Deferrable Load Scheduling in a Stand-alone Power System with Renewable Energy Sources and a Perfect Battery

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***Abstract*—Renewable energy resources are environmentally friendly and have received a lot of attention lately. In this paper, we consider a stand-alone power system with renewable energy sources and a perfect battery. We assume that the loads to the system are deferrable so that they can be scheduled to balance the power supply and demand. The main objective of a scheduling policy in such a system is to effectively utilize renewable energy sources so that the loads can be served as quickly as possible and the amount of wasted energy can be minimized. For such a purpose, we consider three classes of scheduling policies: (i) no lookahead policies, (ii) 1-period lookahead policies, and (iii) 1-period prediction policies. By using a sample path argument, we prove a comparison theorem between a maximal 1-period lookahead policy and a maximal no lookahead policy. We also prove an invariant theorem for the battery capacity of a maximal 1-period lookahead policy. Our simulation results reveal two interesting findings: (i) increasing the battery capacity is very effective in improving the performance of a maximal no lookahead policy, and (ii) prediction can be very effective in improving the system performance without needing to increase battery capacity.**

I. Introduction

Renewable energy resources, such as solar, wind and hydropower, are environmentally friendly, and have received a lot of attention recently. One key problem of renewable energy sources is that the amount of energy produced by renewable sources is not *controllable*. In particular, for photovoltaic panels and wind turbines, their energy outputs depend heavily on weather conditions. When renewable energy sources do not produce enough energy to meet the demand, an electrical system has to draw additional energy either from the power grids or from energy storage devices such as battery banks. When a system is connected to the power grids, it can simply pay for the additional energy, and the Demand-Response (DR) problem [1] in such a setting is commonly formulated as an optimization problem that minimizes the cost of the system (see e.g., [2], [3], [4], [5], [6], [7], [8]).

Recently, it was shown in [9] that it is possible to deploy a large energy storage unit into a grid powered by an arbitrary number of photovoltaic panels and wind turbines. Such a grid can be viewed as a stand-alone system with renewable energy sources and a large "battery." To deal with the fluctuation in both the power supply and demand, Wang et al. [9] extended and applied the stochastic network calculus (see e.g., the book [10] and references therein) to analyze such a system. The stochastic network calculus was also used in [11] for the DR problem in such a setting. In order to use the stochastic network calculus, one has to obtain probabilistic traffic characterizations for the supply by using a *service curve* and the demand by using an *arrival curve*. In all these works, both the service curve and the arrival curve were found based on historical data. Furthermore, it is shown in [12] that there is an equivalent theorem between a distribution network and a queueing system. Such an equivalent theorem enables us to apply the traditional teletraffic theory for analyzing power distribution networks.

Inspired by all these works, we consider in this paper a stand-alone power system with renewable energy sources and a perfect battery. We assume that the loads are *deferrable* so that we can schedule the loads to balance the power supply and demand. Such an assumption has been widely adopted in many previous works (see e.g., [13], [12], [7], [8]). On the other hand, unlike the stochastic network calculus approach in [9], [14], [11], we make no assumptions on the traffic characterizations for the supply and the demand. In such a stand-alone system, the main objective is to schedule deferrable loads to efficiently utilize the renewable energy so that the loads can be served as quickly as possible and the amount of wasted energy can be minimized. We consider three policies: (i) no lookahead policies, (ii) 1-period lookahead policies, and (iii) 1-period prediction policies. No lookahead policies are the most conservative among these three classes of policies as they only schedule loads without exceeding the energy stored in the battery. On the other extreme, 1-period lookahead policies assume that the energy generated by renewable sources in the next period is known and thus they can schedule loads without exceeding the sum of the energy in the battery and the energy generated in the next time period. Intuitively, one would expect that the performance of a 1-period lookahead policy should be better than that of a no lookahead policy. Such an intuition is formally proved by using a sample path argument in the paper. However, 1-period lookahead policies cannot be implemented in practice as it is impossible to know the amount of energy that will be generated by renewable sources in the future. As such, the best one can do is to estimate/predict the amount of energy that will be generated by renewable sources in the next period and that leads to the 1-period prediction policies. For the comparison of these three policies, we consider two key performance metrics: (i) the loads that remain in the system at any time t, and (ii) the cumulative amount of energy wasted by time t (due to overflows of battery capacity).

