

# Real-time Simulation of Biologically-Realistic Stochastic Neurons in VLSI

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Summary of Changes					
Colour convention :					
Black : Reviewers' of	comments				
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neuron with the injection of noisy stimuli. The paper is a major revision of a previously submitted paper: in this new version the authors have reduced - albeit not completely removed - the confusion between channel and synaptic noise. I still think that comparing the effects of channel and synaptic noise – as the authors do, for example, in Fig. 2 - is, at best, misleading for the reader. Moreover, the "explanation" provided with Eqs. (4) and (5) is not very satisfactory.

**RE:** Thanks very much for the valuable feedback. To minimise confusion and to comply with the reviewer's suggestion, we decide to remove Fig.2, avoiding the comparison on the effect of the two types of noise. In addition, Eq.(4) and Eq.(5) are kept to clarify the differences between the two types of noise. The changes made in the manuscript are

Fig.2 is removed. The 2<sup>nd</sup> para. of Sec.III.B is removed

Notwithstanding this remark, I believe that the presented results can be of interest, especially for researchers in the field of the hardware implementation of neuronal (possibly hybrid) networks. Therefore, I think this paper is suitable for publications on the IEEE Transactions on Neural Networks. I also noticed some minor mistakes that should be corrected:

1) After Eq. (1), the authors say that x and y model the fraction of activated (open) and inactivated (closed) channels respectively. In reality, x and y are both variables in the range [0,1] and they both represent the fraction of channels in the open state, be they an activation variable (x) or an inactivation variable (y).

**RE:** Thanks very much for pointing out the imprecise description. The "(open)" and the "(close)" in the sentence have been removed to make it clear.

In Sec. II, line 9 of the 1<sup>st</sup> para., the sentence is modified as "...x is the gating variable modeling the fraction of ion channels that are activated, while y the gating variable modeling the fraction of ion channels that are inactivated."

2) In paragraph IV.B, the authors claim, referring to Fig. 5, that "software tools such as NEURON would require non-negligible time to complete the same simulations, and the required time would increase dramatically as the number of stochastic neurons grows". I do not agree on the first part of the sentence: the simulation of a single compartment neuron model for 1.5s that reproduces the behaviour shown in Fig. 5.a takes a negligible time on a modern machine.

**RE:** Thanks very much for the valuable feedback. We agree with that the NEURON could also complete the single-neuron simulation within negligible time. Therefore,

the text has been modified as

In Sec.IV.B , line 4 of the  $2^{nd}$  para. ".... Although software tools such as NEURON can also complete the simulation in Fig.4a within negligible time, the time required would increase dramatically as the number of neurons grows."

3) In paragraph IV.C, starting from the second line onward, the authors should replace "polarizing" with "hyperpolarizing" when referring to the stimulus required to evoke a post-inhibitory rebound in the model. The same applies to the caption of Fig. 6.

**RE:** Thanks very much for the valuable feedback. "polarizing" has been replaced with "hyperpolarizing" in Sec.IV.C, and the captions of Fig.5 and Fig.6.

4) It would be nice if the authors could present the same histograms as in Fig. 10 for the deterministic stimulus, i.e. for the traces shown in Fig. 8, to better motivate the sentence "the post-synaptic neuron could only detect the frequency but not the waveform of the stimuli".

**RE:** Thanks very much for the valuable feedback. The histograms for deterministic stimuli have been added as Fig.8, with corresponding description in line 13 of the 1<sup>st</sup> para. in Sec.IV.D. "....Let the timing of each spike be calculated as....."

5) I think reference 16 appeared on Nature: I don't know what "Neurosilicon Interface" is.

**RE:** Thanks very much for the valuable feedback. The reference list has been checked and revised thoroughly.

6) There are various typos and language mistakes throughout the paper. **RE:** Thanks very much for the valuable feedback. The final manuscript has been thoroughly checked again to avoid typos and language mistakes.

# **Editor's comments**

1. Please add your author names under the paper title and affiliation information to the footnote on the lower left corner of the first page. See any IEEE Transactions papers for style.

**RE:** The author names and the affiliation have been added in accordance with the IEEE format.

2. Please see if you can do something about Figure 2 since when it is printed black and white, the lines are not distinguishable. Please solve this problem now.

