A BLOCK-BY-BLOCK BLIND POST-FFT BEAMFORMING ALGORITHM FOR MULTIUSER OFDM SYSTEMS BASED ON SUBCARRIER AVERAGING

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ABSTRACT
This paper considers blind beamforming of multiuser orthogonal frequency division multiplexing (OFDM) systems. Assuming that the channel is static within one OFDM block, a blind post-FFT multi-stage beamforming algorithm (MSBFA) based on subcarrier averaging over one OFDM block is proposed, which basically comprises: i) source (path) extraction using a hybrid beamforming algorithm composed of a Fourier beamformer and a kurtosis maximization beamformer, ii) time delay estimation and compensation, iii) classification (path-to-user association) and blind maximum ratio combining (of paths). The designed beamformer effectively utilizes multipath diversity for performance gain, and is robust against the effects of correlated sources. Some simulation results are presented to demonstrate the effectiveness of the proposed MSBFA.

1. INTRODUCTION
The need for high transmission rate and guaranteed quality of service has grown rapidly in wireless communication systems. To meet this need, orthogonal frequency division multiplexing (OFDM) in conjunction with multiple-input multiple-output (MIMO) signal processing has been considered for broadband communications, especially in the beamforming design of the receiver, such as the pre-FFT beamforming structure (pre-FFT BFS) and post-FFT beamforming structure (post-FFT BFS) [1–3]. The nonblind minimum mean square error (MMSE) and maximum ratio combining (MRC) beamformers require the channel information estimated in advance through the use of pilot signals resulting in reduced spectral efficiency. Thus, blind beamforming algorithms without pilot signals are preferable [2, 4].

Some blind beamforming algorithms associated with the post-FFT BFS have been reported such as the subspace-based method proposed by Zeng and Ng [4] and the conventional fast kurtosis maximization algorithm (FKMA) proposed by Chi et al. [5, 6]. However, they require a set of N beamformers (where N is the total number of subcarriers) each for a subcarrier and the channel to be static over multiple OFDM blocks. Recently, Peng et al. [2] proposed a single-user blind beamforming algorithm by kurtosis maximization based on subcarrier averaging over one OFDM block associated with the post-FFT BFS to combat co-channel interference, where the desired signals are significantly stronger than the interfering signals, under a channel condition of no correlated sources. Moreover, users’ data sequences must be independent identically distributed (i.i.d.) binary phase-shift keying (BPSK) symbol sequences.

Our work can be regarded as a multiuser generalization of [2] provided that the carrier frequency synchronization is perfect. We propose a novel block-by-block blind beamforming algorithm associated with the post-FFT BFS for multiuser OFDM systems based on subcarrier averaging. The designed beamformer is exactly the same for all the subcarriers, is applicable to both the BPSK case and the quadrature-phase-shift keying (QPSK) case, effectively utilizes multipath diversity for performance gain, and is also robust against the effects of correlated sources.

2. MIMO MODELS
Consider the uplink of a quasi-synchronous multiuser OFDM system [4] with P active users where each mobile unit has a single transmit antenna and the base station is equipped with a uniform linear array of Q elements equally spaced by half a carrier wavelength of the radio. The baseband discrete-time transmitted OFDM signal from user p after serial-to-parallel (S/P) conversion, N-point IFFT operation, parallel-to-serial (P/S) conversion, and insertion of a guard interval (GI) of length N_g (< N/2) is known to be

\[ s_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} u_p[k] e^{j2\pi kn/N} \]  

for \( n = -N_g, ..., -1 \), where \( u_p[k] \) is the data sequence of user p. For ease of later use, let us define the following notations:

- \( \theta_{p,l} \): direction of arrival (DOA) of the \( l \)th path of user p
- \( a(\theta_{p,l}) : Q \times 1 \) steering vector of user p with DOA \( \theta_{p,l} \)
- \( \alpha_{p,l} : \) path gain of user p with DOA \( \theta_{p,l} \)
- \( \tau_{p,l} : \) time delay (in samples) of user p with DOA \( \theta_{p,l} \)
- \( L_p : \) total number of paths (or DOAs) of user p.

