EE203001 Linear Algebra
Quiz #6   04/01/2003

Quiz Problems:

1. (10 pts). In each case find an orthonormal basis for the subspace of R^4 spanned by the given vectors.

   (a) $x_1 = (0,0,1,1)$, $x_2 = (0,1,1,0)$, $x_3 = (0,-1,0,1)$, $x_4 = (1,0,0,1)$.

   (b) $x_1 = (1,2,-2,1)$, $x_2 = (1,0,2,1)$, $x_3 = (1,1,0,1)$.

2. (10pts). Let x_1, x_2,..., x_k be a finite sequence of elements in an Euclidean space V. Then we can find a corresponding sequence of elements y_1, y_2,...,y_k in V by the Gram-Schmidt process such that for each i, $1 \le i \le k$ (a) y_i is orthogonal to every element in $(y_1, y_2, ..., y_{i-1})$,(b) $L(y_1, y_2, ..., y_i) = L(x_1, x_2, ..., x_i)$. Prove that these corresponding elements y_1,..., y_k are nonzero if and only if the k elements x_1,..., x_k are independent.

3. (10pts). In the linear space of all real polynomials, with inner product $(x, y) = \int_0^1 x(t)y(t)dt$, let $x_n(t) = t^n + 1$ for $n = 0, 1, 2, ...$ Prove that the functions

$$y_0(t) = 1,\ y_1(t) = \sqrt{3}(2t-1),\ y_2(t) = \sqrt{5}(6t^2 - 6t + 1),$$

   form an orthonormal set spanning the same subspace as $\{x_0, x_1, x_2\}$.

Teaching Assistants: EECS 608, ext. 4032

- Meng-Hua Chang, mhchang@abel.ee.nthu.edu.tw
- Chao-Chung Chang, ccchang@abel.ee.nthu.edu.tw
- Chen-Wei Hsu, cwhsu@abel.ee.nthu.edu.tw
- Wen-Yao Chen, wychen@abel.ee.nthu.edu.tw

A Note on the Pythagorean Theorem

A counterexample (given by Chao-Chung Chang) that in a **complex** Euclidean space, two elements x and y satisfying the Pythagorean identity $||x+y|| = ||x||^2 + ||y||^2$, but x and y are not orthogonal.

In $\mathbb{C}^2$, with the usual inner product $(x, y) = x_1\overline{y}_1 + x_2\overline{y}_2$. Let $x = (1,0)$ and $y = (i,0)$, then $(x,y) = 1 \cdot \overline{i} + 0 \cdot \overline{0} = -i \neq 0$. But x and y satisfy the Pythagorean identity, since

$$||x||^2 = (x,x) = 1 \cdot \overline{1} + 0 \cdot \overline{0} = 1,$$

$$||y||^2 = (y,y) = i \cdot \overline{i} + 0 \cdot \overline{0} = 1,$$

and

$$||x+y||^2 = (1+i)\overline{(1+i)} + 0 \cdot \overline{0} = (1+i)(1-i) = 1+1 = 2.$$