**RE:** To comply with the reviewer's comment, we decide to remove Fig.2, avoiding the comparison on the effect of the two types of noise.

3. Please reorganize your reference list so that all your references are listed alphabetically according to the first author's last name.

**RE:** The reference list is reorganized according to the first author's name.

4. I have attached a note prepared for IEEE style of references. Please take a careful look at the note and correct your reference list as needed.

**RE:** The reference list is thoroughly checked and refined to comply with the IEEE style.

5. Please double check your manuscript after your revision so that all your references are cited correctly in your revised paper.

**RE:** The reference list is compiled by the Bibtex with IEEEtranS.bst, and is doubled checked after the final manuscript is compiled.

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# Real-time Simulation of Biologically-realistic Stochastic Neurons in VLSI

Hsin Chen, Member, IEEE, Sylvain Saïghi, Member, IEEE, Laure Buhry, and Sylvie Renaud, Member, IEEE

Abstract-Neuronal variability has been thought to play an important role in the brain. As the variability mainly comes from the uncertainty in biophysical mechanisms, stochastic neuron models have been proposed for studying how neurons compute with noise. However, most studies are limited to simulating stochastic neurons in a digital computer. The speed and the efficiency are thus limited especially when a large neuronal network is of concern. This study explores the feasibility of simulating the stochastic behavior of biological neurons in a Very Large Scale Integrated (VLSI) system which implements a programmable and configurable Hodgkin-Huxley model. By simply injecting noise to the VLSI neuron, various stochastic behaviors observed in biological neurons are reproduced realistically in VLSI. The noise-induced variability is further shown to enhance the signal modulation of a neuron. These results point towards the development of analog VLSI systems for exploring the stochastic behaviors of biological neuronal networks in large scale.

Index Terms—Stochastic behavior, Noise, Hodgkin-Huxley formalism, analog VLSI, stochastic neurons, neuromorphic VLSI

#### I. INTRODUCTION

**B** IOLOGICAL neurons have been found noisy both in the generation of spikes and in the transmission of synaptic signals. The noise comes from the random openings of ion channels, the quantal releases of neural transmitters, the coupling of background neural activity, etc. [25] [19]. As the noise affects neural computation directly, it has been of great interests to study how neurons compute with noise reliably [24]. Interestingly, many studies have indicated that noise plays a beneficial role at least by (a) inducing neuronal variability [7], (b) enhancing the sensitivity of neurons to environmental stimuli [26], (c) inducing synchronization between neurons [1], and (d) facilitating probabilistic inference according to the Bayes' rule in the brain [16]. The effect on synchrony could further relate to neural disorders such as Parkinson's disease [11] and hearing loss [4]. Understanding the effect of noise is thus crucial both for computational neuroscience and for improving the treatments to these neural diseases.

One major approach of theoretical studies is adding white noise to the biologically-plausible, deterministic Hodgkin-Huxley (HH) model [12], either to the dynamics of gating variables of different ion channels, or to the dynamics of the membrane potential [9], [20]. As a result, the neuronal dynamics are modeled by stochastic differential equations

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S. Saïghi, L. Buhry, and S. Renaud are with IMS Lab (CNRS UMR 5218), University of Bordeaux, F-33400 Talence, FRANCE. (SDEs). This leads to at least two challenges for computerbased studies. First, the maximum number of neurons or SDEs a computer simulation can consider is limited. Many simplified models have thus been proposed [10], [13], However, the parameters of the these models no longer relate to real biophysical properties directly, making it more difficult to extract parameter values, or to understand how different parameters affect neuronal behaviors. The second challenge is that the suggestions drawn from theoretical studies are not easy to verify with biological neurons, owing to the difficulty in manipulating a specific property of biological neurons independently.

Contrary to computer simulation, analog circuits are inherently suitable for simulating differential equations in realtime and in parallel [3]. By the merit of the natural, differential current-voltage relationship of a capacitor, noise-induced stochastic dynamics can be simulated by simply applying a noise current to the capacitor and measuring its corresponding voltage dynamics. The hardware simulation further facilitates the building of a hybrid network incorporating both VLSI and biological neurons, allowing the network behavior to be studied efficiently by tuning the properties of VLSI neurons [15]. Therefore, this study explores the feasibility of simulating different types of stochastic neurons in an analog VLSI system called the *Pamina*, which realizes the conductancebased HH model and runs in biologically-realistic time [21].