Our contributions and findings are summarized as follows: (i) The loads that remain in the system for a maximal 1-period lookahead policy is always not greater than that for a maximal no lookahead policy. This can be formally proved by using a sample path argument. Such a result provides theoretical justification for our intuition. Moreover, we show by computer simulations that there is a significant performance gap between a maximal 1-period lookahead policy and a maximal no lookahead policy. (ii) Increasing the battery capacity has little effect on the performance of a maximal 1-period lookahead policy. In fact, we prove by a sample path argument that the cumulative amount of energy wasted by time t for a maximal 1-period lookahead policy is independent of battery capacity. Such a result appears to be quite counterintuitive as one would expect that the performance of the system can be improved by increasing the battery capacity. (iii) Increasing the battery capacity is very effective in improving the performance of a maximal no lookahead policy. By computer simulations, we show that one can narrow the performance gap between a maximal 1-period lookahead policy and a maximal no lookahead policy by increasing the battery capacity of a maximal no lookahead policy. (iv) Prediction can be very effective in improving the system performance without needing to increase battery capacity. By computer simulations, we show that the performance of a 1-period prediction policy is very close to that of a maximal 1-period lookahead policy. On the other hand, for systems with a moderate battery capacity, the performance gap between a 1-period prediction policy and a maximal no lookahead policy can be quite large. This suggests using prediction can greatly improve the system performance.

This paper is organized as follows: In Section II, we describe the model for a stand-alone power system with renewable energy sources and a perfect battery. In Section III, we introduce the three scheduling policies. The analytic results are shown in Section IV. We then report our simulation results in Section V. The paper is concluded in Section VI.

II. The model

In this paper, we consider a stand-alone power system with renewable energy sources and a perfect battery (see Figure 1). As in [9], we consider the discrete-time model with time indexed from $t = 0, 1, 2, \dots$. Let $s(t)$ be the amount of energy generated during the t^{th} time period. Without loss of generality, we assume that $s(t)$'s are nonnegative integers. Also, let $d(t)$ be the amount of energy consumed by the system during the t^{th} time period. Assume that the battery is *perfect* with the charge efficiency $\eta_c = 1$ and the discharge efficiency $\eta_d = 1$. Denote by $b_{\max}$ the capacity of the battery and $b(t)$ the energy level of the battery at the end of the t^{th} time period. Then it was shown in [9] that the energy level of the battery satisfies the following recursive equation:

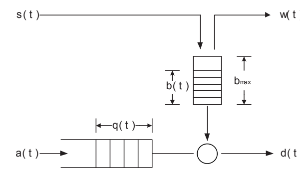


Fig. 1: The model of a stand-alone system with renewable energy sources and a perfect battery.

$$b(t) = \min[b_{\max}, (b(t-1) + s(t) - d(t))^+], \tag{1}$$

where $x^+ = \max(0, x)$.

On the other hand, there are loads (jobs) arriving at the system and it requires a certain amount of energy to complete a load. When a load is completed, it leaves the system. Let $a(t)$ be the total amount of loads that arrive during the t^{th} time period. In this paper, we assume that there are no deadlines for these loads and they can be delayed indefinitely. Denote by $q(t)$ the total amount of energy required by the loads that remain in the system at the end of the t^{th} time period. As the loads can be delayed indefinitely, we have the following queueing equation:

$$q(t) = (q(t-1) - d(t))^+ + a(t). \tag{2}$$

Here we assume the gated service model and loads that arrive during the t^{th} time period are not allowed to be served during the same time period. The two equations in (1) and (2) are the governing equations for our model of a stand-alone power system with renewable energy sources and a perfect battery.

Note that if

$$s(t) > d(t) + (b_{\max} - b(t-1)), \tag{3}$$

then the battery is full at the end of the t^{th} time period, i.e., $b(t) = b_{\max}$, and the additional power generated by renewable energy sources cannot be stored and thus is wasted. Let $w(t)$ be the amount of wasted energy during the t^{th} time period. Then

$$w(t) = (s(t) - d(t) - (b_{\max} - b(t-1)))^+. \tag{4}$$

For our analysis, we also need to consider the cumulative effect of the system. Specifically, we let $S(t) = \sum_{\tau=1}^{t} s(\tau)$ be the cumulative amount of energy generated by time t, $D(t) = \sum_{\tau=1}^{t} d(\tau)$ be the cumulative amount of energy consumed by the system by time t, $A(t) = \sum_{\tau=1}^{t} a(\tau)$ be the cumulative amount of the energy required by the loads that arrive by time t, and $W(t) = \sum_{\tau=1}^{t} w(\tau)$ be the cumulative amount of energy wasted by time t. From the principle of conservation of energy, it is clear that

$$b(t) = b(0) + S(t) - D(t) - W(t). \tag{5}$$

Similarly, we also have

$$q(t) = q(0) + A(t) - D(t). \tag{6}$$

III. Scheduling policies

For the model described in the previous section, the (on-line) load scheduling problem is to select a set of loads to serve at the beginning of every time period. Once the decision is made at the beginning of the t^{th} period, the amount of energy consumed during the t^{th} period, i.e., $d(t)$, is also determined. Certainly, for a scheduling policy to be *feasible*, the amount of energy consumed during a time period cannot be greater than the total amount of energy required by the loads that remain in the system at the beginning of the t^{th} time period, i.e.,

$$d(t) \le q(t-1). \tag{7}$$

Such a constraint is called the *load feasibility constraint* in this paper.

