Nonadaptive Deterministic Asynchronous Conflict Resolution

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***Abstract*—In this letter, we consider the conflict resolution problem in a discrete-time multiple access channel. Our focus is the duration of achieving the first successful transmission, called the *conflict resolution time*. Assume that each device has a unique ID. Our proposed nonadaptive deterministic algorithms can guarantee deterministic upper bounds on the conflict resolution time even when the clocks of the devices in the channel are not synchronized. Furthermore, on average, our proposed algorithms achieve short conflict resolution time on par with existing randomized algorithms (e.g. independent fair coin-flipping) in the literature.**

***Index Terms*—multiple access, nonadaptive algorithms, worst case analysis.**

I. Introduction

We consider the Asynchronous Conflict Resolution (ACR) problem in a multiple access channel. Assume there are (at most) N devices in a multiple access channel and each device has a unique ID, an integer in $[0, N-1]$ [1]–[3]. Time is assumed to be discrete and partitioned into time slots. If two or more active devices transmit in the same time slot, a collision happens. Collision causes the transmitted messages to be corrupted. On the other hand, if none of the active devices transmit in a time slot, then this time slot is simply wasted. A message can only be successfully transmitted only when exactly one active device transmits it in one time slot. The objective of the conflict resolution problem is to schedule these active devices so that messages can be transmitted successfully. In practice, conflict resolution is typically dealt with by random access methods, such as ALOHA, CSMA and Ethernet [4], [5]. In the literature, tree algorithms using channel feedback and coordinating active devices can resolve conflicts within $O\big(n + \log_2(N/n)\big)$ time slots (where $2 \leq n \leq N$) [6], [7]. If n is known, then there exist *non-adaptive* algorithms (i.e. they do not depend on the feedback information) with the same performance guarantee [8]. However, the key assumptions in these deterministic conflict resolution algorithms is that all clocks of these active devices are perfectly synchronized. Clearly, the above deterministic conflict resolution algorithms in the literature usualy cannot be directly applied. One can only apply random access algorithms instead, but the biggest drawback is the lack of a deterministic lower bound for the number of time slots achieving a successful broadcast. Our objective is to propose deterministic conflict resolution algorithms with a guaranteed lower bound on the number of time slots to achieve a successful broadcast. Our result may be useful in upcoming 5G networks [9].

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II. ACR without unique IDs

A unique identifier (ID) refers to a string associated to a device. If each device has a different ID, then this ID can be used to derive a number related to the transmission time in the design of conflict resolution. Therefore, the assumption of unique IDs can greatly simplify the design of asynchronoous conflict resolution. Now we first consider the general case where there are any number of active devices without unique IDs. Suppose that there are n active, unsynchonized devices among all N devices. For simplicity, the transmission schedule of the j^{th} device can be represented as a sequence of binary variables $\{s_j(t), t \geq 1\}$ for $j = 1, 2, \ldots, N$, in which the j^{th} device transmits a message at its local time t if it is *active* and $s_j(t) = 1$. The transmission schedule is *nonadaptive* if it does not depend on the feedback of the channel. First we are to show that the conflict resolution time of n devices cannot be (upper) bounded by a constant if they both follow the same (randomized) conflict resolution algorithm. This result is presented in Theorem 1. We need to make some definitions in order to facilitate the presentation of Theorem 1.

Definition 1: A *stationary* sequence is a random sequence whose joint probability distribution is invariant over time. If there is an algorithm that generates a stationary binary random sequence $\{s(t), t \geq 1\}$.Such an algorithm is called a *stationary algorithm.*

Definition 2: Suppose n active devices then *independently* generate their transmission schedules using such a stationary algorithm. Let d_j, $i = 1, 2, \ldots, n$, be the clock drift of the j^{th} device to the global clock. Then the conflict resolution time T_{syn} is defined as

$$\mathsf{T}_{\text{syn}} = \min\{t \geq 1 | \text{there exists } 1 \leq j \leq n, \text{ such that } s_j(t + d_j) = 1, s_k(t + d_k) = 0, \forall k \neq j\}. \quad (1)$$

Now we introduce our first result.