### II. THE HODGKIN-HUXLEY MODEL IN VLSI

Fig.1a shows the *Pamina* chip [21] containing two HH-type neurons. Let  $C_M$  represent the membrane capacitance, and  $V_M$  the membrane voltage. Each neuron implements the formalism  $C_M(dV_M/dt) = -\sum_i I_{ion,i} + \sum_j I_{syn,j} + I_{stim}$ , where  $I_{ion,i}$  represents an ionic current,  $I_{syn,j}$  a synaptic current, and  $I_{stim}$  the stimulating input. The general form of  $I_{ion,i}$  is given as

$$I_{ion,i} = g_i \cdot x^p \cdot y^q \cdot (V_M - E_i) \tag{1}$$

where  $g_i$  and  $E_i$  are the maximum conductance and the reversal potential of the ionic current, respectively. x is the gating variable modeling the fraction of ion channels that are activated, while y the gating variable modeling the fraction of ion channels that are inactivated. Let  $\lambda$  represent either x or y. The dynamics of  $\lambda$  are guided by

$$\tau_{\lambda} \cdot \frac{d\lambda}{dt} = \lambda_{\infty}(V_M) - \lambda \tag{2}$$

$$\lambda_{\infty}(V_M) = \frac{1}{1 + exp(-(V_M - V_{off,\lambda})/V_{slope,\lambda})}$$
(3)

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The minus sign in front of  $(V_M - V_{off,\lambda})$  is omitted for the inactivation variable y.  $\tau_{\lambda}$  is the time constant for approaching  $\lambda_{\infty}(V_M)$ . Although  $\tau_{\lambda}$  is a function of  $V_M$  in the original HH model, it is a constant value in the *Pamina* chip to simplify circuit design.  $V_{off,\lambda}$  and  $V_{slope,\lambda}$  control the offset and the slope of the sigmoid function, respectively.

As shown by Fig.1b, each *Pamina* neuron contains five ionic currents, eight synaptic inputs and one stimulating input. The five ionic conductances include the sodium current  $(I_{Na})$ , the potassium current  $(I_K)$ , the leakage current  $(I_{leak})$ , the calcium current  $(I_{Ca})$ , and the calcium-dependent potassium current  $(I_{K(Ca)})$ . (p,q) for  $I_{Ca}$  allows users to select between (2, 1) and (1, 0), and the function  $m(V_M, [Ca^{2+}])$  is realized in accordance with [14]. For the synaptic current, the dynamics of  $r(V_{pre,j})$  also obey Eq.(2) and (3) with  $V_M$  replaced by the pre-synaptic potential  $V_{pre,j}$  [6]. As  $I_{syn,j}$  and  $I_{stim}$  have the same form as  $I_{ion,i}$ , all the conductances are implemented with a library of the analog operators detailed in [21]. Finally,  $V_{stim}$  and  $V_{pre,j}$  are externally applied voltage.

The parameters of all the conductances are stored in the analog *parameter memory* (Fig.1a), and the types of conductances or synapses connected to each neuron are controlled by the digital data stored in the *topology memory*. By integrating the *Pamina* chip with a Field-Programmable-Gate-Array (FPGA) and data converters on a customized Peripheral-Component-Interconnect (PCI) card, the neurons can be configured and recorded easily through C programming in a computer. Compared to other conductance-based neurons in VLSI [8], [22], the *Pamina* chip has the advantages that all neuronal parameters are dynamically-tunable over a wide range, together with a flexible topology. These features make the chip particular suitable for exploring the stochastic behaviors observed in different biological neurons.

#### III. MAPPING BIOLOGICAL MODELS INTO VLSI

## A. Parameter extraction

The minimal HH model proposed in [18] is of our particular interests, as different classes of cortical and thalamic neurons have been modeled satisfactorily with a minimal number of ionic conductances. In addition, the conductance models in [18] are similar to those implemented in the *Pamina* chip, allowing most parameters to be adopted directly for VLSI simulation according to the mappings described below.