In this paper, we study a simple version of the scheduling problem by assuming that all the loads have an identical profile and they all take one unit of energy/per time period to complete. The purpose of such an assumption is to avoid the bin packing problem that might further complicate the load scheduling problem. Under such an assumption, the quantity $q(t)$ in (2) is also the number of loads that remain in the system at the end of the t^{th} time period.

In the following subsections, we will introduce three classes of scheduling policies: (i) no lookahead policies (with the superscript (0) on the corresponding notations), (ii) 1-period lookahead policies (with the superscript (1) on the corresponding notations) and (iii) 1-period prediction policies (with the superscript (p) on the corresponding notations).

A. No lookahead policies

A scheduling policy is called a no lookahead policy if it selects a set of loads so that the amount of energy consumed during a time period is not greater than the amount of energy stored in the battery at the beginning of that time period, i.e.,

$$d^{(0)}(t) \le b^{(0)}(t-1). \tag{8}$$

Clearly, a no lookahead policy is a very conservative policy and we have

$$(b^{(0)}(t-1) + s(t) - d^{(0)}(t))^+ = b^{(0)}(t-1) + s(t) - d^{(0)}(t).$$

Let $\tilde{s}(t) = \min[s(t), b_{\max}]$. Then, we can rewrite (1) as follows:

$$\begin{aligned} b^{(0)}(t) &= \min[b_{\max}, b^{(0)}(t-1) + s(t) - d^{(0)}(t)] \\ &= \min[b_{\max}, b^{(0)}(t-1) - d^{(0)}(t) + \min[s(t), b_{\max}]] \\ &= \min[b_{\max}, b^{(0)}(t-1) + \tilde{s}(t) - d^{(0)}(t)]. \end{aligned} \tag{9}$$

This equation shows that if the amount of energy generated during a time period is larger than the capacity of the battery, then the additional amount of energy is simply wasted under a no lookahead policy. In other words, under a no lookahead policy the system subject to the renewable energy sources $\{s(t), t \ge 1\}$ is equivalent to the system subject to the renewable energy sources $\{\tilde{s}(t), t \ge 1\}$.

Similarly, we can also use (7) to rewrite (2) as follows:

$$q^{(0)}(t) = q^{(0)}(t-1) + a(t) - d^{(0)}(t). \tag{10}$$

From (7) and (8), we know that

$$d^{(0)}(t) \le \min[b^{(0)}(t-1), q^{(0)}(t-1)]$$

under a no lookahead policy. Thus, the maximum amount of energy that can be consumed during the t^{th} time period under a no lookahead policy is

$$d^{(0)}(t) = \min[b^{(0)}(t-1), q^{(0)}(t-1)]. \tag{11}$$

A no lookahead policy is said to be *maximal* if it satisfies (11).

B. 1-period lookahead policies

As shown in the previous section, no lookahead policies are not efficient as they waste energy if the amount of energy generated during a time period is larger than the capacity of the battery. To improve the efficiency of using renewable energy sources, let us consider an ideal case that assumes $s(t)$ is known at the beginning of the t^{th} time period for the purpose of performance comparison. Now we can select more loads to serve in every time period for this ideal case. Specifically, a policy is called a 1-period lookahead policy if it satisfies the following two constraints:

$$d^{(1)}(t) \le q^{(1)}(t-1), \tag{12}$$

$$d^{(1)}(t) \le b^{(1)}(t-1) + s(t). \tag{13}$$

The constraint in (12) is the load feasibility constraint in (7). The constraint in (13), called the energy feasibility constraint in this paper, is much more relaxed than that in (8) as we are now allowed to look ahead one period in time. Clearly, the maximum amount of energy that can be consumed during the t^{th} time period under such a policy is

$$d^{(1)}(t) = \min[q^{(1)}(t-1), b^{(1)}(t-1) + s(t)]. \tag{14}$$

A 1-period lookahead policy is said to be *maximal* if it satisfies (14).

Analogous to the derivation in the previous section, we have the following two governing equations for a system under a 1-period lookahead policy:

$$b^{(1)}(t) = \min[b_{\max}, b^{(1)}(t-1) + s(t) - d^{(1)}(t)], \tag{15}$$

and

$$q^{(1)}(t) = q^{(1)}(t-1) + a(t) - d^{(1)}(t). \tag{16}$$

Intuitively, the performance of a maximal 1-period lookahead policy should be better than that of a maximal no lookahead policy. Such an intuition will be formally proved in Theorem 2.

C. 1-period prediction policies

Since the amount of energy generated during a period is in general not known at the beginning of that period, the best we can do is to estimate the amount of energy generated during that period and use that information for load scheduling. Specifically, let $\hat{s}(t)$ be the estimate for the total amount of energy generated during the t^{th} time period (the method to obtain the estimate $\hat{s}(t)$ will be described in details in the simulation section). A policy is called a 1-period prediction policy if it satisfies the following two constraints:

$$d^{(p)}(t) \leq q^{(p)}(t-1), \tag{17}$$

$$d^{(p)}(t) \leq b^{(p)}(t-1)+\hat{s}(t). \tag{18}$$

The first constraint is simply the load feasibility constraint. Since $s(t)$ is not known at the beginning of the t^{th} period, we use its estimate $\hat{s}(t)$ instead. This then leads to the constraint in (18).