The baseband discrete-time received signal can be expressed as

\[ x[n] = \sum_{p=1}^{P} \sum_{l=1}^{L_p} \alpha_{p,l} a(\theta_{p,l}) s_p[n - \tau_{p,l}] + w[n] \]  

for \( n = -N_g, ..., -1 \), where \( w[n] \) is a \( Q \times 1 \) noise vector.

Next, let us make some general assumptions for the received signal \( x[n] \) given by (2) as follows:

(A1) \( u_1[k], ..., u_P[k] \) are i.i.d. QPSK symbol sequences, and \( u_p[k] \) is statistically independent of \( u_q[k] \) for \( q \neq p \).

(A2) \( \theta_{q,m} \neq \theta_{p,l} \) for all \( (q, m) \neq (p, l) \); \( L = L_1 + ... + L_P \) (total number of paths of all the users) and \( Q \geq L \).

(A3) \( \tau_{p,l} \in \{0, ..., N_g\} \) for all p and l (quasi-synchronous OFDM system) and \( \tau_{p,l} < ... < \tau_{p,L_p} \).

(A4) \( w[n] \) is zero-mean Gaussian, and is statistically independent of \( u_p[k] \), \( p = 1, ..., P \).
2.1. MIMO Model for Pre-FFT BFS

For the pre-FFT BFS [3], the received signal $\mathbf{x}[n]$ given by (2) can also be expressed as a $Q \times L$ instantaneous MIMO model:

$$\mathbf{x}[n] = \mathbf{A}s[n] + \mathbf{w}[n], \quad n = -N_y, \ldots, N - 1,$$

(3)

where

$$\mathbf{A} = (\mathbf{A}_1, \ldots, \mathbf{A}_P) = (a^{(1)}, \ldots, a^{(L)})$$

(4)

$$s[n] = (s^T_1[n], \ldots, s^T_p[n])^T$$

(5)

are a $Q \times L$ DOA matrix and an $L \times 1$ input vector consisting of $L$ source signals, respectively, in which

$$\mathbf{A}_p = (a_{p,1} \mathbf{a}(\theta_{p,1}), \ldots, a_{p,L} \mathbf{a}(\theta_{p,L}))$$

(6)

and

$$s_p[n] = (s_p[n - \tau_{p,1}], \ldots, s_p[n - \tau_{p,L}])^T.$$  

(7)

Remark 1: One can observe, from (5), (7), and Assumptions (A1) and (A3), that $s[n]$ for each fixed $n$ is a zero-mean random vector with all the random components being mutually statistically independent. Moreover, $s[n]$ is approximately Gaussian by Central Limit Theorem. On the other hand, the $Q \times L$ DOA matrix $\mathbf{A}$ (see (4) and (6)) is of full column rank by Assumption (A2) and each column $a^{(l)}$ of $\mathbf{A}$ only comprises the energy from a single path. These observations imply that only blind beamforming algorithms using second-order statistics (SOS) can be applied, such as the Capon’s minimum variance (MV) beamformer and Fourier beamformer [7], but their performance is limited due to the lack of path diversity.

2.2. MIMO Model for Post-FFT Beamformer

After the processes of the removal of GI, S/P conversion, N-point FFT operation, and P/S conversion at each receive antenna, for each subcarrier $k$ the MIMO model of the post-FFT BFS can be established as follows [4]:

$$\mathbf{x}[k] = \mathbf{A}^{(k)} \mathbf{u}[k] + \mathbf{w}[k], \quad k = 0, \ldots, N - 1,$$

(8)

where

$$\mathbf{u}^{(k)} = (u_1[k], \ldots, u_p[k])^T \in \mathbb{C}^p$$

(9)

$$\mathbf{A}^{(k)} = (a^{(1)} \mathbf{a}(\theta_{1}), \ldots, a^{(L)} \mathbf{a}(\theta_{L})) \in \mathbb{C}^{Q \times P}$$