All voltage levels in the VLSI neuron are designed to be five times greater than their corresponding values in biological neurons, i.e.  $V_{VLSI} = 5 * V_{BIO}$ , while the time scale is identical for both VLSI and biological neurons. Let  $C_{VLSI}$ and  $C_{BIO}$  represent the membrane capacitances of VLSI and biological neurons, respectively. The conductance mapping is proportional to the capacitance ratio as  $g_{VLSI}/g_{BIO} =$  $C_{VLSI}/C_{BIO}$ . The current mapping then equals the product of the voltage and conductance mappings, i.e.  $I_{VLSI}/I_{BIO} =$  $5 * C_{VLSI}/C_{BIO}$ . In the *Pamina* chip,  $C_{VLSI} = 5nF$  and the biological neurons have  $C_{BIO} = C_M \cdot Area$  with  $C_M$  and *Area* given in Table.I.

The only difference between the VLSI and biological neurons is that the dynamics of gating variables in [18] are



Fig. 1: (a)The microphotograph of the *Pamina* chip fabricated with the  $0.35\mu m$  BiCMOS technology by the Austriamicrosystems. The chip area is  $4170 \times 3480\mu m^2$ . (b)The block diagram of a neuron.

modeled as  $d\lambda/dt = \alpha_{\lambda}(V_M) \cdot (1-\lambda) - \beta_{\lambda}(V_M) \cdot \lambda$  instead of Eq.(2). The parameters  $\tau_{\lambda}$ ,  $V_{off,\lambda}$ , and  $V_{slope,\lambda}$  are thus extracted by (a) calculating  $\alpha_{\lambda}(V_M)$  and  $\beta_{\lambda}(V_M)$  over the range  $V_M = [-100, 100]mV$ , (b) deriving  $\lambda_{\infty}(V_M)$  according to  $\lambda_{\infty} = \alpha_{\lambda}/(\alpha_{\lambda} + \beta_{\lambda})$ , (c) setting the  $V_M$  corresponding to  $\lambda_{\infty} = 0.5$  as  $V_{off,\lambda}$ , (d) and then extracting  $V_{slope,\lambda}$ at a specific  $\lambda_{\infty}$ . In addition,  $\tau_{\lambda}$  is simply calculated from  $\tau_{\lambda}(V_M) = [\alpha_{\lambda}(V_M) + \beta_{\lambda}(V_M)]^{-1}$  at  $V_M = -70mV$ .

Three types of neurons, the *Fast-Spiking* (FS) neurons, the *Regular-Spiking* (RS) neurons, and the *Low-Threshold-Spiking* (LTS) neurons were simulated in our experiments. Table.I summarized the parameter values extracted from [18]. For the RS neuron, the calcium conductance  $(I_{Ca})$  was programmed to realize the slow potassium current  $(I_M)$  with (p,q) = (1,0). For the LTS neuron, (p,q) = (2,1) and  $\tau_s = 0.65ms$  were set to realize the low-threshold calcium current  $(I_T)$  in [18].

TABLE I: The parameters of different neurons simulated in VLSI

	FSneuron	RS neuron	LTS neuron
$C_M(\mu F/cm^2)$	1	1	1
$Area(cm^2)$	$14 \times 10^{-5}$	$29 \times 10^{-5}$	$29 \times 10^{-5}$
$g_{stim}(mS/cm^2)$	1.08	1.08	1.08
$g_{Na}(mS/cm^2)$	44	44	44
$E_{Na}(mV)$	50	50	50
$\tau_m(ms)$	0.07	0.07	0.07
$V_{off,m}(mV)$	-34.42	-34.42	-34.42
$V_{slope,m}(mV)$	6.47	6.47	6.47
$\tau_h(ms)$	0.36	0.36	0.36
$V_{off,h}(mV)$	-39.07	-39.07	-39.07
$V_{slope,h}(mV)$	3.932	3.932	3.932
$g_K(mS/cm^2)$	10	10	5 - 10
$E_K(mV)$	-90	-90	-90
$\tau_n(ms)$	1	1	1
$V_{off,n}(mV)$	-29.08	-29.08	-29.08
$V_{slope,n}(mV)$	7.854	7.854	7.854
$g_{leak}(mS/cm^2)$	0.1	0.1	0.1
$E_{leak}(mV)$	-70	-70	-70
$g_{Ca}(mS/cm^2)$	—	0.35	2
$E_{Ca}(mV)$	—	-90	120
$\tau_s(ms)$	—	200	0.65
$V_{off,s}(mV)$	—	-35	-115
$V_{slope,s}(mV)$	—	10	6.2
$\tau_u(ms)$	—	—	100
$V_{off,u}(mV)$	—	—	-120
$V_{slope,u}(mV)$	—	—	16