There is one potential problem for a 1-period prediction policy. The estimate could be off and we end up with scheduling a set of loads that require more energy than the system can supply. In other words, the energy feasibility constraint could be violated, i.e.,

$$d^{(p)}(t) > b^{(p)}(t-1)+s(t).$$

When this happens, we say there is a loss of power supply. In many manufacturing plants, such as semiconductor plants, all the loads are lost due to a loss of power supply. To model such a scenario, we assume that all the loads that are being served at time t are lost and have to be repeated some time later when there is a loss of power supply at time t. Specifically, we have $d^{(p)}(t)=0$ and

$$q^{(p)}(t)=q^{(p)}(t-1)+a(t), \tag{19}$$

when there is a loss of power supply at time t. Clearly, when there is a loss of power supply at time t, the battery is also empty, i.e.,

$$b^{(p)}(t)=0. \tag{20}$$

In order to keep the Loss of Power Supply Probability (LPSP) within a reasonable level, we propose using a control parameter $0<\alpha\leq 1$ and scheduling the loads to satisfy

$$d^{(p)}(t)=\min[q^{(p)}(t-1), b^{(p)}(t-1)+\alpha\hat{s}(t)]. \tag{21}$$

Such a policy is called a 1-period prediction policy with parameter α in this paper.

IV. Analytical results

As pointed out in [12], one can immediately observe from Figure 1 that the model of a stand-alone system with renewable energy sources and a perfect battery is almost the same as the model of a leaky bucket in [15] that can be governed by the Lindley recursive equation [16]. In the following lemma, we show the Lindley recursive equations for the system under a *maximal no lookahead policy* and the system under a *maximal 1-period lookahead policy*. Such a lemma is an easy extension of Theorem 2.2.12 in [18] for a leaky bucket with a constant token generation rate. Its proof is given in Appendix A of the full report [17] and it is omitted due to space limitation.

Lemma 1: Let $y^{(0)}(t)=q^{(0)}(t)+b_{\max}-b^{(0)}(t)$. For a *maximal no lookahead* policy, we have

$$y^{(0)}(t)=(y^{(0)}(t-1)-\tilde{s}(t))^{+}+a(t). \tag{22}$$

Let $y^{(1)}(t)=q^{(1)}(t)+b_{\max}-b^{(1)}(t)$. For a *maximal 1-period lookahead* policy, we have

$$y^{(1)}(t)=(y^{(1)}(t-1)-s(t))^{+}+a(t). \tag{23}$$

A. The main comparison theorem

In this section, we show in Theorem 2 that the performance of a maximal 1-period lookahead policy should be better than that of a maximal no lookahead policy. Specifically, let us consider two systems: one is operated under a maximal no lookahead policy and the other is operated under a maximal 1-period lookahead policy. Suppose these two systems have the same battery capacity $b_{\max}$ and they are subject to the same load arrivals $\{a(t), t\geq 1\}$ and the same renewable energy sources $\{s(t), t\geq 1\}$. Also, assume that the battery energy levels are the same at time 0, i.e., $b^{(0)}(0)=b^{(1)}(0)$, and both systems have the same number of loads at time 0, i.e., $q^{(0)}(0)=q^{(1)}(0)$. The proof of Theorem 2 is given in Appendix B of the full report [17] and it is omitted here due to space limitation.

Theorem 2: For the two systems described above, the total amount of energy required by the loads that remain in the system under a *maximal no lookahead* policy is always not less than that under a *maximal 1-period lookahead* policy, i.e., $q^{(0)}(t)\geq q^{(1)}(t)$ for all t. Moreover, the cumulative amount of energy consumed by time t in the system under a *maximal no lookahead* policy is always not greater than that under a *maximal 1-period lookahead* policy, i.e., $D^{(0)}(t)\leq D^{(1)}(t)$ for all t.

B. An invariant theorem of battery capacity

In this section, we show that the total amount of energy wasted by time t, i.e., $W(t)$, is independent of the battery capacity for a system under a *maximal 1-period lookahead policy*. Such a result appears to be new in the literature as the number of lost tokens is never a concern in a leaky bucket. It is also quite counterintuitive when it is first observed from our simulations. But, a careful examination in Theorem 3 further shows that the time that a waste of energy occurs is also independent of the battery capacity. Such a result then provides the right insight for $W(t)$ to be independent of the battery capacity.

