B_1(t)-B_1(s)\le F_1(s,t),$$
$$B_2(t)-B_2(s)\le B_1(t)-B_1(s),$$
$$B_2(t)-B_2(s)\le F_2(s,t).$$

This implies that B_2 conforms to the dynamic upper envelope $F_1\oplus F_2$ and that $B_2(t)-B_2(t-1)\le A(t)-A(t-1)$. Thus, $B_2\le B$ and $L(t)\le L_1(t)+L_2(t)$ for all t, by Theorem III.4. In fact, a concatenation of the maximal dynamic F_1-clipper and the maximal dynamic F_2-clipper is a suboptimal implementation of an $F_1\oplus F_2$-clipper. The reason for this, as observed in [16], is that the discarding of packets in the F_2-clipper is not accounted for in the F_1-clipper.

Once we understand that clippers in tandem is suboptimal and may yield more cumulative losses than the optimal one, we may use this to compare the cumulative losses for clippers in parallel. Now suppose both the maximal dynamic F_1-clipper and the maximal dynamic F_2-clipper are fed with the input A. Let B'_1 and B'_2 be the outputs from these two clippers and $L'_1(t)=A(t)-B'_1(t)$ and $L'_2(t)=A(t)-B'_2(t)$ be the cumulative losses at these two clippers by time t. Clearly, $L'_1(t)=L_1(t)$. It is easy to see from (14) that $L'_2(t)\ge L_2(t)$. Thus, we still have $L(t)\le L'_1(t)+L'_2(t)$ for all t. This is previously reported in [16], Corollary 2. We note similar comparison results were also reported in [7].

Now we consider the problem of constrained traffic regulation with buffer and delay constraints. Let A be the input and B be the output from the regulator. We require that the buffer occupancy in the regulator be less than or equal to q, the delay be bounded above by d, and that the output B be conformant to a dynamic envelope F. Due to these constraints, packets may need to be discarded. Let A_1 be the effective input, i.e. $A_1(t)$ counts the total number of packets arriving up to and including slot t which eventually depart the regulator without being discarded. The objective is to maximize the effective input A_1 and the output B, given the buffer and delay constraints and the constraint that B conforms to the dynamic envelope F. More formally, given the input A and a dynamic envelope F, we seek A_1 and B which are as large as possible subject to the following constraints.

(C1) (Clipping constraint) $A_1(t)-A_1(t-1)\le A(t)-A(t-1)$ for all t.

(C2) (Buffer constraint) $A_1(t)-q\le B(t)$ for all t, where q is the buffer size at the regulator.

(C3) (Delay constraint) $A_1(t)\le B(t+d)$ for all t, where d is the maximum tolerable delay at the regulator.

(C4) (Traffic regulation) B conforms to the dynamic upper envelope F.

(C5) (Flow constraint) $B(t)\le A_1(t)$ for all t.

The clipping constraint implies that the packets in the effective input A_1 is a subset of the packets in A for any time t. We note that the clipping constraint does not imply that packets arriving at time t have to be clipped at time t. In fact, they could be clipped at some time later than t. However, as we will show below that optimal clipping can be greedy and only those packets arriving at time t need to be clipped at time t. Note also that we use the buffer constraint $A_1(t)-B(t)\le q$, rather than $A(t)-B(t)\le q$, even though the backlog of packets in the regulator at the end of slot t is $A(t)-B(t)$. Since $A_1\le A$, the constraint C2 is in fact less restrictive than the natural buffer constraint $A(t)-B(t)\le q$. However, as the theorem below shows, the optimal value of $A_1(t)$ can be computed without knowledge of $A(s)$ for $s>t$, so that packets which will eventually be discarded in an optimal clipper can in fact be discarded when they arrive. Assuming this is the case, the backlog of packets in the optimal regulator at the end of slot t is $A_1(t)-B(t)$.

Theorem III.5: Suppose that $A\in\mathcal{F}_0$ and $F\in\tilde{\mathcal{F}}_0$. Let A_1 be the output from the maximal dynamic G-clipper for the input A, where

$$G(s,s)=0,\quad \forall s, \quad (15)$$
$$G(s,t)=\min[F^*(s,t+d),F^*(s,t)+q],\quad \forall s<t. \quad (16)$$

Also, let B be the output from the maximal dynamic F-regulator for the input A_1. Then all the constraints [C1-5] are satisfied. Moreover, for any $\tilde{A}_1,\tilde{B}\in\mathcal{F}_0$ that satisfy [C1-C5], one has $\tilde{A}_1\le A_1$ and $\tilde{B}\le B$.