# B. Noise injection

While the stochastic behaviors of biological neurons were reproduced with remarkable precision in [20] by adding white noise  $(\sigma \cdot W(t))$  to the kinetics of gating variables, the Pamina chip originally designed for simulating deterministic HH models only allowed the noise to be added to the kinetics of the membrane voltage as Eq.(4).

$$C_M \frac{dV_M}{dt} = -\sum_i I_{ion,i} + \sum_j I_{syn,j} + I_{stim} + \sigma \cdot W(t)$$
(4)

Let  $V_M = V_{Md} + V_{Ms}$ , with  $V_{Md}$  and  $V_{Ms}$  representing the deterministic and the stochastic components, respectively. By Taylor expansion,  $\lambda_{\infty}(V_{Md} + V_{Ms})$  can be expressed as

$$\lambda_{\infty}(V_{Md} + V_{Ms}) = \lambda_{\infty}(V_{Md}) + \lambda_{\infty}'(V_{Md}) \cdot V_{Ms} + o(V_{Ms})$$
(5)

where  $o(V_{Ms})$  represents high-order terms of  $V_{Ms}$ . Eq.(5) indicates that, although the white noise in Eq.(4) can be transferred to the dynamics of  $\lambda$  in Eq.(2) via  $V_M$ , the transferred noise,  $\lambda'_{\infty}(V_{Md}) \cdot V_{Ms} + o(V_{Ms})$ , is no longer white due to the filtering effect by Eq.(4). Furthermore,  $V_{Ms}$ is nonlinearly transformed by the sigmoid function. Adding white noise to the kinetics of  $V_M$  could thus result in different responses from adding white noise to the kinetics of  $\lambda$ .

With this note in mind, we superimposed the noise signal  $V_n$  on the stimulating signal  $V_S$  in the *Pamina* chip to obtain



Fig. 2: The responses of a stochastic FS neuron in VLSI with  $V_n = 800mV_{pp}$  and  $(top)V_S = 2.33V$  (middle) $V_S = 2.34V$  (bottom) $V_S = 2.37V$ .



Fig. 3: The superimpose of 37 spikes generated by a stochastic FS neuron in VLSI with  $V_S = 2.34V$  and  $V_n = 800mV_{pp}$ 

 $V_{stim} = V_S + V_n$  (Fig.1b).  $V_{stim}$  was then converted into the current  $I_{stim} = g_{stim}(V_{stim} - V_{ref})$ , wherein the stochastic component of  $I_{stim}$  corresponded to  $\sigma \cdot W(t)$  in Eq.(4). The effect of the noise on different types of neurons was then explored and discussed as follows.

# IV. SIMULATING STOCHASTIC NEURONS IN VLSI

# A. Fast-Spiking neurons

The FS neuron is a major class of neurons in the cerebral cortex, involving only  $I_{Na}$ ,  $I_K$ , and  $I_{leak}$ . In the absence of noise injection, the FS neuron simulated in the *Pamina* chip generates spikes only when  $V_S \ge 2.34V$ . With  $V_n = 800mV_{pp}$  superimposed on  $V_S$ , the measured responses of the FS neuron to (a)subthreshold ( $V_S = 2.33V$ ) (b)suprathreshold<sup>1</sup> ( $V_S = 2.34V$ ), and (c)above-threshold ( $V_S = 2.37V$ ) stimulation are shown in Fig.2. Under subthreshold stimulation, the noise induced spontaneous firings. Suprathreshold stimulation then leads to increased spiking frequency and reduced frequency variation. As  $V_S$  is well above the threshold, the spiking frequency approaches constant while the spiking amplitude remains slightly variable due to the presence of noise. These phenomena have been reported in biological experiments both *in-vivo* and *in-vitro* [2], [5].