Theorem 3: For a system under a maximal 1-period lookahead policy, if *the battery is fully charged at time 0*, i.e., $b^{(1)}(0)=b_{\max}$, and the system is started from some fixed number of loads, i.e., $q^{(1)}(0)=q_0$ for some q_0, then the first time that a waste of energy occurs is at time

$$\tau^{*}=\inf\{t\geq 1: S(t)>A(t-1)+q_0\}. \tag{24}$$

Moreover, the total amount of energy wasted by time t is also independent of the battery capacity $b_{\max}$.

Proof. Since we assume that the initial energy level of the battery is full, i.e., $b^{(1)}(0) = b_{\max}$, we then have from (5) that

$$S(t) + b_{\max} = D^{(1)}(t) + W^{(1)}(t) + b^{(1)}(t). \quad (25)$$

Also, since the system is started from some fixed number of loads, i.e., $q^{(1)}(0) = q_0$, we have from (6) that

$$A(t) = D^{(1)}(t) + q^{(1)}(t) - q_0. \quad (26)$$

Suppose that a waste of energy occurs for the first time at some time τ, i.e., $w^{(1)}(\tau) > 0$ and $w^{(1)}(t) = 0$ for all $t < \tau$. We will show that $\tau = \tau^*$ by first showing $\tau^* \le \tau$ and then $\tau^* \ge \tau$.

Since a waste of energy occurs for the first time at some time τ, the battery must be full at the end of the τ^{th} time period, i.e.,

$$b^{(1)}(\tau) = b_{\max}. \quad (27)$$

Also, it follows from (4) and $w^{(1)}(\tau) > 0$ that

$$s(\tau) + b^{(1)}(\tau - 1) > d^{(1)}(\tau) + b_{\max}. \quad (28)$$

For a maximal 1-period lookahead policy, $d^{(1)}(\tau)$ is either $q^{(1)}(\tau - 1)$ or $s(\tau) + b^{(1)}(\tau - 1)$. In view of (28), it cannot be the latter as we would have a contradiction $0 > b_{\max}$. Thus, we know $d^{(1)}(\tau) = q^{(1)}(\tau - 1)$ when a waste of energy occurs for the first time at time τ. It then follows from (16) that

$$q^{(1)}(\tau) = a(\tau). \quad (29)$$

Thus, we have from (26) that

$$D^{(1)}(\tau) = A(\tau) - a(\tau) + q_0 = A(\tau - 1) + q_0. \quad (30)$$

From (27), the equation in (25) can be rewritten as

$$\begin{aligned} S(\tau) &= D^{(1)}(\tau) + W^{(1)}(\tau) \\ &= D^{(1)}(\tau) + w^{(1)}(\tau), \quad (31) \end{aligned}$$

where we use the fact that $W^{(1)}(\tau) = w^{(1)}(\tau)$ as τ is the first time that a waste of energy occurs. In conjunction with (30), we have

$$w^{(1)}(\tau) = S(\tau) - A(\tau - 1) - q_0. \quad (32)$$

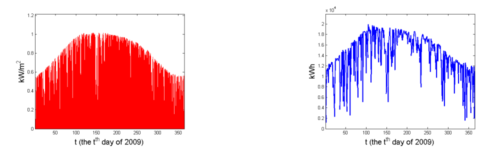
Since $w^{(1)}(\tau) > 0$, we know from the definition of τ^* that $\tau^* \le \tau$.

On the other hand, note from (26) and the fact $q^{(1)}(t) \ge a(t)$ that

$$\begin{aligned} D^{(1)}(\tau^*) &= A(\tau^*) - q^{(1)}(\tau^*) + q_0 \\ &\le A(\tau^*) - a(\tau^*) + q_0 \\ &= A(\tau^* - 1) + q_0. \quad (33) \end{aligned}$$

It then follows from (25) and the fact $b^{(1)}(t) \le b_{\max}$ that

$$\begin{aligned} & W^{(1)}(\tau^*) = S(\tau^*) + b_{\max} - D^{(1)}(\tau^*) - b^{(1)}(\tau^*) \\ & \ge S(\tau^*) - D^{(1)}(\tau^*) \\ & \ge S(\tau^*) - A(\tau^* - 1) - q_0 > 0, \quad (34) \end{aligned}$$



(a) Hourly GHI for 2009 in Long Beach, CA.

(b) Daily DC output for 2009 in Long Beach, CA.

Fig. 2: The solar information for 2009 in Long Beach, CA.

where we use the definition of τ^* in the last inequality. This shows that a waste of energy occurs no later than τ^* and thus $\tau^* \ge \tau$.