Theorem 1: If n unsynchronized devices with clock drifts d_j $(j = 1, 2, \ldots, n)$ *independently* generate their transmission schedules using a stationary algorithm, then the tail distribution of the conflict resolution time T_{syn} in (1) has the following lower bound:

$$\mathsf{P}(\mathsf{T}_{\text{syn}} \geq t + 1) \geq \frac{1}{2^t}. \quad (2)$$

As a result, Theorem 1 clearly shows that it not even possible to find a stationary algorithm such that the conflict resolution time of the two active devices is bounded by a constant (since (2) implied that $\mathsf{P}(\mathsf{T}_{\text{syn}} \geq c) \neq 0$ for any constant $c > 0$), let alone $n \geq 3$.

Proof. Let $Y_j(t) = \big(s_j(1), s_j(2), \ldots, s_j(t)\big)$, $j = 1, 2, \ldots, n$, and $\Gamma(t) = \{0, 1\}^t$. Since all devices use the same algorithm *independently*, these transmission schedules $\{s_j(t), t \geq 1\}$, $j = 1, 2, \ldots, n$ are independent and have the same stationary joint distribution. If $Y_1(t+d_1) = Y_2(t+d_2) = \ldots = Y_n(t+d_n)$, then the conflict resolution time T in (1) must be at least $t+1$, which implies that $\mathsf{P}(\mathsf{T}_{\text{syn}} \geq t+1) \geq \mathsf{P}\big(Y_1(t+d_1) = Y_2(t+d_2) = \ldots = Y_n(t+d_n)\big)$.

$$\begin{aligned} & \mathsf{P}(\mathsf{T}_{\text{syn}} \geq t+1) \\ \geq \;& \mathsf{P}\big(Y_1(t+d_1) = Y_2(t+d_2) = \ldots = Y_n(t+d_n)\big) \\ = \;& \sum_{y \in \Gamma(t)} \mathsf{P}\big(Y_1(t+d_1) = y, \ldots, Y_n(t+d_n) = y\big) \\ = \;& \sum_{y \in \Gamma(t)} \mathsf{P}\big(Y_1(t+d_1) = y\big) \cdots \mathsf{P}\big(Y_n(t+d_n) = y\big) \\ = \;& \sum_{y \in \Gamma(t)} \mathsf{P}\big(Y_1(t) = y\big) \cdots \mathsf{P}\big(Y_n(t) = y\big) \\ = \;& \sum_{y \in \Gamma(t)} \mathsf{P}\big(Y_1(t) = y\big)^n. \end{aligned} \tag{3}$$

Note that $|\Gamma(t)| = 2^t$. Using the Jensen inequality in (3) yields

$$\begin{aligned} & \mathsf{P}(\mathsf{T}_{\text{syn}} \geq t+1) \geq (2^t) \sum_{y \in \Gamma(t)} \frac{1}{2^t} \mathsf{P}\big(Y_1(t) = y\big)^n \\ & \geq (2^t) \left(\sum_{y \in \Gamma(t)} \frac{1}{2^t} \mathsf{P}\big(Y_1(t) = y\big) \right)^n = \frac{1}{2^{(n-1)t}}. \end{aligned} \tag{4}$$

Note that the lower bound in (2) is in fact achieved by flipping independent fair coins at each time. This result shows that flipping independent fair coins is the best strategy for these devices to resolve their conflict if they follow the same stationary algorithm independently.

Theorem 1 shows that the conflict resolution time of any number of devices cannot be bounded by a constant if they all follow the same conflict resolution algorithm. However, since each device has a unique ID, if we take advantage of that the conflict resolution time can be bounded. More precisely, if we can use this unique ID as an input to the stationary algorithm to generate *dependent* transmission schedules, the conflict resolution time may be upper bounded by a constant. In fact, our proposed method (Algorithm 1, to be introduced later) is the **first deterministic** *asynchronous conflict resolution algorithm with a theoretically* **proven upper bound**.

III. ACR WITH UNIQUE IDS

In order to establish an upper bound for the conflict resolution time, we make use of the unique ID property and propose a nonadaptive deterministic conflict resolution algorithm. For simplicity, we focus on a special type of algorithms such that each device transmits exactly twice with a predefined time gap between these two transmissions as follows. Assume that the j^{th} device has ID $j-1$, $j = 1, 2, \ldots, N$. For the j^{th} device with ID $j-1$, it is also given two positive integers: the gap between two transmissions u_j and the period p. The transmission schedule of the j^{th} device according to its own local time t_j is given by

$$s_j(\mathsf{t}_j) = \begin{cases} 1, & \text{if if } \mathsf{t}_j \equiv 0, \text{ or } \mathsf{t}_j \equiv u_j (\text{mod } p) \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Clearly, for any p consecutive time slots, an active device j transmits exactly twice during that time period, and the gap between these two transmissions in that time period is either u_j or $p - u_j$. Now we define the concept of an *effective set of gaps* as follows.