As  $V_M$  is polarized to around the same minimum voltage after each spike generation, the minimum voltage can be thought as the initial state from which the neuron is discharged

<sup>&</sup>lt;sup>1</sup>The quantitative definition for suprathreshold stimulation is that the probability of generating spikes under suprathreshold stimulation is 0.5 [25]



Fig. 4: (a)The response of a stochastic RS neuron in VLSI to a step-input stimulation rising from  $V_S = 1.3V$  to  $V_S = 2.4V$  at t = 0.2s (b)The inverse of inter-spike-interval of a stochastic RS neuron in VLSI in response to the same depolarizing stimulation lasting for 1600ms with various levels of noise.

by  $V_S$  to generate the next spike. Fig.3 superimposes 37 spikes generated under the suprathreshold stimulation, aligning their initial states with t = 0. With a constant  $V_S$ , the time required for discharging the membrane over the spiking threshold varies from one spike to another. Such noise-induced variability has been widely observed in biological neurons [17]. Although the variability could impede neurons from coding information as spike timing precisely, it has been found useful for auditory neurons, for example, to encode distinct features efficiently [4]. Therefore, the results here demonstrate the feasibility of reproducing the stochastic behaviors of biologically-realistic neurons in VLSI by simply adding noise to the neuronal membrane.

#### B. Regular-Spiking neurons

The RS neuron has been the largest class of neurons in the neocortex. The slow potassium current  $(I_M)$  is activated by the depolarization of neuronal membranes. Once activated,  $I_M$  functions as an extra polarizing current, causing the spiking frequency to adapt towards a minimum.

With  $V_S$  stepping from 1.3V (inhibition) to 2.4V (above threshold) at t=0.2s and  $V_n = 300mV_{pp}$ , the measured responses of the stochastic RS neuron in the Pamina chip are shown in Fig.4a. The frequency adaptation is clearly shown, and the noise distorts the spiking frequency during adaptation. Let the inverse of the inter-spike-interval (ISI) between consecutive spikes approximate the instantaneous spiking frequency. Fig.4b plots the spiking frequency of the RS neuron during 1600ms of the above-threshold stimulation  $(V_S = 2.4V)$ . Without noise, the spiking frequency adapts from 137Hz to 25Hz gradually. The variability around 25Hz is attributed to the clockfeedthroughs in the PCI system. As the noise is increased, the adaptation process becomes distorted. The initial firing frequency further reduces when  $V_n$  is greater than  $300mV_{pp}$ , owing to the serious threshold variations induced by the noise. On the contrary, the adaptation rate is nearly constant for different  $V_n$ . This is because  $I_M$ with a large  $\tau_s$  (200ms) is less affected by noise.



Fig. 5: The (a) $V_M$  and (b) $I_T$  of a LTS neuron in VLSI in response to a hyperpolarizing stimulation which was dismissed by changing  $V_S = 2.22V$  to  $V_S = 2.3V$  at t = 0.2s.



Fig. 6: The responses of a LTS neuron in VLSI to  $V_n = 1.8V_{pp}$  superimposed on a hyperpolarizing stimulation which was dismissed abruptly by changing  $V_S = 2.22V$  to  $V_S = 2.3V$  at t = 0.2s.

This experiment demonstrates that the effect of noise can be studied efficiently by VLSI simulation in real-time, and the same should hold as a large network of neurons is of concern. Although software tools such as NEURON can also complete the simulation in Fig.4a within negligible time, the time required would increase dramatically as the number of neurons grows.

## C. Low-Threshold-Spiking neurons

The major distinctive behavior of the LTS neuron is the generation of a burst of spikes at the "off-set" of a hyperpolarizing current stimulus. This property has been shown related to the low-threshold calcium current  $(I_T)$ . With the *Pamina* chip programmed to simulate the LTS neuron, the neuron generates post-inhibitory rebounds after the release of a hyperpolarizing stimulation (at t = 0.2s), as shown in Fig.5. The corresponding  $I_T$  is shown to function as a depolarizing current, inducing the spikes during its slow inactivation.