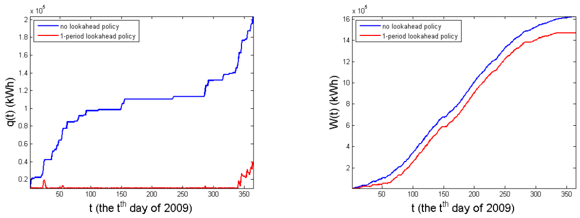
As τ^* is independent of the battery capacity, the first time that a waste of energy occurs is independent of the battery capacity. When this happens, the battery is fully charged and it can be viewed as a renewal point. Thus, the second time that a waste of energy occurs is also independent of the battery capacity. From induction, it is easy to see that the total amount of energy wasted by time t is also independent of the battery capacity $b_{\max}$. ■

V. Simulation results

A. Experimental setup

For our simulations, we need to generate two sequences: the total amount of energy generated in each time period, i.e., $\{s(t), t \ge 1\}$, and the total number of load arrivals in each time period, i.e., $\{a(t), t \ge 1\}$. In our simulations, each time period corresponds to a day in a stand-alone photovoltaic system. To simulate the sequence of the total amount of energy generated in each day, we first obtain the complete solar information for year 2009 in Long Beach, California, from the National Renewable Energy Laboratory (NREL) [19]. We then use that information as the input of the System Advisor Model (SAM) [20] to calculate the hourly DC output of arrays of solar cells for the system. By summing up the hourly DC outputs of a day, we then obtain the total amount of energy generated in each day by the renewable energy resources, i.e., solar cells here. In Figure 2a. we plot the hourly Global Horizontal Irradiance (GHI) for year 2009 in Long Beach, California. The GHI is one component of the complete solar information from NREL. In Figure 2b, we also plot the corresponding daily DC output. From Figure 2a and Figure 2b, one can see that there is strong correlation between the GHI and the DC output. Using this model, we also calculate the average amount of energy generated per day from year 2006 to year 2008. The average is roughly 14000 kWh per day.

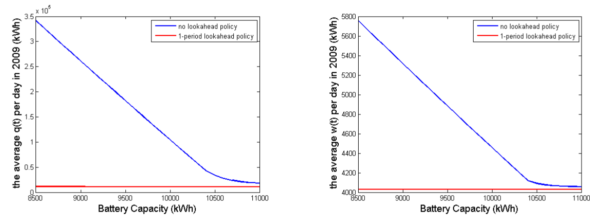
On the other hand, the total number of load arrivals in each time period, i.e., $\{a(t), t \ge 1\}$ is modelled by a Poisson process. To ensure that the power supply can meet the demand, we set the Poisson arrival rate λ to be 10000 kWh per day.



(a) The total number of loads that remain in the system at the end of t^{th} time period.

(b) The total amount of energy wasted by time t.

Fig. 3: Performance comparison between a no lookahead policy and a 1-period lookahead policy with $b_{\max} = 10000kWh$.



(a) The average number of loads that remain in the system per day.

(b) The average amount of wasted energy per day.

Fig. 4: The effect of battery capacity.

B. Comparison results between a maximal no lookahead policy and a maximal 1-period lookahead policy

In Theorem 2, we have already shown a qualitative result for the comparison between a maximal no lookahead policy and a maximal 1-period lookahead policy. In this section, we further show by simulations that the gain from using a maximal 1-period lookahead policy could be very substantial. In Figure 3, we show the comparison results between a no lookahead policy and a 1-period lookahead policy for a fixed battery capacity $b_{\max} = 10000kWh$. As clearly shown in Figure 3(a), the number of loads that remain in the system under a *maximal 1-period lookahead* policy is substantially smaller than that under a *maximal no lookahead* policy. Also, as shown in Figure 2b, the daily DC output for year 2009 is a unimodal function that has lower than average daily DC outputs in the first month and in the last month. As such, the energy supply is unable to meet the demand for the first month and the last month. Thus, the queue for the loads that remain in the system grows during these two months. From Figure 3(a), one can also see that a maximal 1-period lookahead policy is more robust to the change to daily DC outputs and its queue is stable for most of the days in that year.

In Figure 3(b), we show the comparison results for the total amount of energy wasted by time t. Once again, a maximal 1-period lookahead policy wastes less energy than a maximal no lookahead policy. Note that for a maximal 1-period lookahead policy, $W(t)$ grows more slowly in the first month and in the last month when the power supply cannot meet the demand. This shows that a maximal 1-period lookahead policy uses the energy more efficiently during that period of time.

C. The impact of battery capacity

In this section, we study the effect of battery capacity on the performance of the system under a maximal no lookahead policy and a maximal 1-period lookahead policy, respectively. In Figure 4(a), we show the average number of loads that remain in the system per day under both policies. The results are obtained by averaging over 365 days for 1000 simulations of load arrivals. Clearly, the performance of a maximal 1-period lookahead policy is substantially better than that of a maximal no lookahead policy. However, as the battery capacity increases, the gap becomes smaller. This shows that it is possible to improve the performance of a maximal no lookahead policy by increasing the battery capacity. But increasing the battery capacity seems to have minimal effect on the performance of a maximal 1-period lookahead policy. Similar results are found for the average amount of wasted energy per day in Figure 4(b).