Definition 3: Let there be n unsynchornized devices and each device j transmits exactly twice with gaps u_j $(1 \leq j \leq n)$ according to Algorithm 1. Let t_j denote the j^{th} local time. Then the local time t_j is equivalent to the global time $\mathsf{t}_j + d_j$, where d_j is the clock drift of the j^{th} device. (Note that the j^{th} device has no information about d_j.) Then $\{u_j | 1 \leq j \leq n\}$ is an effective set of gaps with respect to p if, for any $\{d_j | 1 \leq j \leq N\}$, there exists $1 \leq j' \leq N; 1 \leq \mathsf{t}_{j'} \leq p$ such that

$$s_{j'}(\mathsf{t}_{j'}) = 1, s_i(\mathsf{t}_{j'} + d_{j'} - d_i) = 0, \ \forall i \neq j'. \tag{6}$$

Example: Choose $N = 4$. There are only a few combinations and we observe that $\{1, 2, 4, 6\}$ is an effective set of gaps with respect to $p = 7$.

Given any number of devices N. The largest element in the effective set of gaps corresponds to the maximum time to resolve the conflict. In other words, N is an upper bound for the conflict resolution time. We present our nonadaptive deterministic conflict resolution algorithm in Algorithm 1.

Algorithm 1 The nonadaptive deterministic conflict resolution algorithm for any arbitrary number of active devices

Input: $N \in \mathbb{N}$ number of devices. Each device has a unique ID. An effective set of gaps $\mathcal{U} = \{u_j | 1 \leq j \leq N\}$.
Output: A local transmission schedule $s_j(\mathsf{t}_j)$ for each device $j = 1, 2, \ldots, N$ with its local time t_j.

1: Define $s_j(\mathsf{t}_j) = \begin{cases} 1, & \text{if } \mathsf{t}_j \equiv 0, \text{ or } \mathsf{t}_j \equiv u_j (\text{mod } p) \\ 0, & \text{otherwise.} \end{cases}$
2: Schedule the j^{th} device to transmit at its local time t_j if it is active and $s_j(\mathsf{t}_j) = 1$.

The correctness of Algorithm 1 is presented in the following theorem.

Theorem 2: If $\mathcal{U}$ is an effective set of gaps with respect to p, then Algorithm 1 produces a conflict-resolving schedule with a guaranteed conflict resolution time $p - 1$.

Proof. Since $\mathcal{U} = \{u_j | 1 \leq j \leq N\}$ is an effective set of gaps, there exists j' such that, for any $\{d_j | 1 \leq j \leq N\}$, there exists $1 \leq j' \leq N; 1 \leq \mathsf{t}_{j'} \leq p$ such that $s_{j'}(\mathsf{t}_{j'}) = 1, s_i(\mathsf{t}_{j'} + d_{j'} - d_i) = 0, \ \forall i \neq j'$. Since $s_{j'}(\mathsf{t}_{j'}) = 1$, device j' transmits at locat time $\mathsf{t}_{j'}$. According to the definition of $d_{j'}$, device j' actually transmits at global time $\mathsf{t}_{j'} + d_{j'}$. Moreover, none of the devices transmits at global time $d_{j'}$ since $s_i(\mathsf{t}_{j'} + d_{j'} - d_i) = 0, \ \forall i \neq j'$. Conflict is thus resolved at global time $d_{j'}$. Now since every active device transmits exactly *twice* in a period p, it is impossible to have exactly one successful transmission within a period p. Thus, we conclude that there are at least *two* successful transmissions within a period p. This then implies that the conflict resolution time of Algorithm 1 is upper bounded by $p - 1$. ■

Algorithm 1 is nonconstructive since its correctness depends on the selection of effective sets of gaps. In fact, it is very difficult to find such a set. However, the following theorem guarantees the existence as it provides a method to construct one.