(10)

and each column of $\mathbf{A}^{(k)}$ is given by

$$a^{(k)}_{p,l} = \sum_{l=1}^{L_p} \alpha_{p,l} \mathbf{a}(\theta_{p,l}) e^{-j2\pi k \tau_{p,l}/N}.$$  

(11)

Remark 2: By (9) and Assumption (A1), one can observe that all the $u_p[k]$’s in $\mathbf{u}^{(k)}$ are zero-mean non-Gaussian statistically independent. Moreover, $\mathbf{A}^{(k)}$ is of full column rank by Assumption (A2) and each column $a^{(l)}_{p}$ of $\mathbf{A}^{(k)}$ comprises multipath energy implying that a path diversity gain in the estimation of $u_p[k]$ can be foreseen. Thus, blind beamforming algorithms either using SOS such as blind subspace methods [4] or using higher-order statistics (HOS) such as the conventional FKMA [5, 6] are applicable to the estimation of $u_p[k]$. But in general, they require many OFDM data blocks under the assumption that the channel is static over these OFDM data blocks, and meanwhile a set of $N$ beamformers is needed because of $\mathbf{A}^{(k)} \neq \mathbf{A}^{(j)}$ for all $k \neq j$.

Next, let us present a hybrid source extraction algorithm (Section 3) followed by the blind beamforming algorithm (Section 4) based on subcarrier averaging, called the Multistage Beamforming Algorithm (MSBA).

3. SOURCE EXTRACTION BY SUBCARRIER AVERAGING

Let

$$u_{p,l}[k] = u_{p,l}^{(1)}[k] e^{-j2\pi k \tau_{p,l}/N},$$  

(12)

$$u_{p,l}[k] = (u_{p,l}^{(1)}[k], \ldots, u_{p,l}^{(L)}[k])^T,$$

(13)

$$\mathbf{u}[k] = (u^{(1)}[k], \ldots, u^{(L)}[k])^T.$$  

(14)

Then the MIMO model $\mathbf{x}[k]$ given by (8) can be rewritten as

$$\mathbf{x}[k] = \mathbf{A} \mathbf{u}[k] + \mathbf{w}[k], \quad k = 0, \ldots, N - 1,$$

(15)

where $\mathbf{x} \in \mathbb{C}^{Q \times L}$.

Remark 3: By treating the subcarrier $k$ as the time index, it can be seen that $u[k]$ involves correlated components associated with the same user because $E\{u_{p,l}[k]u_{p,l}^{(i)}[k]\} = e^{-j2\pi k (\tau_{p,l} - \tau_{p,l}^{(i)})/N} \neq 0$ for all $j \neq i$ (by (12) through (14) and Assumptions (A1) and (A3)), implying that none of blind beamforming algorithms including the conventional FKMA can be applied.

For ease of later use, let

$$\{\langle a^{(r)}, \theta^{(r)}, r^{(r)} \rangle, r = 1, \ldots, L\} = \{\langle a_{p,l}(\theta_{p,l}), r \rangle, p = 1, \ldots, P, l = 1, \ldots, L_p\},$$

(16)

where the mapping between $r$ and $(p, l)$ is one-to-one. The subcarrier average of $u^{(i)}[k]$ is defined as

$$\langle u^{(i)}[k] \rangle = \frac{1}{N} \sum_{k=0}^{N-1} u^{(i)}[k].$$

(17)

The following lemma is needed for DOA estimation and input-output cross-correlation (IOCC) channel estimation in the source extraction to be presented below.