The construction of A_1 and B, based on a concatenation of the maximal dynamic G-clipper and the maximal dynamic F-regulator, is called the maximal dynamic F-regulator with delay d and buffer q.

Proof. Suppose that A_1 and B are as stated in the theorem. Theorem III.4 then implies [C1], and also that $A_1(t) \le (A_1 \star G)(t)$. Conditions [C4-C5] follow from Theorem III.3 (i) and (ii). To establish [C2], note that

$$\begin{aligned} A_1(t) - q &\le (A_1 \star G)(t) - q \\ &\le \min_{0 \le s \le t} [A_1(s) + F^*(s,t) + q] - q \\ &= (A_1 \star F^*)(t) \\ &= B(t) . \end{aligned}$$

Similarly, to establish [C3], note that

$$\begin{aligned} A_1(t) &\le (A_1 \star G)(t) \\ &\le \min_{0 \le s \le t} \{A_1(s) + F^*(s, t+d)\} . \end{aligned}$$

Since $A_1(t) \le A_1(s)$ for $s > t$ and F^* is non-negative, it therefore follows that $A_1(t) \le (A_1 \star F^*)(t+d) = B(t+d)$, which establishes [C3]. Thus, [C1-C5] are satisfied as claimed.

Next, suppose that $\tilde{A}_1, \tilde{B} \in \mathcal{F}_0$ satisfy [C1-C5]. From Theorem III.3 (iii), we know that under the flow constraint in [C5] and the traffic constraint [C4], we have

$$\tilde{B} \le \tilde{A}_1 \star F^*. \tag{17}$$

Moreover, combining this with [C2] and [C3], we obtain

(C2′) (Buffer constraint) $\tilde{A}_1(t) - q \le (\tilde{A}_1 \star F^*)(t)$ for all t.

(C3′) (Delay constraint) $\tilde{A}_1(t) \le (\tilde{A}_1 \star F^*)(t+d)$ for all t.

The buffer constraint in (C2′) can be rewritten as

$$\tilde{A}_1 \le \tilde{A}_1 \star F_2 \tag{18}$$

with $F_2(s,t) = F^*(s,t) + q$. Since $\tilde{A}_1(t) \in \mathcal{F}_0$ is non-decreasing in t and $F^*(s,t)$ is nonnegative,

$$\tilde{A}_1(s) + F^*(s, t+d) \ge \tilde{A}_1(t), \quad s = t+1, \ldots, t+d. \tag{19}$$

Thus, the conditions in (19) are redundant and the delay constraint in (C3′) can be rewritten as

$$\tilde{A}_1 \le \tilde{A}_1 \star F_3 \tag{20}$$

with $F_3(s,t) = F^*(s, t+d)$. Using the idempotency and distributivity, the constraints in (18) and (20) are equivalent to

$$\tilde{A}_1 = \tilde{A}_1 \oplus \tilde{A}_1 \le (\tilde{A}_1 \star F_2) \oplus (\tilde{A}_1 \star F_3) = \tilde{A}_1 \star (F_2 \oplus F_3) \tag{21}$$

Note that $(F_2 \oplus F_3)(s,t) = G(s,t)$ for all $s < t$, where G is defined in (15). Thus, $\tilde{A}_1$ conforms to the dynamic envelope G. Using Theorem III.4(iii) and the assumption that $\tilde{A}_1$ satisfies [C1], it therefore follows that $\tilde{A}_1(t) \le (\hat{A} \oplus G)^*(0,t) = A_1(t)$. From the monotonicity of $\star$, we also have from (17) that

$$\tilde{B} \le \tilde{A}_1 \star F^* \le A_1 \star F^* = B.$$

■

For the special cases that $d = \infty$ (without delay constraint) and that $q = \infty$ (without buffer constraint), our result in Theorem III.5 recovers the results of Examples 4 and 5 in [7]. We note that the results in [7] only find a representation of the optimal traffic regulator that satisfies either the delay constraint or the buffer constraint. They did not address the implementation issue of such a regulator. In [17], the buffer constraint and the delay constraint are treated separately and it is shown that the optimal solution can be implemented by the greedy flow controller which discards packets only when needed. As shown in Theorem III.5, the maximal dynamic F-regulator with delay d and buffer q is still the greedy flow controller as the maximal dynamic G-clipper discards packets only when needed.