Theorem 3: If we choose $\mathcal{U} = \{u_1, \ldots, u_N\}$ such that

$$u_1 = 1, \quad u_j > \sum_{k=1}^{j-1} u_k \quad \forall\ 2 \le j \le N, \quad p = u_N + 1, \quad (7)$$

Then $\mathcal{U}$ is an effective set of gaps with respect to p. In particular, $\{1, 2, 4, \ldots, 2^n\}$ is an effective set of gaps for any $n \in \mathbb{N}$.

Proof. We first show that there is at least one successful transmission within a period p for any number of active devices $2 \le n \le N$. We prove this by contradiction. Suppose there is no successful transmission within a period p. Since each device transmits twice in a period, these transmissions must collide with others. Specifically, if we start from some device j_1, then its two transmissions in the period must collide with the transmissions from other devices. Suppose the first transmission of the j_1^{th} device in the period is at some global time T_0, i.e., $s_{j_0}(\mathsf{T}_0 - d_{j_1}) = 1$. According to the transmission policy, its second transmission is time $\mathsf{T}_1 \equiv \mathsf{T}_0 + u_{j_1} (\text{mod } p)$, i.e., $s_{j_0}(\mathsf{T}_0 - d_{j_1} + u_{j_1}) = 1$. As there is no successful transmission within this period, there must be another device j_2 that also transmits at global time T_1. Since the transmission gap for device j_2 is u_{j_2}, we know that device j_2 transmits another message at global time T_2. Either $\mathsf{T}_2 \equiv \mathsf{T}_1 + u_{j_2} (\text{mod } p)$ or $\mathsf{T}_2 \equiv \mathsf{T}_1 - u_{j_2} (\text{mod } p)$ in this period. In either case, we can represent $\mathsf{T}_2 \equiv \mathsf{T}_1 + (-1)^{k_2} u_{j_2} (\text{mod } p)$ for some binary variable k_2. Again, at global time T_2, there must be another device j_3 that also transmits a message. Once again, device j_3 transmits another message at time T_3 with $\mathsf{T}_3 \equiv \mathsf{T}_2 + (-1)^{k_3} u_{j_3} (\text{mod } p)$ for some binary variable k_3. Since there are only a finite number of active devices, if we repeat this process there must be a repetition in the set $\{\mathsf{T}_j | j = 1, \ldots, N\}$ for the following reason. If there exists $j' \le N$ such that $\mathsf{T}_{j'} \not\equiv \mathsf{T}_i (\text{mod } p)$ for any i. Let $\mathsf{t}_{j'} = \mathsf{T}_{j'} - d_{j'}$ Then we have $s_{j'}(\mathsf{t}_{j'}) = 1$, $s_j(\mathsf{t}_{j'} + d_{j'} - d_i) = 0$ for all i. This leads to a contradition. Therefore, let $i < i' \le N$ be the first repetition such that $\mathsf{T}_i \equiv \mathsf{T}_{i'} (\text{mod } p)$. Therefore, we have

$$\begin{cases} \mathsf{T}_i \equiv \mathsf{T}_0 + (-1)^{k_1} u_{j_1} + \cdots + (-1)^{k_i} u_{j_i} (\text{mod } p). \\ \mathsf{T}_{i'} \equiv \mathsf{T}_0 + (-1)^{k_1} u_{j_1} + \cdots + (-1)^{k_{i'}} u_{j_{i'}} (\text{mod } p). \end{cases} \quad (8)$$

$$\mathsf{T}_{i'} \equiv \mathsf{T}_i + (-1)^{k_i} u_{j_i} + \cdots + (-1)^{k_{i'}} u_{j_{i'}} (\text{mod } p). \quad (9)$$

Since $\mathsf{T}_i \equiv \mathsf{T}_{i'} (\text{mod } p)$, we have

$$(-1)^{k_{i+1}} u_{j_{i+1}} + \cdots + (-1)^{k_{i'}} u_{j_{i'}} \equiv 0 \ (\text{mod } p). \quad (10)$$