Lemma 1. Under the assumptions (A1) through (A3) and the noise-free assumption, the following subcarrier averages converge in probability:

$$\frac{\langle |\mathbf{x}[k]|^2 \rangle}{\langle |\mathbf{u}[k]|^2 \rangle} \xrightarrow{p} \mathbf{a}^{(r)} = \mathbf{a}(\theta^{(r)}).$$

(18)

where $\mathbf{a} \xrightarrow{p} \mathbf{a}^{(r)}$ denotes “convergence in probability” as $N \rightarrow \infty$, and for sufficient large $Q$,

$$\bar{\theta} = \arg \max_{|\hat{\theta}| = 1} \left\{ \frac{1}{\sqrt{Q}} \mathbf{a}^H(\theta) \mathbf{x}[k] \right\} \xrightarrow{p} \theta' \in \{\theta^{(1)}, \ldots, \theta^{(L)}\}. $$

(19)

The proof of Lemma 1 is omitted due to page limit. Let $\mathbf{v}$ (a $Q \times 1$ vector) denote a beamformer to be designed and let $e[k]$ denote the corresponding beamformer output, i.e.,

$$e[k] = \mathbf{v}^H \mathbf{x}[k].$$

(20)

Suppose that in the presence of noise, $e[k]$ is an estimate of $u^{(r)}[k]$ up to an unknown scale factor, and $\bar{\theta}$ is also a DOA estimate of $\theta^{(r)}$ (by Lemma 1). Then the associated IOCC channel estimate can be obtained as

$$\tilde{\mathbf{a}}^{(r)} = \frac{\langle |\mathbf{x}[k]|^2 \rangle}{\langle |\mathbf{e}[k]|^2 \rangle} \mathbf{a}^{(r)}.$$  

(21)

Moreover, a “blind performance index” is suggested as follows:

$$\rho(\mathbf{a}(\hat{\theta}), \tilde{\mathbf{a}}^{(r)}) = \frac{\| \mathbf{a}(\hat{\theta}) \| \cdot \| \tilde{\mathbf{a}}^{(r)} \| }{\| \mathbf{a}^{(r)} \|}.$$

(22)
which is nothing but the magnitude of the normalized cross correlation between $a(\hat{\theta})$ and $a^{(r)}$ and $0 \leq \rho(a(\hat{\theta}), a^{(r)}) \leq 1$. As mentioned above, $\hat{\theta}$ and $a^{(r)}$ are estimates of $\theta^{(r)}$ and $a^{(r)}a(\theta^{(r)})$, respectively, implying the following remark.

**Remark 4:** The better the estimation accuracy of both $\hat{\theta}$ and $a^{(r)}$ (i.e., the better performance of the associated blind beamformer), the larger the value of $\rho(a(\hat{\theta}), a^{(r)})$.

Next, let us present how to extract a path signal in $u[k]$ based on subcarrier averaging over one OFDM block, involving a Fourier beamformer and a kurtosis maximization beamformer (KMBF).

### 3.1. Fourier Beamformer by Subcarrier Averaging

The Fourier beamformer by subcarrier averaging is given by

$$v_{FB} = \frac{1}{\sqrt{\|a(\hat{\theta})\|^2}}$$

and its output is $e_{FB}[k] = v_{FB}^H x[k]$. Then the associated channel estimate $\hat{a}_{FB}$ can be obtained by (21) with $e[k]$ replaced by $e_{FB}[k]$ and the corresponding blind performance index

$$\rho_{FB} = \rho(a(\hat{\theta}), \hat{a}_{FB}). \quad \text{(by (22))}$$

### 3.2. KMBF by Subcarrier Averaging

The kurtosis of $u^{(i)}[k]$ by subcarrier averaging is defined as

$$\gamma\{u^{(i)}[k]\} = \|u^{(i)}[k]\|^4 - 2\|u^{(i)}[k]\|^2 - 2\|u^{(i)}[k]\|^2. \quad \text{(25)}$$

The objective function to be maximized for the design of the KMBF is defined as [2]

$$J(v) = J(e[k]) = \frac{|\gamma\{e[k]\}|^2}{(\|e[k]\|^2)^2}$$

which is also called the magnitude of normalized kurtosis of beamformer output $e[k]$. The optimum KMBF, denoted $v_{KMBF}$, by maximizing $J(v)$ (see (26)) is supported by Theorem 1 below for which three more assumptions are needed as follows:

(A5) $\tau_{p,j} \neq (\tau_{p,i} + \tau_{p,j})/2, \forall p$, if $L_p \geq 3$, where $i, j, l$ are distinct integers.

