EE203001 Linear Algebra
Quiz #8   04/22/2003

1. A function $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by the formula given for $T(x, y, z)$, where (x, y, z) is an arbitrary point in $\mathbb{R}^3$. In the two cases below, determine whether T is one-to-one on $\mathbb{R}^3$.

   (a) $T(x, y, z) = (x + y, y + z, x + z)$.

   (b) $T(x, y, z) = (x, y, x + y)$.

2. Let V be a linear space. Let S and T be in $\mathcal{L}(V, V)$ and assume that $ST - TS = I$.

   (1) Prove that $ST^n - T^nS = nT^{n-1}$ for all $n \geq 1$.

   (2) If we let V be the linear space of all real polynomials $p(x)$, and T be the linear transformation that maps $p(x)$ onto $xp(x)$. Let D denote the differentiation operator. Prove that $DT - TD = I$ and that $DT^n - T^nD = nT^{n-1}$ for $n \geq 2$.

3. Let V be the linear space of all real polynomials $p(x)$. Let R, S, T be the functions that map an arbitrary polynomial $p(x) = c_0 + c_1x + \cdots + c_nx^n$ to the polynomials $r(x)$, $s(x)$, and $t(x)$, respectively, where

$$r(x) = p(0), \qquad s(x) = \sum_{k=1}^{n} c_k x^{k-1}, \qquad t(x) = \sum_{k=0}^{n} c_k x^{k+1}.$$

   (a) Prove that R, S, T are linear operators on V and T is ono-to-one.

   (b) If $n \geq 1$, express $(TS)^n$ and S^nT^n in terms of I and R.

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