Let $J = \max\{j_l | i < l \le i'\}$ be the largest ID of these devices. We have

$$u_J \equiv \sum_{i<l\le i', j_l \ne J} (-1)^{k_J + k_l} u_{j_l} \ (\text{mod } p). \quad (11)$$

$$p > u_J > \sum_{i<l\le i'} u_{j_l} > \sum_{i<l\le i', j_l \ne J} (-1)^{k_J + k_l} u_{j_l} \quad (12)$$

$$-p < -u_J < -\sum_{i<l\le i'} u_{j_l} < \sum_{i<l\le i', j_l \ne J} (-1)^{k_J + k_l} u_{j_l} \quad (13)$$

$$0 < u_J < p \quad (14)$$

(11),(12),(13),(14) together imply

$$u_J = \sum_{i<l\le i', j_l \ne J} (-1)^{k_J + k_l} u_{j_l}, \quad (15)$$

which contradicts to (7). Thus we know that $\mathcal{U}$ is an effective set of gaps with respect to p. ■

The condition (7) is vital as it provides a constructive method for finding an effective set of gaps, which guarantees a deterministic upper bound for the n-party conflict resolution problem. The problem of finding the optimal effecitve set of gaps is quite complicated. We now show some partial results. We need to make the following definition to facilitate our discussions.

Definition 4: in a set of gaps $\mathcal{U}$, a subset $\mathcal{H} = \{u_1, \ldots, u_n\} \subset \mathcal{U}$ forms a collision subset if $\mathcal{H}$ can be partitioned into two subsets $\mathcal{H}_1, \mathcal{H}_2$ such that $\sum_{u \in \mathcal{H}_1} u = \sum_{u \in \mathcal{H}_2} u$.

We introduce the following result.

Theorem 4: Let $\mathcal{U}$ be a set of gaps. If there exists two subsets $\mathcal{H}_1, \mathcal{H}_2$ of $\mathcal{U}$, such that $\mathcal{H}_1 \cap \mathcal{H}_2 = \emptyset$, $\mathcal{H}_1 \cup \mathcal{H}_2 \subset \mathcal{U}$, and $\mathcal{H}_1, \mathcal{H}_2$ both form collision subsets. Then $\mathcal{U}$ is not an effective set of gaps with respect to any $p > 0$.

Proof. Let $\mathcal{H}_1, \mathcal{H}_2$ be two disjoint collision subsets. Without loss of generality, we may assume $\mathcal{H}_1 = \{v_1, \ldots, v_h, v_{h+1}, \ldots, v_{h'}\}$ such that $v_1 + \cdots + v_h = v_{h+1} + \cdots + v_{h'}$, and $\mathcal{H}_2 = \{v'_1, \ldots, v'_l, v'_{l+1}, \ldots, v'_{l'}\}$ such that $v'_1 + \cdots + v'_l = v'_{l+1} + \cdots + v'_{l'}$. We divide into two cases (1) $\mathcal{U} - \mathcal{H}_1 \cup \mathcal{H}_2 \ne \emptyset$ (2) $\mathcal{U} - \mathcal{H}_1 \cup \mathcal{H}_2 = \emptyset$. For case (1), we sort $\mathcal{U}$ as $\mathcal{U} = \{v_1, \ldots, v_{h'}, v'_1, \ldots, v'_{l'}, w_1, \ldots, w_q\}$. More precisely, $\mathcal{U} = \{u_j\}_{j \in I}$ $\left(I = [1, h' + l' + q]\right)$ such that

$$u_j = \begin{cases} v_j, & \text{if } 1 \le j \le h' \\ v'_{j-h'}, & \text{if } h'+1 \le j \le h'+l' \\ w_{j-h'-l'}, & \text{otherwise.} \end{cases} \quad (16)$$

Moreover, we define $\{d_j\}_{j \in I}$ such that

$$d_j = \begin{cases} 0, & \text{if } j = 1, h+1, h'+1, h'+l+1 \\ \sum_{i=1}^{j-1} u_i, & \text{if } 2 \le j \le h \\ \sum_{i=h+1}^{j-1} u_i, & \text{if } h+2 \le j \le h' \\ \sum_{i=h'+1}^{j-1} u_i, & \text{if } h'+2 \le j \le h'+l' \\ \sum_{i=h'+l'+2}^{j-1} u_i, & \text{if } h'+l'+2 \le j \le h'+l'+q. \end{cases} \quad (17)$$