Now consider the maximal dynamic F-regulator with delay d and buffer q when

$$F(s, s+t) = \min_{1 \le i \le K} [\rho_i t + \sigma_i], t > 0.$$

This corresponds to the case of multiple leaky buckets with the delay constraint d and the buffer constraint q. In this case,

$$\begin{aligned} G(s,t) &= \min \Big[\min_{1 \le i \le K} [\rho_i (t+d-s) + \sigma_i], \\ &\qquad \min_{1 \le i \le K} [\rho_i (t-s) + \sigma_i] + q \Big] \\ &= \min_{1 \le i \le K} \Big[\rho_i (t-s) + \sigma_i + \min[q, \rho_i d] \Big]. \end{aligned}$$

Thus, one can construct the maximal dynamic G-clipper by feeding the input to K parallel bufferless $(\sigma_i + \min[q, \rho_i d], \rho_i)$-leaky buckets. A packet is discarded (or clipped) if it cannot be admitted to one of these K leaky buckets. The output from the maximal dynamic G-clipper is then fed into another K parallel (σ_i, ρ_i)-leaky buckets with buffer q.

IV. Dynamic service guarantees

To guarantee end-to-end deterministic quality-of-service for an input, the concept of service curves is developed in [14], [1], [6] to work with the static envelopes in Cruz [12], [13]. A static f-server ($f \in \mathcal{F}_0$) for an input sequence A if its output sequence $B \equiv \{B(t), t = 0, 1, 2, \ldots\}$ satisfies

$$B(t) \ge \min_{0 \le s \le t} [A(s) + f(t-s)] \tag{22}$$

for all t. Based on this, there is an associated filtering theory (under the $(\min, +)$-algebra) in [10], [1], [6] that eases design and computation of deterministic QoS. Our main objective of this section is to extend the concept of service curves and the associated filtering theory to the time varying setting so that *dynamic* QoS can be guaranteed. The theory is based on the following definition of a dynamic F-server.

Definition IV.1: (Dynamic F-server) A server is called a dynamic F-server ($F \in \tilde{\mathcal{F}}_0$) for an input sequence A if its output sequence satisfies $B \ge A \star F$, i.e.,

$$B(t) \ge \min_{0 \le s \le t} [A(s) + F(s,t)] \tag{23}$$

for all t. If the inequality in (23) is satisfied for all input sequences, then we say the dynamic F-server is universal. If the inequality in (23) is an equality, we say the dynamic F-server is exact.

Analogous to the filtering theory for static service curves, one may view the right hand side of (23) as the output from a linear filter with the time varying impulse response $F(s,t)$ under the $(\min, +)$-algebra. If $F(s,t) = f(t-s)$ for some non-decreasing function f, then a dynamic F-server is equivalent to a static f-server.

Clearly, the maximal dynamic F-regulator is a *universal* and *exact* dynamic F^*-server. Analogous to the time invariant case, one has the following properties in Theorems IV.2-IV.5 for dynamic F-servers. The proofs are omitted as they are identical to those in [10], [11].

Theorem IV.2: (Concatenation) A concatenation of a dynamic F_1-server for an input sequence A and a dynamic F_2-server for the output from the dynamic F_1-server is a dynamic F-server for A, where $F = F_1 \star F_2$.

Theorem IV.3: (Filter bank summation) Consider an input sequence A. Let B_1 (resp. B_2) be the output from a dynamic F_1-server (resp. F_2-server) for A. The output from the "filter bank summation", denoted by B, is $B_1 \oplus B_2$. Then the "filter bank summation" of a dynamic F_1-server for A and a dynamic F_2-server for A is a dynamic F-server for A, where $F = F_1 \oplus F_2$.

Theorem IV.4: (Feedback) Consider an input sequence $A \in \mathcal{F}_0$ and a dynamic F-server for B, where $B = A \oplus A_1$, and A_1 is the output from the dynamic F-server. If $\inf_t F(t,t) > 0$, then the feedback system is a dynamic F^*-server for A.

Theorem IV.5: Consider a dynamic F_2-server for A. Let B be the output. Also, let $q = \sup_{t\geq 0}[A(t) - B(t)]^+$ be the maximum queue length at the server, where $x^+ = \max(0, x)$. Let $d = \inf\{d \geq 0 : B(t+d) \geq A(t) \quad \text{for all} \quad t\}$ be the maximum delay at the server. Suppose that A conforms to the dynamic upper envelope F_1.
(i) (Queue length) $q \leq \sup_s \max_{t\geq s}[F_1^*(s,t) - F_2(s,t)]^+$.
(ii) (Output burstiness) If $B \leq A$, then B conforms to the dynamic upper envelope F_3^*, where

$$F_3(s,t) = \max_{0\leq\tau\leq s}[F_1^*(\tau,t) - F_2(\tau,s)]^+.$$

(iii) (Delay) $d \leq \inf\{d \geq 0 : \sup_s \max_{t\geq s}[F_1^*(s,t) - F_2(s,t+d)] \leq 0\}$.