To summarize, there is a significant performance gap between a maximal no lookahead policy and a maximal 1-period lookahead policy. It is possible to narrow the gap by increasing the battery capacity as the performance of a maximal no lookahead policy can be greatly improved by increasing the battery capacity. On the other hand, increasing the battery capacity seems to have little effect on the performance of a maximal 1-period lookahead policy. In fact, as shown in Theorem 3, the amount of wasted energy is invariant to the battery capacity under a maximal 1-period lookahead policy. In practice, the battery capacity cannot be increased without a limit as there are budget constraints. Thus, it is of importance to look for good scheduling policies that can further improve the performance without needing to increase the battery capacity. In the next section, we will propose a prediction method that can be used for a 1-period prediction policy that yields comparable performance to a maximal 1-period lookahead policy.

D. The prediction method

As prediction has to be made at the beginning of each working day, we select the hourly GHI's only from 7 am to 9 am on that day as input features and the corresponding daily DC output power of that day as the output of the prediction model. Specifically, we formulate the prediction model with the following prediction function

$$\hat{s}(t) = f(t, GHI_7(t), GHI_8(t), GHI_9(t)), \tag{35}$$

where t is the index of the day in a year and $GHI_i(t)$, $i = 7, 8$ and 9, is the GHI at i a.m. on the t^{th} day. To estimate the prediction function in (35), we use the GHI's and the corresponding DC output from year 2006 to year 2008 as our training data. Once we obtain the prediction function, it will be used for testing the amount of daily energy generated in year 2009.

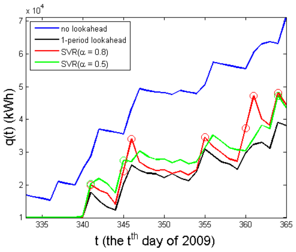


Fig. 5: Simulation results for the number of loads that remain in the system under various scheduling policies.

In this paper, our prediction method is to use the support vector regression (SVR) to obtain the estimate $\hat{s}(t)$. For our experiments, we use libSVM [21], a library for Support Vector Machines, to run nu-SVR for the estimate of the output.

E. Comparison results for 1-period prediction policies

In this section, we study the effect of α for a system under a 1-period prediction policy with parameter α. In Figure 5, we show the number of loads that remain in the system at the end of the t^{th} time period, i.e., $q(t)$, under a maximal no lookahead policy, a maximal 1-period lookahead policy and two 1-period prediction policies with $\alpha = 0.8$ and $\alpha = 0.5$, respectively. Each circle in the curves for the two 1-period prediction policies in Figure 5 indicates the time when a loss of power supply occurs. We can see that sometimes a 1-period prediction policy with $\alpha = 0.8$ is better than that with $\alpha = 0.5$ and sometimes it is the other way around. That is because a larger α allows the system to serve more loads when there is no loss of power supply. However, when there is a loss of power supply, all the loads are lost and they have to be repeated later. Clearly, a larger α implies a higher LPSP and we face a tradeoff between serving more loads and reducing the LPSP. Even with such a tradeoff for choosing α, we can see from Figure 5 that the performance of both 1-period prediction policies are very close to that of a maximal 1-period lookahead policy and there is a substantial gain over a maximal no lookahead policy. This result shows it is possible to improve the system performance by using prediction methods and it can be done without needing to increase battery capacity.

VI. Conclusions

In this paper, we considered a stand-alone power system with renewable energy sources and a perfect battery. We compared three classes of scheduling policies: (i) no lookahead policies, (ii) 1-period lookahead policies and (iii) 1-period prediction policies. We proved a comparison theorem between a maximal 1-period lookahead policy and a maximal no lookahead policy. We also proved an invariant theorem for the battery capacity of a maximal 1-period lookahead policy. In addition to these theoretical results, our simulation results revealed two interesting findings: (i) increasing the battery capacity is very effective in improving the performance of a maximal no lookahead policy, and (ii) prediction can be very effective in improving the system performance without needing to increase battery capacity.

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APPENDIX A

In this section, we prove Lemma 1

We first prove (22) for a *maximal no lookahead* policy. Using (9) and (10) yields

$$\begin{aligned} y^{(0)}(t) &= \max[q^{(0)}(t), y^{(0)}(t-1) + a(t) - \tilde{s}(t)] \\ &= a(t) + \max[q^{(0)}(t-1) - d^{(0)}(t), y^{(0)}(t-1) - \tilde{s}(t)]. \end{aligned} \quad (36)$$

Consider the following two cases: (i) $b^{(0)}(t-1) \geq q^{(0)}(t-1)$ and (ii) $b^{(0)}(t-1) < q^{(0)}(t-1)$. For the first case, we have from (11) that $d^{(0)}(t) = q^{(0)}(t-1)$. It then follows from (36) that $y^{(0)}(t) = (y^{(0)}(t-1) - \tilde{s}(t))^+ + a(t)$.