(A6) $\tau_{p,i} + \tau_{p,j} \neq \tau_{p,m} + \tau_{p,n}, \forall p$, if $L_p \geq 4$, where $i, j, l, m$ are distinct integers.

(A7) $\tau_{p,j} - \tau_{p,n} \neq \tau_{q,m} - \tau_{q,l}, \forall p \neq q, j \neq i, l, m \neq l$, if $L_p \geq 2$ and $L_q \geq 2$.

**Theorem 1.** Under the assumptions (A1) through (A3), (A5) through (A7), and the noise-free assumption, $J(v_{KMBF}) = \max\{J(v)\}$ - $\rho_{KMBF} = \rho(a(\hat{\theta}), \hat{a}_{KMBF})$, and $\hat{a}_{KMBF}^H x[k] = \beta^{(r)} u^{(r)}[k]$ where $\beta^{(r)} \neq 0$ is an unknown constant and $r \in \{1, \ldots, L\}$ is an unknown integer.

The proof of Theorem 1 is also omitted due to page limit. Although the objective function $J(v)$ given by (26) is a highly nonlinear function of $v$ without closed-form solutions, fortunately, the optimum $v_{KMBF}$ and $e_{KMBF}[k] = v_{KMBF}^H x[k]$ can be efficiently obtained through the same fashion of the conventional iterative FKMA [5, 6].

### 3.3. Fourier-KMBF by Subcarrier Averaging

We empirically found that the KMBF performs better than the Fourier beamformer (both based on subcarrier averaging), but the former requires three extra assumptions (A5), (A6) and (A7). It may occur that these three assumptions may not be valid all the time in practice, thus limiting the use of the KMBF. This motivates the following two-step hybrid Fourier-KMBF which is free from these three assumptions:

1. Obtain $e_{FB}[k] = v_{FB}^H x[k], \hat{a}_{FB}$ and $\rho_{FB}$ as presented in Section 3.1. and obtain $e_{KMBF}[k] = v_{KMBF}^H x[k], \hat{a}_{KMBF}$ and $\rho_{KMBF}$ as presented in Section 3.2.

2. Let $\rho_{max} = \max\{\rho_{FB}, \rho_{KMBF}\}$. If $\rho_{max} > \eta$, then obtain

$$e[k], \hat{a} = \begin{cases} (e_{KMBF}[k], \hat{a}_{KMBF}), & \text{if } \rho_{KMBF} > \rho_{FB}, \\ (e_{FB}[k], \hat{a}_{FB}), & \text{if } \rho_{FB} > \rho_{KMBF}, \end{cases}$$

otherwise no source (with enough power) can be extracted, where $\eta$ is a preassigned threshold.

**Remark 5:** This two-step Fourier-KMBF is an automatic selection scheme by the performance of the Fourier beamformer and the KMBF by Remark 4. When any of the assumptions (A5), (A6) and (A7) does not hold true, the selection scheme (28) will switch to the Fourier beamformer since $\rho_{FB} > \rho_{KMBF}$ in this case. Therefore, the three extra assumptions (A5), (A6) and (A7) can be relaxed (i.e., removed).

### 4. PROPOSED BLIND MSBFA

Under Assumptions (A1) through (A4), the proposed blind MSBFA basically includes source (path) extraction, time delay estimation and compensation, classification (path-to-user association) and blind maximum ratio combining (BMRC) (of paths) which are presented next, respectively.