As the maximal dynamic F-regulator is a dynamic F^*-server, there is an intuitive explanation why the maximal dynamic F-regulator with delay d and buffer q is a concatenation of the maximal dynamic G-clipper (with G being defined in (15)) and the maximal dynamic F-regulator. As shown in Theorem III.4, the output from the maximal dynamic G-clipper conforms to the dynamic envelope $G(s,t) = \min[F^*(s,t+d), F^*(s,t)+q]$. When such an output is fed to the maximal dynamic F-regulator, one has from Theorem IV.5 that the delay at the maximal dynamic F-regulator is bounded above by d and the queue length is also bounded above by q. Thus, both the delay constraint and the buffer constraint are satisfied.

In the following, we illustrate the use of dynamic service guarantees by a work conserving link and a dynamic window flow control problem.

Example IV.6: **(Work conserving link with a time varying capacity)** *Consider a server with a time varying capacity. Let $c(t)$ be the maximum number of packets that can be served at time t, and $C(s,t) = \sum_{\tau=s+1}^{t} c(\tau)$ be the cumulative capacity in the interval $[s+1,t]$. Let $A(t)$ and $B(t)$ be the input and the output from the work conserving link. Denote by $q(t)$ the number of packets at the link at time t. Then the work conserving link is governed by Lindley's equation*

$$q(t+1) = [q(t) + A(t+1) - A(t) - c(t+1)]^+.$$

Suppose $q(0) = 0$. Recursive expansion of Lindley's equation yields

$$q(t) = \max_{0\leq s\leq t}[A(t) - A(s) - C(s,t)].$$

Since $q(t) = A(t) - B(t)$, we have

$$B(t) = \min_{0\leq s\leq t}[A(s) + C(s,t)].$$

Thus, the work conserving link is a universal and exact dynamic C-server. Moreover, as C is linear, $C^{(2)} = C$ and thus $C^* = C$. It is also the maximal dynamic C-regulator.

Example IV.7: **(Dynamic window flow control)** *Consider a network with the input A and the output B. Suppose that the network enforces a dynamic window flow control for the input A with the dynamic window size $w(t)$. We assume that $\inf_t w(t) > 0$. For the dynamic window flow control system, the effective input to the network, denoted by A_1, satisfies*

$$A_1(t) = \min[A(t), B(t) + w(t)]. \tag{24}$$

Observe that $B(t)+w(t) = (B\star H)(t)$, where H is the function with $H(s,t) = \infty$ for $s < t$ and $H(t,t) = w(t)$. One may rewrite (24) as follows:

$$A_1 = A \oplus (B \star H). \tag{25}$$

Also, we assume that the network is a dynamic F-server for the effective input A_1, i.e.,

$$B \geq A_1 \star F. \tag{26}$$

In conjunction with (25),

$$B \geq A_1 \star F = (A \oplus (B \star H)) \star F = (A \star F) \oplus (B \star (H \star F)),$$

where we apply the distributive property and the associativity of $\star$. Since we assume that $\inf_t w(t) > 0$,

$$\inf_t (H \star F)(t,t) = \inf_t [H(t,t) + F(t,t)] \geq \inf_t w(t) > 0.$$

We then have from Lemma II.3(iii) that

$$B \geq A \star F \star (H \star F)^*.$$

Thus, the dynamic window flow control system is a dynamic $F \star (H \star F)^*$-server.

V. THE DYNAMIC SCED SCHEDULING ALGORITHM

In this section, we define a scheduling algorithm, called the dynamic SCED algorithm, which we will show achieves the dynamic service guarantees in Section IV.

Consider a server with a time varying capacity. Let $c(t)$ be the maximum number of packets that can be served at time t, and $C(s,t) = \sum_{\tau=s+1}^{t} c(\tau)$ be the cumulative capacity in the interval $[s+1,t]$. A policy is called the Earliest Deadline First (EDF) if the server schedules the packets according to their deadlines. Note that the EDF policy is work conserving, i.e., the server serves packets whenever there are packets at the server.