Now we show the following property. For any j, if $s_j(\mathsf{t}_j) = 1$, then there exists i such that $s_i(\mathsf{t}_j + d_j - d_i) = 1$, implying that $\mathcal{U}$ is not an effective set of gaps. According to (16) and (17), device #0 collides with device #1 at global time u_1. Device #1 collides with device #2 at global time $u_1 + u_2$. Device #j collides with device #$j+1$ at global time $u_1 + \cdots + u_j$ for all $j = 1, \ldots, h-1$. Moreover, device #$h+1$ collides with device #0 at global time 0. Device #j collides with device #$j+1$ at global time u_j for $j = h+1, \ldots, h'-1$. Device #h collides with device $\#h'$ at global time $\sum_{j=1}^{h} u_j$ since $\sum_{j=1}^{h} u_j = \sum_{j=h+1}^{h'} u_j$. Note that this result holds for any $p > 0$ as $x = y$ implies $x \equiv y (\text{mod } p)$ in all equations above. Similarly, all transmissions are collided. The second case can be proved in a similar way. We omit the details due to similarity and repetition. ■

IV. Empirical Study of Average Sync Time

Here we empirically study the average conflict resolution time. We consider the scenario with N active devices. The clock drift of each device is uniformly chosen from $[0, 2^N - 1]$. In Figure 1, we simulate the average conflict resolution time, referred to as *Alg 1* in the legend, by averaging over 10,000 experiments for each N. We compare this curve with the average conflict resolution time of the random conflict resolution algorithm (in which each device flips an independent fair coin at each time). This curve of random conflict resolution is referred to as *Rand* in the legend. The average conflict resolution time of the random conflict resolution algorithm is simply $2^N/N$. In Figure 1(a), Alg 1's average conflict resolution time is almost identical to Rand's when N is large. In Figure 1(b), we plot the average conflict resolution times of these two algorithms for $N \leq 10$. Alg 1 is even slightly better for $5 \leq N \leq 10$. According to our experiments, the average conflict resolution time of both algorithms are roughly the same for a wide range of N. Nevertheless, our algorithm has a guaranteed worst-case bound for the conflict resolution time, whereas there is no such guarantee for the random conflict resolution algorithm.

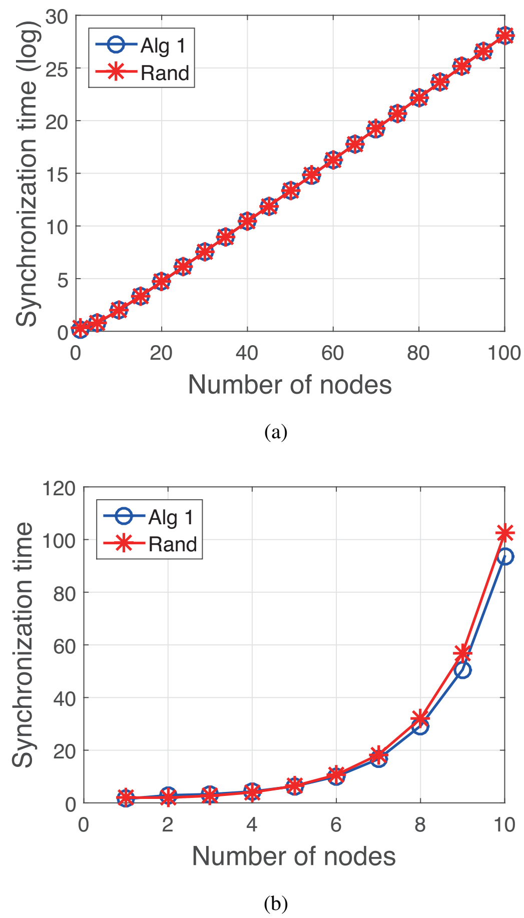

Fig. 1: Comparison of the average conflict resolution time of Algorithm 1 against random conflict resolution algorithm

V. Conclusion

In this letter, we proposed nonadaptive deterministic algorithms that establishes a guaranteed deterministic upper bound on the conflict resolution time even when the clocks of the devices are not synchronized in a discrete-time multiple access channel. In comparison to coin-flipping random access algorithms, our proposed method provides an upper bound for the worst case and has a comparable performance for the average case. For channels with $n \leq N$ devices (where n is arbitrary), our worst-case bound is $2^N - 1$. We have shown that our proposed method is equivalent to finding effective sets of gaps, which is a very complicated problem. We present theoretical results regarding explict conditions to construct them.

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