As  $V_n = 1.8V_{pp}$  is superimposed on the same hyperpolarizing stimulation, the LTS neuron responds as shown in Fig.6. Before the hyperpolarization ended (t < 0.2s), the neuron generates no spikes even if the noise amplitude plus the hyperpolarizing stimulation already exceeds the firing threshold (2.34V). This is because the noise has a maximum amplitude with a very low likelihood and in a short period of time. After t > 0.2s, the post-inhibitory rebounds are evoked by the stimulation off-set, but the spiking frequency is distorted. The magnified window further reveals dynamics analogous to the afterdepolarization (ADP) and afterhyperpolarization (AHP) observed in biological neurons. The ADP and AHP could play an important role in affecting the synaptic plasticity in the hippocampus [23], and has been simulated



Fig. 7: The responses of a FS neuron to (a)a sinusoidal and (b)a square inputs with an offset of 2.34V and an amplitude of 30mV. The  $V_{stim}$  has been shifted by -0.7V.



Fig. 8: The statistical firing probability of a deterministic FS neuron in response to (a)the sinusoidal and (b)the square waves.

with a more complex HH model with noise added to gating variables in [20]. The feasibility of simulating sophisticated stochastic behaviors such as ADP and AHP in real-time in VLSI is thus demonstrated. Nevertheless, adding noise to the gating variables would be much more effective, as discussed in Sec.III-B.

# D. Noise-enhanced signal modulation

Except for the rich stochastic behaviors explored above, noise has been shown useful for enhancing neurons' sensitivity to weak signals by the mechanism called *stochastic resonance* 



Fig. 9: The responses  $(V_M)$  of a stochastic FS neuron with  $V_n = 800mV_{pp}$  superimposed on (a)the sinusoidal and (b)the square stimuli,  $V_S$ . The total input  $V_{stim} = V_S + V_n$  has been shifted by -0.7V in the plots.

[26]. We here demonstrate the noise-enhanced sensitivity as the responses of a stochastic FS neuron to two weak stimuli, one with a sinusoidal waveform and the other with a square waveform. Both stimuli have an amplitude of 30mV, an offset of 2.34V, and a frequency of 5Hz. The offset level introduces suprathreshold stimulation to the neuron. Without noise, the neuron only fires when the stimulating waveform exceeds its firing threshold, as shown in Fig.7. The firing frequency and the separation between consecutive groups of spikes are very similar for both stimuli. Let the timing of each spike be calculated as its phase with respect to the stimulating waveforms. By recording the response to each stimulus for 2s, the statistical distributions of the spike timing for both stimuli are obtained and shown in Fig.8. The square waveform simply results in a wider distribution than the sinusoidal wave. Given the two spike trains are received by a post-synaptic neuron, the post-synaptic neuron could only detect the frequency but not the waveform of the stimuli.

By contrast, with  $V_n = 800mV_{pp}$  added to the input, the FS neuron exhibits dramatically-different responses, as shown in Fig.9. Although the sinusoidal or square waveform is masked off by the large noise, the ISIs are modulated in accordance with the waveforms. From a prolonged recording of 20s for each stimulus, Fig.10 plots the statistical distributions of the spike timing for the two stimulating waveforms. The histograms reconstruct the waveforms of the input stimuli, indicating that the modulated ISIs allow post-synaptic neurons

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Fig. 10: The statistical firing probability of a stochastic FS neuron in response to (a)the sinusoidal and (b)the square waves.

to perceive the waveforms from spike timing. The advantage and the utility of noise-induced stochastic behavior in neurons is first demonstrated in VLSI. Certainly, an optimum level of noise exists for maximizing the sensitivity, and the optimum level is different from one case to another.

# V. CONCLUSION

This study demonstrates the feasibility of simulating various stochastic neurons in VLSI by simply injecting noise into the membrane capacitor of a HH model in VLSI. Various stochastic behaviors observed in biological neurons have been reproduced in VLSI realistically. The effect of noise on different neurons has thus been studied efficiently. These promising results point towards the development of analog VLSI systems able to simulate stochastic neuronal networks in real or accelerated time. The influence of noise on synaptic connections and network behaviors will then be explored In addition, hybrid silicon-neuron networks could be built to ease the investigation on how individual parameters affect stochastic neural computation, as well as to verify the suggestions drawn from computer-based simulations. The the understanding on how neurons compute with noise reliably would further inspire novel neuromorphic computation. For example, as the intrinsic noise of transistors will become nonnegligible in the deep-submicron era, it will be interesting to explore the possibility of using the intrinsic VLSI noise for computation like biological neurons do.

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