For the second case, we have from (11) that $d^{(0)}(t) = b^{(0)}(t-1)$. Since $\tilde{s}(t) \leq b_{\max}$,

$$\begin{aligned} &y^{(0)}(t-1) - \tilde{s}(t) \\ &= q^{(0)}(t-1) + b_{\max} - b^{(0)}(t-1) - \tilde{s}(t) \\ &\geq q^{(0)}(t-1) - b^{(0)}(t-1) > 0. \end{aligned}$$

It then follows from (36) that

$$\begin{aligned} y^{(0)}(t) &= a(t) + y^{(0)}(t-1) - \tilde{s}(t) \\ &= (y^{(0)}(t-1) - \tilde{s}(t))^+ + a(t). \end{aligned}$$

Now we prove (23) for a *maximal 1-period lookahead* policy. Using (15) and (16) yields

$$\begin{aligned} &y^{(1)}(t) \\ &= \max[q^{(1)}(t), y^{(1)}(t-1) + a(t) - s^{(1)}(t)] \quad (37) \\ &= a(t) + \max[q^{(1)}(t-1) - d^{(1)}(t), y^{(1)}(t-1) - s(t)]. \quad (38) \end{aligned}$$

Consider the following two cases: (i) $b^{(1)}(t-1) + s(t) \geq q^{(1)}(t-1)$ and (ii) $b^{(1)}(t-1) + s(t) < q^{(1)}(t-1)$. For the first case, we have from (14) that $d^{(1)}(t) = q^{(1)}(t-1)$. It then follows from (38) that $y^{(1)}(t) = (y^{(1)}(t-1) - s(t))^+ + a(t)$.

For the second case, we have from (14) that $d^{(1)}(t) = b^{(1)}(t-1) + s(t)$. Thus,

$$\begin{aligned} &y^{(1)}(t-1) - s(t) \\ &= q^{(1)}(t-1) + b_{\max} - b^{(1)}(t-1) - s(t) \\ &\geq q^{(1)}(t-1) - b^{(1)}(t-1) - s(t) > 0. \end{aligned}$$

Thus,

$$\max[q^{(1)}(t-1) - d^{(1)}(t), y^{(1)}(t-1) - s(t)] = y^{(1)}(t-1) - s(t),$$

and

$$(y^{(1)}(t-1) - s(t))^+ = y^{(1)}(t-1) - s(t).$$

It then follows from (37) that

$$\begin{aligned} y^{(1)}(t) &= a(t) + y^{(1)}(t-1) - s(t) \\ &= (y^{(1)}(t-1) - s(t))^+ + a(t). \end{aligned}$$

■

APPENDIX B

In this section, we prove Theorem 2.

Since $b^{(0)}(0) = b^{(1)}(0)$ and $q^{(0)}(0) = q^{(1)}(0)$, we have $y^{(0)}(0) = y^{(1)}(0)$. Note that $\tilde{s}(t) = \min[s(t), b_{\max}] \leq s(t)$ for all t. It then follows from the two Lindley equations in (22) and (23) that for all $t > 0$

$$y^{(0)}(t) \geq y^{(1)}(t). \quad (39)$$

To show $q^{(0)}(t) \geq q^{(1)}(t)$, we consider the following two cases:

Case 1. $b^{(1)}(t-1) + s(t) \geq q^{(1)}(t-1)$:

For this case, we have from (14) that $d^{(1)}(t) = q^{(1)}(t-1)$. It then follows from (16) that $q^{(1)}(t) = a(t)$. Also, from (7) and (10), we know that $q^{(0)}(t) \geq a(t)$. Thus, $q^{(0)}(t) \geq q^{(1)}(t)$.

Case 2. $b^{(1)}(t-1) + s(t) < q^{(1)}(t-1)$:

For this case, we have from (14) that

$$d^{(1)}(t) = b^{(1)}(t-1) + s(t).$$

It then follows from (16) that

$$q^{(1)}(t) = q^{(1)}(t-1) + a(t) - b^{(1)}(t-1) - s(t). \quad (40)$$

From (39) and $y^{(i)}(t) = q^{(i)}(t) + b_{\max} - b^{(i)}(t)$, $i = 0$ and 1, we know that

$$q^{(1)}(t-1) - b^{(1)}(t-1) \leq q^{(0)}(t-1) - b^{(0)}(t-1).$$

Using this in (40) yields

$$\begin{aligned} q^{(1)}(t) &\leq q^{(0)}(t-1) + a(t) - b^{(0)}(t-1) - s(t) \\ &\leq q^{(0)}(t-1) + a(t) - b^{(0)}(t-1). \end{aligned} \quad (41)$$

From (11), we know for a no lookahead policy that

$$d^{(0)}(t) \leq b^{(0)}(t-1).$$

In conjunction with (41), we have

$$q^{(1)}(t) \leq q^{(0)}(t-1) + a(t) - d^{(0)}(t) = q^{(0)}(t), \quad (42)$$

where we use (10) in the last equality.

That $D^{(0)}(t) \leq D^{(1)}(t)$ follows directly from (6) and $q^{(0)}(t) \geq q^{(1)}(t)$.