EE203001 Linear Algebra
Quiz #9   04/29/2003

1. (10 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T(\boldsymbol{i} + 2\boldsymbol{j}) = 4\boldsymbol{i} + \boldsymbol{j}, \quad T(\boldsymbol{j}) = \boldsymbol{i} + \boldsymbol{j}.$$

   (a) Compute $T(4\boldsymbol{i} - 3\boldsymbol{j})$ and $T^2(4\boldsymbol{i} - 3\boldsymbol{j})$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

   (b) Determine the matrix of T.

2. (10 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T(\boldsymbol{i}) = (1, 1, 0), \quad T(\boldsymbol{j}) = (-1, 0, 1).$$

   Find bases (v_1, v_2) for $\mathbb{R}^2$ and (w_1, w_2, w_3) for $\mathbb{R}^3$ relative to which the matrix of T will be in diagonal form.

3. (10 pts) In the space of all real-valued functions, let V be the subspace spanned by $\{e^x, xe^x\}$. Let D be the differentiation operation on V. Find the matrix representations of D and D^2 relative to the given basis.

4. (10 pts) Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Prove or disprove that $A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$.