#### 4.1. Multistage Source Extraction Using Fourier-KMBF

Assume that the source estimate $e_{\ell-1}[k]$ and the channel estimate $\hat{a}_{\ell-1}$ are obtained at stage $\ell - 1$ and that $x_{\ell-1}[k]$ is the MIMO signal deflated from $x[k]$ at stage $\ell - 1$. At the $\ell$th stage,

$$x_{\ell}[k] = x_{\ell-1}[k] - \hat{a}_{\ell-1} e_{\ell-1}[k]. \quad \text{(29)}$$

First, obtain a distinct DOA estimate $\hat{\theta} = \hat{\theta}^{(r)}$ by (19) with $x[k]$ replaced by $x_{\ell}[k]$, and then calculate $v_{FB}$ by (23). Then obtain $e_{\ell}[k], \hat{a}_{\ell}$ (see (28)) using the proposed Fourier-KMBF, where

$$e_{\ell}[k] = \tilde{u}^{(r)}[k] \approx \beta_r v_{\ell(r)}[k] e^{-j2\pi k r}/N \quad \text{(by (12))}$$

in which $\beta_r$ is an unknown constant, $r \in \{1, \ldots, L\}$ is unknown and $r$ maps to $p(r)$ uniquely (by (16)). Note that $x_{\ell}[k] = x[k]$ and the extracted source $e_{\ell}[k]$ is free from error propagation since $x[k]$ instead of $x_{\ell}[k]$ is processed by the Fourier-KMBF at each stage.

#### 4.2. Time Delay Estimation and Compensation (TDEC)

The unknown time delay $\tau^{(r)}$ in the extracted source $e_{\ell}[k]$ (see (30)) can be estimated by subcarrier averaging as

$$\hat{\tau}^{(r)} = \arg\max_{0 \leq \tau \leq T} \left\{ \phi(e_{\ell}[k], e^{j2\pi k \tau}/N) \right\},$$

where $\phi$ is a function of $\tau$.
where
\[ \phi(e_k[\ell], e^{j2\pi k r/N}) = \left| \left( \frac{e_k[\ell] e^{j2\pi k r/N}}{[e_k]} \right) \right|^2, \]
simply because substituting (30) into (32) leads to
\[ \phi(e_k[\ell], e^{j2\pi k r/N}) \geq \begin{cases} 1, & \text{for } \tau = \tau^{(r)} \\ 0, & \text{for } \tau \neq \tau^{(r)} \end{cases}. \]

Therefore, by (30) and (31), the time delay compensated source estimate can be obtained in a straightforward manner as follows:
\[ e_k[\ell] = e_k[\ell] e^{j2\pi k r/N} + \beta_k p(\tau^{(r)})[k] + \varpi_k[\ell], \]
where \( p(\tau) \) is unknown and \( \varpi_k[\ell] \) is the estimation error.

### 4.3. Classification and BMRC

Assume that at stage \( \ell - 1 \), the time delay compensated source estimates \( \{ e_k[\ell], \ldots, e_{k-1}[\ell] \} \) (see (34)) have been optimally combined into a set of \( P \) symbol sequence estimates \( u_{\ell}[k] \) given by
\[ u_{\ell}[k] = \alpha_\ell u_{\ell}(\ell) + \varpi_{\ell}[\ell], \quad q = 1, \ldots, P \leq P, \]
where \( \alpha_\ell \) is unknown, \( \beta(\ell) \in \{ p(\tau) \} \) obtained at stages up to \( \ell - 1 \), \( \varpi_{\ell}[\ell] \) is the estimation error, and the “\( q \)” is a “class number” (associated with a distinct user for each \( q \)).

Next, let \( \varrho_q \) denote the magnitude of normalized cross-correlation between \( e_k[\ell] \) and \( u_{\ell}[k] \) defined as
\[ \varrho_q = \frac{|\langle e_k[\ell] u_{\ell}[k] \rangle|}{\sqrt{\langle |e_k[\ell]|^2 \rangle \langle |u_{\ell}[k]|^2 \rangle}}. \]
and let \( p = \arg \max_q \varrho_q \leq P \).