Now consider feeding n streams of inputs to such a server. Let $A_i(t)$ be the cumulative number of packet arrivals of the i^{th} stream up to time t. Each packet is assigned a deadline. We assume that the deadlines within the same stream is *non-decreasing*. Also, let $N_i(t)$ be the number of packets from the i^{th} stream that have deadlines not greater than t. As we assume the deadlines for each stream is non-decreasing, packet k from stream i is assigned the deadline $D_{i,k}$ from the following inverse mapping

$$D_{i,k} = \inf\{t : t \geq 0 \text{ and } N_i(t) \geq k\}. \tag{27}$$

Theorem V.1: Suppose that the server is operated under the EDF policy.
(i) A necessary condition for every packet to be served not later than its deadline is

$$\sum_{i=1}^{n} N_i(t) \leq \min_{0\leq s\leq t}[\sum_{i=1}^{n} A_i(s) + C(s,t)], \tag{28}$$

for all t.
(ii) A sufficient condition for every packet to be served not later than its deadline is

$$\sum_{i\in S} N_i(t) \le \min_{0\le s\le t}[\sum_{i\in S} A_i(s) + C(s,t)], \quad (29)$$

for all t and for every S that is a subset of $\{1, 2, \dots, n\}$.

Lemma V.2: Suppose we choose $N_i = A_i \star F_i$ some $F_i \in \tilde{\mathcal{F}}_0, i = 1, \dots, n$. If $\sum_{i=1}^n F_i(s,t) \le C(s,t)$ for all $0 \le s \le t$, then all the packets are served not later than their deadlines.

Such a deadline assignment scheme is called the dynamic SCED algorithm in this paper.

The next lemma implies that deadlines in the dynamic SCED algorithm can be assigned in real-time. Specifically, if packet k from stream i arrives during slot t, $D_{i,k}$ can be computed without knowledge of $A_i(s)$ for $s > t$.

Lemma V.3: Suppose packet k from stream i arrives during slot t. Then under the dynamic SCED algorithm, $D_{i,k} = D_{i,k}(t)$ where

$$D_{i,k}(t) = \inf\{\Delta : \Delta \ge t \text{ and } \min_{0\le u\le t-1}[A_i(u)+F_i(u,\Delta)] \ge k\}. \quad (30)$$

In Theorem V.4, we state the admission criteria for the dynamic SCED algorithm for a server with a time varying capacity.

Theorem V.4: A set of n arrival streams, indexed $i = 1, \dots, n$, arrives to a server. The arrival sequence of the i^{th} stream is denoted by A_i, and is known to conform to the dynamic upper envelope G_i. The server has a time varying capacity to serve up to $c(t)$ packets during slot t. Under the dynamic SCED algorithm, the server is a dynamic F_i-server for A_i, for all $i = 1, \dots, n$ if the following condition is satisfied for all $s \le t$:

$$\sum_{i=1}^n (G_i \star F_i)(s,t) \le C(s,t). \quad (31)$$

VI. Conclusions

By extending the filtering theory under the $(\min, +)$-algebra to the time varying setting, we solved the problem of constrained traffic regulation. For a constrained traffic regulation problem with maximum tolerable delay d and maximum buffer size q, we showed that the optimal regulator that generates the output traffic conforming to a dynamic envelope F and minimizes the number of discarded packets is a concatenation of the maximal dynamic G-clipper with $G(s,t) = \min[F^*(s,t+d), F^*(s,t)+q]$ and the maximal dynamic F-regulator. To provide dynamic service guarantees in a network, we developed the concept of the dynamic F-server as a basic network element. We showed that dynamic servers can be joined by concatenation, "filter bank summation," and feedback to form a composite dynamic server. We also proposed the dynamic SCED scheduling algorithm to achieve dynamic service guarantees for a work conserving link subject to multiple inputs.

One possible application of the time varying filtering theory is dynamic admission control. For a given connection i, we may define a service curve f_i to be guaranteed over the interval $[a_i + 1, b_i]$ if a dynamic service curve F_i is guaranteed, where

$$F_i(s,t) = \begin{cases} 0 & \text{, if } s \le a_i \text{ and } t \le a_i \\ f_i(t-a_i) & \text{, if } s \le a_i \text{ and } a_i \le t \le b_i \\ f_i(t-s) & \text{, if } a_i \le s \le b_i \text{ and } a_i \le t \le b_i \\ f_i(b_i - s) & \text{, if } a_i \le s \le b_i \text{ and } t > b_i \\ 0 & \text{, if } s \ge b_i \text{ and } t \ge b_i \\ f_i(b_i - a_i) & \text{, if } s < a_i \text{ and } t > b_i \text{ .} \end{cases}$$

For such a definition for dynamic service guarantees, an interesting problem is to find the relaxation time r_i such that connection i has virtually no impact on the admission criteria in Theorem V.4 after $b_i + r_i$.

Finally, we note that the bivariate function F could be *random*. By specifying the probabilistic characteristics of the bivariate function F, it is possible to provide probabilistic guarantees. Previous results along this line could be found in [9], [15].

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