Next, the conventional FKMA can be used to process \( e_k[\ell] \) to obtain the optimum \( 2 \times 1 \) linear combiner \( \nu \) and the associated estimate \( \bar{u}_{\ell}[k] = \nu^\dagger e_k[\ell] \approx \nu u_{\ell}[k] \) and then update \( u_{\ell}[k] \) by \( \bar{u}_{\ell}[k] \). The process of obtaining \( \bar{u}_{\ell}[k] \) using the conventional FKMA is exactly the BMRC because the obtained \( \bar{u}_{\ell}[k] \) except for a scale factor is exactly the same as obtained by the nonblind MRC combiner which maximizes the output SNR (i.e., maximum multipath diversity gain) [5, 6].

All the users’ symbol sequence estimates \( \{ \bar{u}_{\ell}[k], q = 1, \ldots, P \} \) are obtained by the proposed blind MSBFA through \( L \) stages.

### 4.4. Robustness to Correlated Sources

It is possible in practice that some source signals associated with the same user arrive at the receiver antenna array with different DOAs but at the same time delay (i.e., violation of the assumption (A3)), and they form a “cluster” of correlated sources [7]. Assume that \( \{ \alpha_{p,l}^{(m)} a(\theta_{p,l}^{(m)}) \} \) \( \forall \ell \) is a cluster of \( L_{p,l} \) correlated sources impinging on the receive antennas where \( \alpha_{p,l}^{(m)} \) and \( \theta_{p,l}^{(m)} \) are the path gain and DOA of each correlated signal in the cluster, respectively. The model \( x[n] \) (see (2)) remains valid except that \( \alpha_{p,l} a(\theta_{p,l}) \) must be replaced by \( \alpha_{p,l} = \sum_{\ell=1}^{L_{p,l}} \alpha_{p,l}^{(m)} a(\theta_{p,l}^{(m)}) \). The resultant MIMO model \( x[n] \) (see (15)) for the post-FFT BFS also keeps valid except that the mixing matrix \( A \) is different due to the replacement of \( \alpha_{p,l} a(\theta_{p,l}) \) by \( \alpha_{p,l} \). Therefore, the proposed blind MSBFA is able to estimate \( u_{p,l}[k] = u_{p,l}[k] e^{j2\pi k r_p/N} \) as long as \( \| \bar{u}_{p,l} \| \) is sufficiently large, implying its robustness to correlated signals. However, the Capon’s MV beamformer is incapable of extracting the associated source \( u_{p,l}[k] \) for the pre-FFT BFS regardless of the value of \( \| \bar{u}_{p,l} \| \) because \( \alpha_{p,l} \) is no longer a steering vector of a certain DOA required by the Capon’s MV beamformer [7].

### 5. SIMULATION RESULTS

Consider a four-user (\( P = 4 \)) OFDM system with \( N = 1024, N_g = 20, \) and \( Q = 20 \). The synthetic received signals \( x[n] \) were generated by (2) for users’ symbol sequences \( u_{p,l}[k] \) being i.i.d. QPSK signals with \( E\|u_{p,l}[k]\|^2 = 1 \) and noise vector \( w[n] \) being i.i.d. zero-mean Gaussian with \( E\|w[n]\|^2 = \sigma_w^2 \). Then the proposed blind MSBFA with the threshold \( \xi = 0.5 \) and \( \eta = 0.75 \) was employed to process the MIMO signal \( x[k] \) (see (15)) to estimate all the users’ symbols. For performance comparison, the same data (either pre-FFT data \( x[n] \) or post-FFT data \( x[k] \)) were processed by theoretical nonblind MMSE and MRC beamformers and Capon’s MV beamformer. Twenty five hundred independent runs were performed for different values of input SNR, defined as
\[ \text{Input SNR} = \frac{E\|x[n] - w[n]\|^2}{P \cdot E\|w[n]\|^2}, \]
and calculated the averaged symbol error rate (SER) of all the users’ symbol sequence estimates as the performance index.

**Example 1. (Environment without Correlated Sources)**

In this example, multipath channel parameters used are as follows: \( (L_1, L_2, L_3, L_4) = (4, 2, 2, 2); p_{1,l} \) ’s (path gains) are real numbers with \( \sum_{\ell=1}^{L_1} \|p_{1,l}\|^2 = 1; \) \( p_{1,l} \)’s \( \in \{ 0, 1, \ldots, N_p \} \) are uniformly distributed.

Figure 1 shows performance insights of the proposed blind MSBFA without BMRC (dashed lines), namely its performance (averaged SER) as the Fourier beamformer (Section 3.1), or the KMBF (Section 3.2), or the Fourier-KMBF (Section 3.3) used in the source extraction. In addition, the performance of the proposed blind MSBFA with BMRC is also presented (solid line) where the Fourier-KMBF is used in the source extraction. From this figure one can see that the performance of the blind MSBFA using the Fourier-KMBF (dashed line) is better than the one using either the Fourier beamformer \( \Delta \) and the KMBF \( \Delta \) and dashed line) or the KMBF (dashed line). Moreover, the blind MSBFA with BMRC (solid line) performs much better than the one without BMRC (dashed line).

Next, Fig. 2 shows the performance of the blind MSBFA, nonblind MRC and MMSE beamformers associated with the pre-FFT BFS or post-FFT BFS, and Capon’s MV beamformer associated with the pre-FFT BFS. One can see, from this figure, that the beamformers associated with the post-FFT BFS (solid lines) significantly outperform the beamformers associated with the pre-FFT BFS (dashed lines).
lines) because of path diversity gain only achieved by the former (see Remark 1). Note that the performance of the Capon’s MV beamformer using the true covariance matrix, called theoretic Capon’s MV beamformer (○ and dashed line), is actually the same as the MMSE beamformer (△ and dashed line) associated with the pre-FFT BFS and they perform better than the Capon’s MV beamformer using the estimated covariance matrix, called actual Capon’s MV beamformer (▽ and dashed line). Moreover, the proposed blind MSBFA (○ and solid line) works well with better performance than the MRC beamformer (□ and solid line) associated with the post-FFT BFS, although the performance of the former is slightly worse than that of the MMSE beamformer associated with the post-FFT BFS (△ and solid line).

**Example 2. (Environment with Correlated Sources)**

For this example, the simulation was conducted with each path in Example 1 replaced by a cluster of four paths (i.e., \( L_{p,l} = 4 \) for all \( p \) and \( l \)), and the corresponding results are shown in Fig. 3. Basically, all the observations from Fig. 2 also apply to Fig. 3. Besides, all the beamformers under test perform better for higher input SNR except the Capon’s MV beamformer (either theoretical one or actual one), which performs worse (better) for higher input SNR as input SNR is higher (lower) than 4 dB, demonstrating that it is not applicable in the presence of correlated sources.

The above simulation examples demonstrate the efficacy of the proposed blind MSBFA, whereas no comparison with other block-by-block post-FFT beamforming algorithms for multuser OFDM systems since, to our best knowledge, none of them can be found in the open literature.

### 6. CONCLUSION

Under Assumptions (A1) through (A4), we have presented a block-by-block blind post-FFT beamforming algorithm based on subcarrier averaging (the blind MSBFA), which is well suited to outdoor rural environments with some but not too many multiple paths, for the estimation of symbol sequences of all the users of an OFDM system. It is also a multistage blind beamforming algorithm consisting of source extraction using the proposed blind Fourier-KMBF, TDEC processing, classification and BMRC processing at each stage. Some simulation results were provided to support that the proposed blind MSBFA performs well no matter whether correlated path signals are present or not, and its performance is close to the “optimal” nonblind MMSE beamformer associated with the post-FFT BFS.

### 7. REFERENCES


